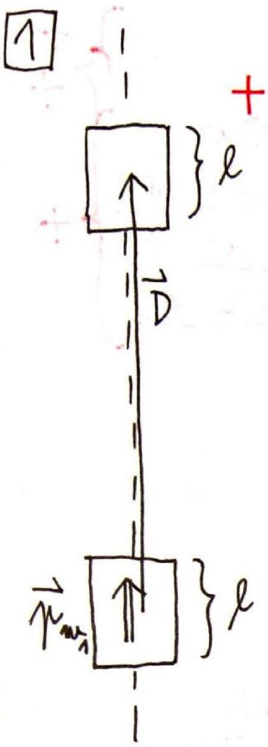
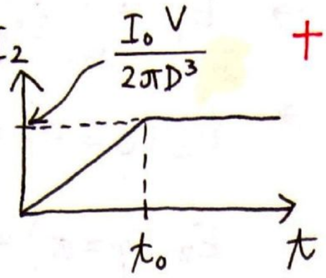
1. PISNI IZPIT

1



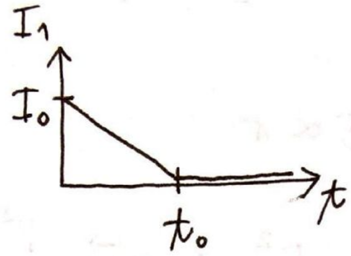


$$+\left\{\vec{B}_2 = \vec{B}_{DIP} = \frac{\mu_0}{4\pi}\,\frac{3(\vec{p}_{m1}\cdot\vec{D})\vec{D} - D^2\vec{p}_{m1}}{D^5}\right.$$

$$\vec{B}_2(\vec{D}\parallel\vec{p}_{m1}) = \frac{\mu_0}{4\pi}\,\frac{2D^2\vec{p}_{m1}}{D^5} = \frac{\mu_0 p_{m1}}{2\pi D^3}\hat{e}_z$$

$$+\{\ p_{m1} = N I_1 S$$

$$+\{\ \Phi_2 = N B_2 S = \overbrace{\frac{\mu_0 N^2 S^2}{2\pi D^3}}^{L_{21}} I_1$$



$$+\left\{\begin{cases} U_2 = -\dot{\Phi}_2 = -L_{21}\dot{I}_1 \\ U_2 = L_2 \dot{I}_2 \end{cases}\right.$$

$$\Rightarrow \dot{I}_2 = -\frac{L_{21}}{L_2}\dot{I}_1 = -\frac{\frac{\mu_0 N^2 S^2}{2\pi D^3}}{\underbrace{\frac{\mu_0 N^2 S}{\ell}}}\dot{I}_1 \qquad \frac{1}{4}$$

$$\dot{I}_1 = \frac{I_0}{t_0}\begin{cases} -1, & t < t_0 \\ 0, & t \geq t_0 \end{cases}$$

$$\Downarrow$$

$$\underline{\dot{I}_2 = \frac{-V}{2\pi D^3}\dot{I}_1} \qquad \frac{S\ell}{2\pi D^3} = \frac{V}{2\pi D^3}$$

$$\dot{I}_2 = \frac{I_0 V}{2\pi D^3 t_0}\begin{cases} 1, & t < t_0 \\ 0, & t \geq t_0 \end{cases}$$

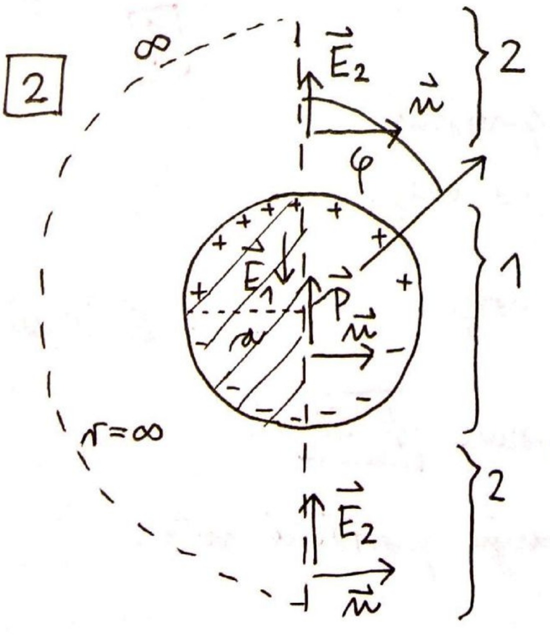
$t < t_0$ → linearno naraščanje

$t \geq t_0$ → konstantni tok

$$+\left\{ I_{2\,FIN} = \dot{I}_2 t_0 = \boxed{\frac{I_0 V}{2\pi D^3}}\right.$$

1

2



$$+\left\{\sigma_v = \vec{P}\cdot\vec{n} = P\cos\varphi\right.$$

$$U(r,\varphi) = \begin{cases} A r\cos\varphi, & r < a \\ \frac{B}{r}\cos\varphi, & r > a \end{cases}$$

