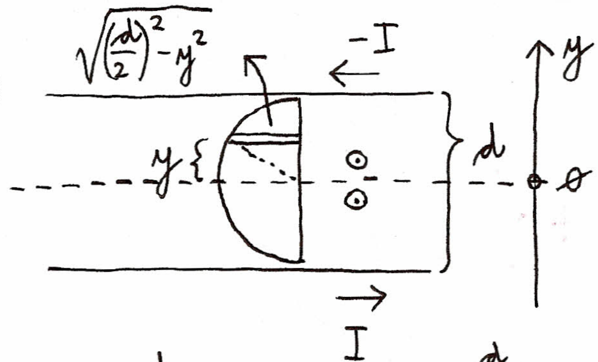
2. PISNI IZPIT

1 a)



polji se seštejeta:

$$+\left\{ B = \frac{\mu_0 I}{2\pi}\left[\frac{1}{\frac{d}{2}+y}+\frac{1}{\frac{d}{2}-y}\right]\right.$$

$$+\left\{ dS = \sqrt{\left(\frac{d}{2}\right)^2-y^2}\,dy\right.$$

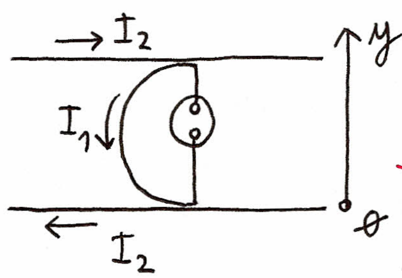
$$\Phi = \int_{y=-\frac{d}{2}}^{y=\frac{d}{2}} B\,dS = \frac{\mu_0 I}{2\pi}\int_{-\frac{d}{2}}^{\frac{d}{2}} \underbrace{\left[\frac{1}{\frac{d}{2}+y}+\frac{1}{\frac{d}{2}-y}\right]}_{\frac{d}{\left(\frac{d}{2}\right)^2-y^2}} \sqrt{\left(\frac{d}{2}\right)^2-y^2}\,dy =$$

$$= \frac{\mu_0 I d}{2\pi}\int_{-\frac{d}{2}}^{\frac{d}{2}} \frac{dy}{\sqrt{\left(\frac{d}{2}\right)^2-y^2}} = \frac{\mu_0 I d}{2\pi}\underbrace{\int_{-1}^{1}\frac{d\tilde{y}}{\sqrt{1-\tilde{y}^2}}}_{\pi\,=\,\arcsin 1-\arcsin(-1)} = \underline{\frac{\mu_0 I d}{2}}$$

$$\Phi = \underbrace{\boxed{\frac{\mu_0 d}{2}}}_{L_{12}}\cdot I \quad \Big\}+$$

$\boxed{\frac{1}{2}}$ +

b)



samoinduktivnost para vzporednih vodnikov:

$$+\left\{ \Phi_2 = \int B\,\underbrace{dS}_{l\,dy} = \frac{\mu_0 I_2 l}{2\pi}\underbrace{\int_a^{d-a}\left(\frac{1}{y}+\frac{1}{d-y}\right)dy}_{\approx \ln\frac{d}{a}-\ln\frac{a}{d}} = \right.$$

$$+\left\{ \begin{aligned} U_{i2} &= -\frac{d\tilde{\Phi}_2}{dt} = -L_{12}\dot{I}_1 \\ U_{i2} &= L_2\dot{I}_2 \quad \text{za drugi tokovni krog (Ohmov zakon)} \end{aligned}\right.$$

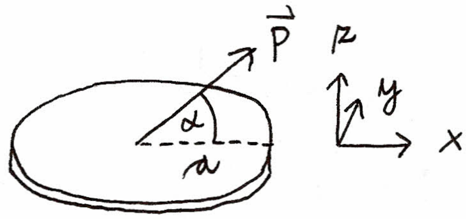
$$= \underbrace{\frac{\mu_0 l}{\pi}\ln\frac{d}{a}}_{L_2}\cdot I_2 \Rightarrow \underbrace{\boxed{L_2 = \frac{\mu_0 l}{\pi}\ln\frac{d}{a}}}_{+}$$

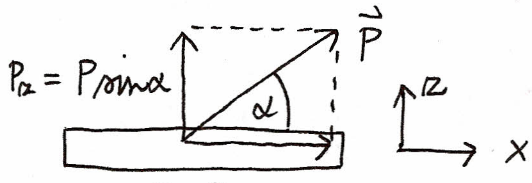
$\Downarrow$

$$\boxed{\left|\frac{I_2}{I_1}\right| = \frac{L_{12}}{L_2} = \frac{\pi}{2}\,\frac{d}{l}\,\frac{1}{\ln\frac{d}{a}} = \frac{\pi}{2}\,\frac{1}{\frac{l}{d}\ln\frac{d}{a}} = 0.068} \quad \Big\}+$$

