2 SEVANJE OKROGLE TOKOVNE ZANKE

$$\vec{B}(\vec{r},t) = -\frac{\mu_0}{4\pi c_0 r}\,\frac{\vec{r}}{r}\times\frac{\partial}{\partial t}\int \vec{j}\left(\vec{r}\,', t-\frac{r}{c_0}+\frac{\vec{r}}{r}\cdot\frac{\vec{r}\,'}{c_0}\right) d^3\vec{r}\,'$$

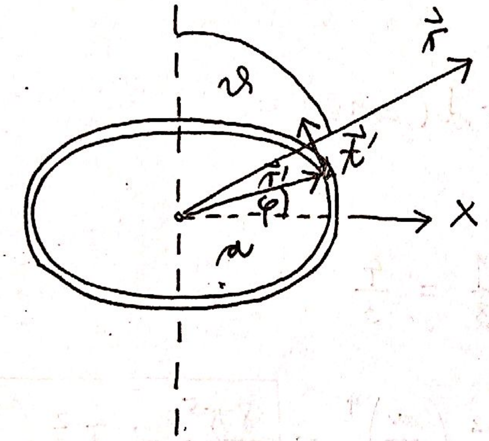
zanemaritev r' $\Rightarrow$ $\int \vec{j}(\vec{r}\,', t_r)\, d^3\vec{r}\,' = 0$ po poljubni ZANKI

↳ ne smemo zanemariti

$$\vec{j}\left(\vec{r}\,', t_r+\frac{\vec{r}}{r}\cdot\frac{\vec{r}\,'}{c_0}\right) \approx \vec{j}(\vec{r}\,', t_r) + \frac{\vec{r}\cdot\vec{r}\,'}{r c_0}\,\dot{\vec{j}}(\vec{r}\,', t_r) \quad \rightarrow \text{Taylor}$$

↳ integral vodi do 0!

$$\frac{\partial}{\partial t}\int \vec{j}\, d^3\vec{r}\,' \rightarrow \underbrace{\int \frac{\vec{r}\cdot\vec{r}\,'}{r c_0}\,\ddot{\vec{j}}(\vec{r}\,', t_r)\, d^3\vec{r}\,'}$$



$$\frac{\vec{r}}{r} = (\sin\vartheta, 0, \cos\vartheta)$$

$$\vec{r}\,' = a(\cos\varphi, \sin\varphi, 0)$$

$$\frac{\vec{r}\cdot\vec{r}\,'}{r c_0} = \frac{a}{c_0}\sin\vartheta\cos\varphi$$

$$d^3\vec{r}\,' = dS'\cdot a\, d\varphi$$

$$\int \vec{j}\, d^3\vec{r}\,' = \int_L I\,\vec{t}\,' a\, d\varphi$$

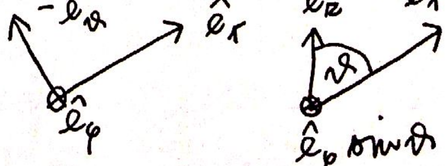
$$\vec{t}\,' = (-\sin\varphi, \cos\varphi, 0)$$

$$\int = \int_0^{2\pi} \frac{a^2 \ddot{I}}{c_0}\underbrace{\sin\vartheta\cos\varphi\,(-\sin\varphi, \cos\varphi, 0)}_{\hat{e}_y \sin\vartheta \underbrace{\cos^2\varphi}_{\frac{1}{2}\cdot 2\pi = \pi} \text{ samo } \neq 0} d\varphi = \underbrace{\frac{\pi a^2 \ddot{I}}{c_0}}_{\frac{\ddot{p}_m}{c_0}}\,\hat{e}_y \sin\vartheta$$

$\hat{e}_y$ ↓ $\hat{e}_\varphi$ v resnici

$$\vec{B}(\vec{r},t) = -\frac{\mu_0}{4\pi c_0^2 r}\,\hat{e}_r\times\hat{e}_\varphi \sin\vartheta\, \ddot{p}_m$$

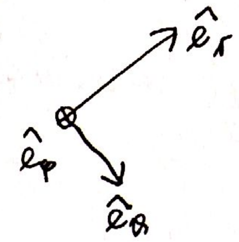
$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi c_0^2 r}\,\hat{e}_\vartheta \sin\vartheta\, \ddot{p}_m$$



$$\hat{e}_z\times\hat{e}_r = \hat{e}_\varphi \sin\vartheta$$

$$\vec{B}(\vec{r},t) = -\frac{\mu_0}{4\pi c_0^2 r}\,\hat{e}_r \times \underbrace{(\hat{e}_R \times \hat{e}_r)\,\ddot{p}_m}_{\ddot{\vec{p}}_m} = \frac{\mu_0}{4\pi c_0^2 r}\,\hat{e}_r \times (\hat{e}_r \times \ddot{\vec{p}}_m)$$

$$\vec{E}(\vec{r},t) = -\hat{e}_r \times c_0 \vec{B} = -\frac{\mu_0}{4\pi c_0 r}\,\underbrace{\hat{e}_r \times \hat{e}_\vartheta}_{\hat{e}_\varphi}\,\sin\vartheta\,\ddot{p}_m$$



$$\vec{P} = \frac{1}{\mu_0}\vec{E} \times \vec{B} = \frac{\mu_0}{16\pi^2 c_0^3 r^2}\sin^2\vartheta\,\ddot{p}_m^2\,\underbrace{(-\hat{e}_\varphi)\times\hat{e}_\vartheta}_{\hat{e}_r}$$

$$\vec{P} = \frac{\mu_0 \sin^2\vartheta\,\ddot{p}_m^2}{16\pi^2 c_0^3 r^2}\,\hat{e}_r \quad , \quad p_m = I_0 \pi a^2 \sin\omega t_r$$

