2. PISNI IZPIT

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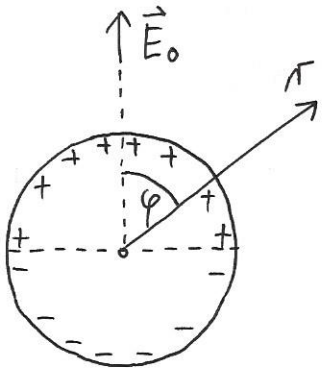

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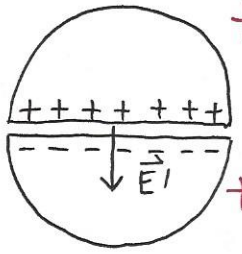

- neprerezan valj:

$$+\{\ U(r,\varphi) = -E_0 r\cos\varphi + \frac{B}{r}\cos\varphi$$

$$+\{\ U(a,\varphi) = \left(-E_0 a + \frac{B}{a}\right)\cos\varphi = 0$$

$$\Downarrow$$

$$B = E_0 a^2$$

Naboj x sloda se enakomerno razporedi po prerezu.

$$+\{\ \underline{U(r,\varphi) = E_0\cos\varphi\left(\frac{a^2}{r} - r\right)}$$

- naboj, ki se inducira na sloda:

$$+\{\ \sigma = \varepsilon_0 \vec{E}\big|_{r=a} = -\varepsilon_0 \vec{\nabla} U\big|_{r=a} = -\varepsilon_0 E_0\cos\varphi\left(-\frac{a^2}{r^2} - 1\right)\Big|_{r=a} = 2\varepsilon_0 E_0\cos\varphi$$

$$+\{\ e = \int \sigma\, dS = 2\varepsilon_0 E_0 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\varphi \cdot a l\, d\varphi = 2\varepsilon_0 E_0 a l \sin\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \underline{4\varepsilon_0 E_0 a l}$$

$$+\{\ \text{- prerez deluje kot ploščati kondenzator}$$

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$$\frac{1}{4}\{\ E' = \frac{\sigma'}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{e}{S'} = \frac{1}{\varepsilon_0}\frac{4\varepsilon_0 E_0 a l}{2 a l} = \boxed{2E_0}$$ kaže NAVZDOL!

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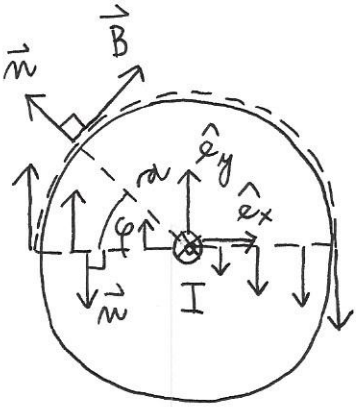

levo desno

$$\vec{F}_m = \frac{1}{\mu_0}\oint\left[\vec{B}(\vec{B}\cdot\vec{n}) - \frac{1}{2}B^2\vec{n}\right] dS$$

$$+\{\ \text{- sloda}: \quad \vec{B}\cdot\vec{n} = 0 \Rightarrow \vec{F}_{m1} = -\frac{1}{2\mu_0}\int B^2\vec{n}\, dS$$

$$+\{\ \vec{F}_{m1} = -\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi a}\right)^2 \int_0^{\pi} \vec{n}\, \underbrace{l a\, d\varphi}_{dS}$$

$$+\{\ \vec{F}_{m1} = -\frac{\mu_0 I^2 l}{8\pi^2 a} \underbrace{\int_0^{\pi}\vec{n}\, d\varphi}_{\int_0^{\pi}\sin\varphi\, d\varphi\, \hat{e}_y = 2\hat{e}_y}$$

$$+\{\ \underline{\vec{F}_{m1} = -\frac{\mu_0 I^2 l}{4\pi^2 a}\hat{e}_y}$$

- presek

+ { - levo: $\vec{B}\cdot\vec{n} = -B \Rightarrow \vec{B}(\vec{B}\cdot\vec{n}) = -B\vec{B} = B^2\vec{n}$

- desno: $\vec{B}\cdot\vec{n} = B \Rightarrow \vec{B}(\vec{B}\cdot\vec{n}) = B\vec{B} = B^2\vec{n}$

$\vec{F}_{m2} = \frac{1}{\mu_0}\int\left[B^2\vec{n} - \frac{1}{2}B^2\vec{n}\right]dS = \frac{1}{2\mu_0}\int B^2\vec{n}\,dS$ }