$\boxed{\frac{1}{2}}$

$\boxed{2}$



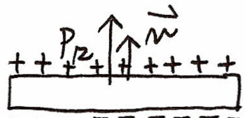


$\Downarrow$

neodvisno rešujemo dva problema:

a) v ravnini xy

b) vzdolž osi z



+ b) $\sigma_{v,z} = \vec{P}_z \cdot \vec{n} = P_z = P\sin\alpha$

+ v smeri z gre torej za problem ploščatega kondenzatorja!

$$\boxed{E_z = \frac{\sigma_{v,z}}{\varepsilon_0} = \frac{P\sin\alpha}{\varepsilon_0}}$$

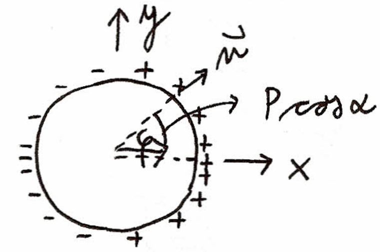
– skupno polje znotraj:

+
$$E = \sqrt{E_{xy}^2 + E_z^2}$$
$$E = \frac{P}{\varepsilon_0}\sqrt{\sin^2\alpha + \frac{\cos^2\alpha}{4}}$$
$$\boxed{E = \frac{P}{\varepsilon_0}\sqrt{1 - \frac{3}{4}\cos^2\alpha}}$$

– kot glede na ploščo:

+
$$\operatorname{tg}\alpha' = \left|\frac{E_z}{E_{xy}}\right| = 2\operatorname{tg}\alpha$$
$$\boxed{\alpha' = \operatorname{arctg}(2\operatorname{tg}\alpha)}$$

a)



+ $\sigma_{v,xy} = \vec{P}_{xy} \cdot \vec{n} = P\cos\alpha \cdot \cos\varphi$

v rešitvi preživijo le členi s $\cos\varphi$!

$\frac{1}{4}$

$$U_{xy}(r,\varphi) = \begin{cases} A r \cos\varphi, & r < a \\ \frac{B}{r}\cos\varphi, & r > a \end{cases}$$

robna pogoja:

1) zveznost U_{xy} $\Rightarrow$ $Aa = \frac{B}{a}$

2) Gauss za $E_\perp$ $\Rightarrow$

$$\frac{\sigma_{v,xy}}{\varepsilon_0} = E_\perp(r=a^+) - E_\perp(r=a^-)$$

$$\frac{P\cos\alpha\cos\varphi}{\varepsilon_0} = \left(+\frac{B}{a^2} + A\right)\cos\varphi$$

$$\frac{P\cos\alpha}{\varepsilon_0} = 2A \Rightarrow \begin{cases} A = \frac{P\cos\alpha}{2\varepsilon_0} \\ B = \frac{Pa^2\cos\alpha}{2\varepsilon_0} \end{cases}$$

$\Downarrow$

polje znotraj:

+
$$U_{xy}(r,\varphi) = \frac{P\cos\alpha}{2\varepsilon_0}\underbrace{r\cos\varphi}_{x}$$
$$\Downarrow$$
$$\boxed{E_{xy} = -\frac{P\cos\alpha}{2\varepsilon_0}\hat{e}_x}$$ v smeri osi x

$\boxed{1}$

$\boxed{3}$ a) $\vec{E}_\pm = E_0 \left(\hat{e}_x \pm i \hat{e}_y\right) e^{i(kz-\omega t)}$ → EM val za dve krožni polarizaciji ($\pm$)

enačba gibanja elektrona v plazmi:

$+\left\{ \underline{m\ddot{\vec{r}} = -m\omega_0^2 \vec{r} - e\vec{E} - e\dot{\vec{r}} \times B\hat{e}_z} \right.$

$\uparrow$

$\vec{E}_\pm \Rightarrow$ nastavek

$\underline{\vec{r}_\pm = r_0 \left(\hat{e}_x \pm i\hat{e}_y\right) e^{i(kz-\omega t)}}$

$$-m\omega^2\vec{r} = -m\omega_0^2\vec{r} - e\vec{E} + ie\omega\vec{r} \times B\hat{e}_z$$

$$(\hat{e}_x \pm i\hat{e}_y) \times \hat{e}_z = -\hat{e}_y \pm i\hat{e}_x = \pm i(\hat{e}_x \pm i\hat{e}_y)$$

$$-m\omega^2 r_0 (\hat{e}_x \pm i\hat{e}_y) = -m\omega_0^2 r_0 (\hat{e}_x \pm i\hat{e}_y) - eE_0 (\hat{e}_x \pm i\hat{e}_y) \pm (-1)e\omega B r_0 (\hat{e}_x \pm i\hat{e}_y)$$