$$\underbrace{RP1}: \quad U(a^-,\varphi) = U(a^+,\varphi) \Rightarrow \underline{A a = \frac{B}{a}}$$

$$\underbrace{RP2}: \quad \sigma_v = \varepsilon_0\left[E_r(a^+) - E_r(a^-)\right] =$$

$$= \varepsilon_0\left(A\cos\varphi + \frac{B}{a^2}\cos\varphi\right) = 2\varepsilon_0 A\cos\varphi$$

$$\sigma_r = 2\varepsilon_0 A\cos\varphi = P\cos\varphi \Rightarrow A = \frac{P}{2\varepsilon_0}, \quad B = \frac{Pa^2}{2\varepsilon_0}$$

$$\underbrace{U(r,\varphi) = \begin{cases} \frac{P}{2\varepsilon_0} r\cos\varphi, & r<a \\ \frac{Pa^2}{2\varepsilon_0}\frac{1}{r}\cos\varphi, & r>a \end{cases}}_{+} \quad \begin{matrix} \to E_1 = -\underline{\frac{P}{2\varepsilon_0}} \quad \}+ \\ \to E_2(\varphi=0) = \underline{\frac{Pa^2}{2\varepsilon_0}\frac{1}{r^2}} \\ E_2(\varphi=\pi) = \underline{\frac{Pa^2}{2\varepsilon_0}\frac{1}{r^2}} \end{matrix} \Big\}+$$

$$\vec{F}_e = \varepsilon_0 \oint \Big[\vec{E}(\underbrace{\vec{E}\cdot\vec{n}}_{0}) - \frac{1}{2}E^2\vec{n}\Big]dS = -\frac{\varepsilon_0}{2}\vec{n}\oint E^2 dS$$

$F_\infty = 0$, ker gre $E^2 \propto \frac{1}{r^4}$, $S \propto r$

$$+\Big\{\ \vec{F}_1 = -\frac{\varepsilon_0}{2}\vec{n}\, E_1^2\, 2al = -\vec{n}\,\varepsilon_0 a l E_1^2 = \underline{-\vec{n}\frac{P^2 a l}{4\varepsilon_0}}$$

$$\frac{1}{4}\Big\{\ \vec{F}_2 = -\frac{\varepsilon_0}{2}\vec{n}\cdot 2\cdot\int_a^\infty E_2^2\, l\, dr = -\vec{n}\,\varepsilon_0\frac{P^2 a^4}{4\varepsilon_0^2} l \underbrace{\int_a^\infty \frac{dr}{r^4}}_{-\frac{1}{3}\frac{1}{r^3}\Big|_a^\infty = \frac{1}{3a^3}}$$

$$\vec{F}_2 = \underline{-\vec{n}\frac{P^2 a l}{12\varepsilon_0}}$$

$$\vec{F} = -\vec{n}\frac{P^2 a l}{\varepsilon_0}\left(\frac{1}{4}+\frac{1}{12}\right) = -\vec{n}\frac{P^2 a l}{3\varepsilon_0}$$

$+\Big\{\ \boxed{\frac{\vec{F}}{l} = -\vec{n}\frac{P^2 a}{3\varepsilon_0}} \to$ sila kaže v smeri $-\vec{n}$, kar je proti LEVI (istovrstni naboji na površini se ODBIJAJO)

$\boxed{1}$

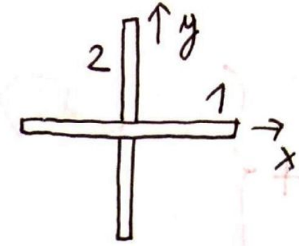
KOMENTAR: Električno polje tik ob površini valja NI PRAVOKOTNO na valj.

$$\left.\begin{aligned} E_r &= -\frac{\partial U}{\partial r}\Big|_{r=a} = \frac{P}{2\varepsilon_0}\cos\varphi \\ E_\varphi &= -\frac{1}{r}\frac{\partial U}{\partial \varphi}\Big|_{r=a} = \frac{P}{2\varepsilon_0}\sin\varphi\ \boxed{\neq 0} \end{aligned}\right\} \Rightarrow \vec{E} \not\parallel \vec{n}$$

Izračun sile preko integrala po zunanji površini valja je zato nekoliko zapletenejši.