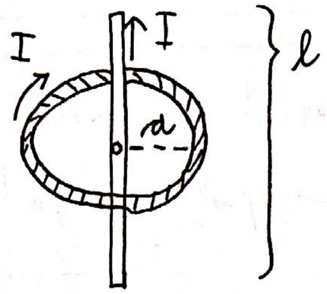
$$\ddot{p}_m = -I_0 \pi a^2 \omega^2 \sin\omega t_r$$

$$\left\langle \int P\,dS \right\rangle = \int \frac{\mu_0 \sin^2\vartheta\, I_0^2 a^4 \omega^4}{32 c_0^3 r^2}\, r^2\, 2\pi\, d(\cos\vartheta) =$$

$$\int \sin^2\vartheta\, d(\cos\vartheta) = 2 - 2\cdot\frac{1}{3} = \frac{4}{3}$$

$$= \frac{\pi}{12}\,\frac{\overbrace{c_0\mu_0}^{Z_0}\, I_0^2 a^4\; 4\cdot 4\pi^4}{\lambda^4} = \frac{4\pi^5}{3} Z_0 I_0^2 \left(\frac{a}{\lambda}\right)^4 = \boxed{\frac{8\pi^5}{3} Z_0 I_{ef}^2 \left(\frac{a}{\lambda}\right)^4}$$

3 KOMBINIRANA ANTENA



$$I = I_0 \sin\omega t$$

a) $\underline{p_e} \to$ le palica

$$e = \int_{t_0}^{t} I\,dt = \frac{I_0}{\omega}\int_{t_0}^{t} \sin\omega t\, d(\omega t) = -\frac{I_0}{\omega}\cos\omega t\Big|_{t_0}^{t} \to -\frac{I_0}{\omega}\cos\omega t$$

$$p_e = e\frac{l}{2} - (-e)\frac{l}{2} = \underline{-\frac{I_0 l}{\omega}\cos\omega t}$$

$\underline{p_m} \to$ le zanka

$$p_m = IS = \underline{I_0 \pi a^2 \sin\omega t}$$

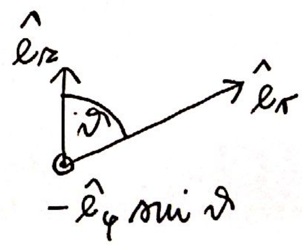
b) glede električnega dipolnega sevanja:

$$\vec B = -\frac{\mu_0}{4\pi c_0 r}\hat e_r \times \dot I l \hat e_z, \qquad \dot p_e = +I_0 l \sin\omega t = Il$$

$$\hookrightarrow \dot I l = \ddot p_e, \quad \dot I l \hat e_z = \ddot{\vec p}_e$$

$$\underline{\vec B = -\frac{\mu_0}{4\pi c_0 r}\hat e_r \times \ddot{\vec p}_e} \to$$ velja kar splošno za električni dipol, ki niha

$$\vec B_e = -\frac{\mu_0}{4\pi c_0 r}\hat e_r \times I_0 l\omega \cos\omega t\, \hat e_z =$$



$$= \frac{\mu_0}{4\pi c_0 r}\underline{I_0 l\omega \cdot \cos\omega t} \sin\vartheta\, \underline{\hat e_\varphi}$$

$$\vec B_m = +\frac{\mu_0}{4\pi c_0^2 r}\underbrace{\hat e_r \times (\underbrace{\hat e_r \times \hat e_z}_{-\hat e_\varphi \sin\vartheta}}_{\hat e_\vartheta} I_0 \pi a^2 \omega^2 \sin\omega t) =$$

$$= \frac{\mu_0}{4\pi c_0 r}\underline{\underbrace{I_0 \pi a^2 \omega^2 \frac{1}{c_0}}_{I_0 l\omega \cdot \frac{\pi a^2 \omega}{l c_0} = I_0 l \omega \cdot \frac{2\pi^2 a^2}{l\lambda}} \cdot \sin\omega t} \sin\vartheta\, \underline{\hat e_\vartheta}$$

$$\vec{B} = \vec{B}_e + \vec{B}_m = \frac{\mu_0}{4\pi c_0 r} I_0 l \omega \sin\vartheta \underbrace{\left[\cos\omega t\, \hat{e}_\varphi + \frac{2\pi^2 a^2}{l\lambda} \sin\omega t\, \hat{e}_\vartheta\right]}$$

$\hat{e}_\varphi \perp \hat{e}_\vartheta$, $\cos\omega t$ vs $\sin\omega t$

↳ eliptična polarizacija!

eliptična → KROŽNA polarizacija: $\dfrac{2\pi^2 a^2}{l\lambda} = 1 \Rightarrow \boxed{l = \dfrac{2\pi^2 a^2}{\lambda}}$

malce težje uresničljivo,
če naj bosta $l, a \ll \lambda$