+ { $B = \frac{\mu_0}{2\pi r}\cdot I\left(\frac{r}{a}\right)^2 = \frac{\mu_0 I r}{2\pi a^2}$ }

+ { $\vec{F}_{m2} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi a^2}\right)^2 \vec{n}\int_{-a}^{a} r^2\,\underbrace{l\,dr}_{dS} = \frac{\mu_0 I^2 l}{8\pi^2 a^4}\vec{n}\cdot\frac{2a^3}{3} = \underline{\frac{\mu_0 I^2 l}{12\pi^2 a}(-\hat{e}_y)}$ }

+ { - skupaj: $\vec{F}_m = \vec{F}_{m1} + \vec{F}_{m2} = \left(-\frac{1}{4} - \frac{1}{12}\right)\frac{\mu_0 I^2 l}{\pi^2 a}\hat{e}_y$

$\boxed{\frac{\vec{F}_m}{l} = -\frac{\mu_0 I^2}{3\pi^2 a}\hat{e}_y}$ $\Rightarrow$ sila je PRIVLAČNA!

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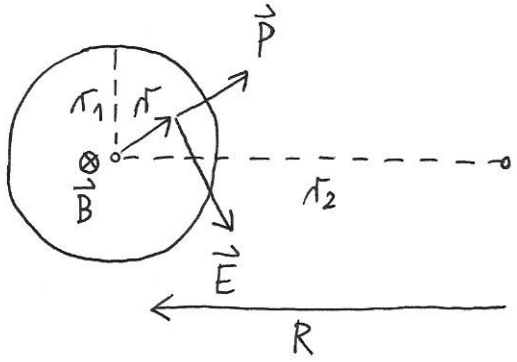

a) – magnetno polje

$$+\begin{cases} \vec{\nabla}\times\vec{B} = \mu_0\vec{j} \rightarrow B\cdot 2\pi R = \mu_0 NI \\ B = \dfrac{\mu_0 NI}{2\pi R} \approx \underline{\dfrac{\mu_0 NI}{2\pi r_2}}, \text{ker } r_2 \gg r_1 \end{cases}$$

– električno polje

$$+\begin{cases} \vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t} \rightarrow 2\pi r E = -\dot{B}\pi r^2 \quad \text{(krožnica s polmerom } r) \\ E = -\dot{B}\dfrac{r}{2} = \boxed{-\dfrac{\mu_0 N\dot{I} r}{4\pi r_2}} \end{cases}$$

$\dot{I} < 0 \Rightarrow \vec{E}$ ima v levem delu toroidne tuljave tangentno smer, in sicer v smislu urinega kazalca

b) Poyntingov vektor $\rightarrow \vec{P} = \dfrac{1}{\mu_0}\vec{E}\times\vec{B}$ ima smer VEN iz plašča tuljave

$$+\begin{cases} P = \dfrac{1}{\mu_0}EB = -\underbrace{\dfrac{NI}{2\pi r_2}}_{\frac{B}{\mu_0}}\cdot\underbrace{\dfrac{\mu_0 N\dot{I} r_1}{4\pi r_2}}_{E} = -\dfrac{\mu_0 N^2 r_1}{8\pi^2 r_2^2} I\dot{I} \end{cases}$$

← na površini

$$+\begin{cases} \int P\,dS = P\cdot\underbrace{2\pi r_1\cdot 2\pi r_2}_{\text{površina tuljave}} = \boxed{-\dfrac{\mu_0 N^2 r_1^2}{2r_2} I\dot{I}} \end{cases}$$

c) energija EM valovanja

$$+\begin{cases} W_{ne} = \dfrac{B^2}{2\mu_0}\cdot\underbrace{\pi r_1^2\cdot 2\pi r_2}_{\text{prostornina tuljave}} = \dfrac{\mu_0 N^2 I^2}{8\pi^2 r_2^2}\cdot 2\pi^2 r_1^2 r_2 = \dfrac{\mu_0 N^2 I^2 r_1^2}{4r_2} \end{cases}$$

$$+\begin{cases} \dot{W}_{ne} = \dfrac{\mu_0 N^2 r_1^2}{4r_2}\underbrace{(I^2)^{\cdot}}_{2I\dot{I}} = \boxed{\dfrac{\mu_0 N^2 r_1^2}{2r_2} I\dot{I}} = \underline{-\int P\,dS} \quad \checkmark \end{cases}$$

$$+\begin{cases} \dot{W}_e = 0, \text{ saj je } \dot{I} = \text{konst} \Rightarrow E^2 = \text{konst} \end{cases}$$

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