$\boxed{3}$

$+$ $$\begin{cases} I_1 = I_0 \sin \omega t \\ I_2 = I_0 \sin(\omega t + \delta) \end{cases} \qquad \vec{\mu}_{m1} = I_1 \pi a^2 \hat{e}_y \qquad \vec{\mu}_{m2} = I_2 \pi a^2 \hat{e}_x$$



$+$

$$\ddot{\vec{\mu}}_m = - I_0 \pi a^2 \omega^2 \left[\hat{e}_y \sin \omega t + \hat{e}_x \sin(\omega t + \delta) \right] \longrightarrow \begin{bmatrix} \sin(\omega t + \delta) \\ \sin \omega t \\ 0 \end{bmatrix}$$

$$\vec{B} = - \frac{\mu_0}{4\pi c_0^2 r} \underbrace{\hat{e}_r \times \left[\hat{e}_r \times \ddot{\vec{\mu}}_m (t_r) \right]}_{\hat{e}_r (\hat{e}_r \cdot \ddot{\vec{\mu}}_m) - \ddot{\vec{\mu}}_m}$$

$+$

$$\vec{E} = c_0 \vec{B} \times \hat{e}_r$$

$$\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{c_0}{\mu_0} \underbrace{(\vec{B} \times \hat{e}_r) \times \vec{B}}_{\hat{e}_r B^2 - \vec{B} \underbrace{(\hat{e}_r \cdot \vec{B})}_{0}} = \frac{c_0}{\mu_0} B^2 \hat{e}_r$$

$$B^2 \propto \left[\hat{e}_r (\hat{e}_r \cdot \ddot{\vec{\mu}}_m) - \ddot{\vec{\mu}}_m \right]^2 = (\hat{e}_r \cdot \ddot{\vec{\mu}}_m)^2 - 2 (\hat{e}_r \cdot \ddot{\vec{\mu}}_m)^2 + \ddot{\vec{\mu}}_m^{\,2} =$$

$$= \underline{\ddot{\vec{\mu}}_m^{\,2} - (\hat{e}_r \cdot \ddot{\vec{\mu}}_m)^2}$$

$\frac{1}{4}$

$$\hat{e}_r = \begin{bmatrix} \sin\vartheta \cos\varphi \\ \sin\vartheta \sin\varphi \\ \cos\vartheta \end{bmatrix} \Rightarrow \hat{e}_r \cdot \ddot{\vec{\mu}}_m \propto \sin\vartheta \left[\cos\varphi \sin(\omega t_r + \delta) + \sin\varphi \sin \omega t_r \right]$$

$$P \propto B^2 \propto \overbrace{\sin^2(\omega t_r + \delta) + \sin^2 \omega t_r}^{\ddot{\vec{\mu}}_m^{\,2}} -$$

$$(\hat{e}_r \cdot \ddot{\vec{\mu}}_m)^2 \begin{cases} - \sin^2\vartheta \left[\cos^2\varphi \sin^2(\omega t_r + \delta) + \sin^2\varphi \sin^2 \omega t_r + \right. \\ \left. \qquad + 2 \sin\varphi \cos\varphi \underbrace{\sin(\omega t_r + \delta) \sin \omega t_r}_{\sin^2 \omega t_r \cos\delta + \sin \omega t_r \cos \omega t_r \sin\delta} \right] \end{cases}$$

$+$

$$\langle P \rangle \propto \frac{1}{2} + \frac{1}{2} - \sin^2\vartheta \left[\frac{1}{2} \cos^2\varphi + \frac{1}{2} \sin^2\varphi + \sin 2\varphi \, \frac{1}{2} \cos\delta + 0 \right] =$$

$$= \underline{1 - \frac{1}{2} \sin^2\vartheta \, (1 + \sin 2\varphi \cos\delta)}$$

$$\langle \int \vec{P}\cdot d\vec{S}\rangle \propto \int d\varphi\, d(\cos\vartheta)\left[1-\frac{1}{2}\sin^2\vartheta\,(1+\sin 2\varphi\cos\delta)\right] =$$

$$= 4\pi - \frac{1}{2}\,2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\cos^2\vartheta)\,d(\cos\vartheta) -$$

$$- \frac{1}{2}\underbrace{\int_0^{2\pi}\sin 2\varphi\, d\varphi}_{0}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\cos^2\vartheta)\,d(\cos\vartheta)\cos\delta$$

$$\langle \int \vec{P}\cdot d\vec{S}\rangle \propto 4\pi - \pi\left(2-\frac{2}{3}\right) = \boxed{\frac{8\pi}{3}}$$

$\rightarrow$ rezultat je NEODVISEN od δ