$\downarrow$ $\hat{e}_x \pm i\hat{e}_y$ se pokrajša (nastavek je pravilen)

$$-m(\omega^2 - \omega_0^2) r_0 = -e(E_0 \pm \omega r_0 B)$$

$$\underline{\left(\omega_0^2 - \omega^2 \pm \omega \underbrace{\frac{eB}{m}}_{\omega_B}\right) r_0 = -\frac{e}{m} E_0}$$

polarizacija plina elektronov na dva načina:

$$\vec{P} = n(-e)\vec{r} = \varepsilon_0(\varepsilon - 1)\vec{E} \Rightarrow -ner_0 = \varepsilon_0(\varepsilon-1)E_0$$

$$\varepsilon - 1 = -\frac{ne}{\varepsilon_0}\boxed{\frac{r_0}{E_0}}$$

$$\varepsilon = 1 + \underbrace{\frac{ne^2}{\varepsilon_0 m}}_{\omega_p^2} \frac{1}{\omega_0^2 - \omega^2 \pm \omega_B\omega}$$

$\downarrow$

$$\boxed{\varepsilon_\pm = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 \pm \omega_B\omega}}$$

$\boxed{\frac{1}{2}}$

b) $n_\pm = \sqrt{\varepsilon_\pm} \approx 1 + \frac{1}{2}\frac{\omega_p^2}{\omega_0^2 - \omega^2 \pm \omega_B \omega}$ lomna količnika

$+\left\{ k_\pm = \frac{\omega}{c_\pm} = \frac{\omega}{c_0} n_\pm \right.$ $\Rightarrow$ posamezni krožni polarizaciji se na dolžini l razmaknete za fazni faktor $\underbrace{e^{i(k_- - k_+)l}}$,

$+\left\{ e^{i\underbrace{\frac{\omega}{c_0}\Delta n l}_{\delta}} = \underbrace{e^{i\delta}} \right.$ $\leftarrow$ polovico ga pripišemo prvi, drugo polovico drugi

$$\underbrace{\vec{E}_1 = E_1 \hat{e}_x}_{} = \frac{E_1}{2}(\hat{e}_x + i\hat{e}_y) + \frac{E_1}{2}(\hat{e}_x - i\hat{e}_y)$$

ZAČETNA polarizacija

$\downarrow$ po prehodu medija

$+\Bigg\{$

$$\frac{E_1}{2}(\hat{e}_x + i\hat{e}_y)\underbrace{e^{-i\frac{\delta}{2}}} + \frac{E_1}{2}(\hat{e}_x - i\hat{e}_y)\underbrace{e^{i\frac{\delta}{2}}} =$$

$$= \frac{E_1}{2}\Big[\hat{e}_x \underbrace{\left(e^{i\frac{\delta}{2}} + e^{-i\frac{\delta}{2}}\right)}_{2\cos\frac{\delta}{2}} - i\hat{e}_y \underbrace{\left(e^{i\frac{\delta}{2}} - e^{-i\frac{\delta}{2}}\right)}_{2i\sin\frac{\delta}{2}}\Big] =$$

$$= \underbrace{E_1\left(\hat{e}_x \cos\frac{\delta}{2} + \hat{e}_y \sin\frac{\delta}{2}\right)}_{}$$

KONČNA polarizacija $\rightarrow$ ZASUK za kot $\boxed{\frac{\delta}{2}}$

$+\Bigg\{$

$$\frac{\delta}{2} = \frac{1}{2}\frac{\omega}{c_0} l \Delta n = \frac{\omega l}{2c_0}\Delta n$$

$$\Delta n = n_- - n_+ = \frac{1}{2}\omega_p^2 \underbrace{\left[\frac{1}{\omega_0^2 - \omega^2 - \omega_B\omega} - \frac{1}{\omega_0^2 - \omega^2 + \omega_B\omega}\right]}_{\frac{2\omega_B\omega}{(\omega_0^2-\omega^2)^2 - \omega_B^2\omega^2}} =$$

$$= \frac{\omega_B \omega \omega_p^2}{(\omega_0^2 - \omega^2)^2 - \omega_B^2\omega^2}$$

$$\boxed{\frac{\delta}{2} = \frac{l}{2c_0}\frac{\omega_B \omega^2 \omega_p^2}{(\omega_0^2-\omega^2)^2 - \omega_B^2\omega^2}}$$

$\boxed{\frac{1}{2}}$