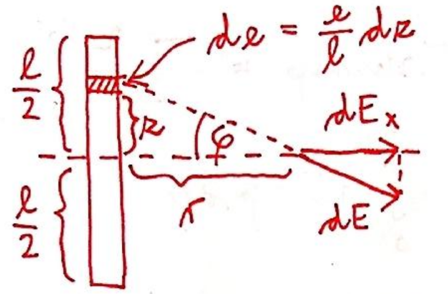
2. PISNI IZPIT IZ VAJ

$\boxed{1}$



a) $\boxed{+}$ $$dE = \frac{de}{4\pi\varepsilon_0 (r^2+z^2)}$$

$\boxed{+}$ $$\begin{cases} dE_x = \dfrac{r}{\sqrt{r^2+z^2}}\, dE = \dfrac{1}{4\pi\varepsilon_0}\dfrac{r}{(r^2+z^2)^{\frac{3}{2}}}\dfrac{e}{l}\,dz \\ dE_x = \dfrac{e}{4\pi\varepsilon_0 l}\, r\, \dfrac{dz}{(r^2+z^2)^{\frac{3}{2}}} \end{cases}$$

$\boxed{\frac{1}{2}}$

$\boxed{+}$ integral

$$\int \frac{dz}{(r^2+z^2)^{\frac{3}{2}}} = \frac{z}{r^2}\frac{1}{\sqrt{z^2+r^2}}$$ $\leftarrow$ integral lahko preberemo iz priročnika, lahko pa ga rešimo s substitucijo

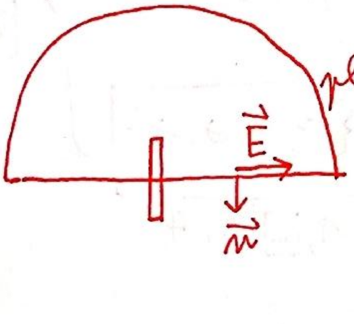
$$\begin{cases} z = r\,\mathrm{tg}\,\varphi \Rightarrow dz = r\dfrac{d\varphi}{\cos^2\varphi} \\ z^2+r^2 = r^2(1+\mathrm{tg}^2\varphi) = \dfrac{r^2}{\cos^2\varphi} \end{cases}$$

$$\int \frac{dz}{(r^2+z^2)^{\frac{3}{2}}} = \frac{1}{r^2}\int \cos\varphi\, d\varphi = \frac{1}{r^2}\int d(\sin\varphi) = \frac{\sin\varphi}{r^2} = \frac{z}{r^2}\frac{1}{\sqrt{z^2+r^2}}$$, seveda isto!

$\boxed{+}$

$$E_x = \frac{e}{4\pi\varepsilon_0 l}\, r \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{dz}{(r^2+z^2)^{\frac{3}{2}}} = \frac{e}{4\pi\varepsilon_0 l}\, r\, 2\frac{\frac{l}{2}}{r^2}\frac{1}{\sqrt{r^2+\left(\frac{l}{2}\right)^2}} = \boxed{\frac{e}{4\pi\varepsilon_0}\frac{1}{r\sqrt{r^2+\left(\frac{l}{2}\right)^2}}}$$

b)



ploskev sklenemo v neskončnosti $\rightarrow$ integral je NIČ, saj E^2 pada kot $\frac{1}{r^4}$, S pa gre kot r^2

$\boxed{\frac{1}{2}}$

$$\vec{F} = \varepsilon_0 \oint \Big[\vec{E}\underbrace{(\vec{E}\cdot\vec{n})}_{0} - \frac{1}{2}E^2\vec{n}\Big]\, dS \Rightarrow \underbrace{F = \frac{\varepsilon_0}{2}\int E_x^2\, dS}_{\text{po simetrijski ravnini}}$$

$\boxed{+}$ $$F = \frac{\varepsilon_0}{2}\frac{e^2}{16\pi^2\varepsilon_0^2}\int_a^{\infty}\frac{2\pi r\, dr}{r^2\left[r^2+\left(\frac{l}{2}\right)^2\right]} = \frac{e^2}{16\pi\varepsilon_0}\int_a^{\infty}\frac{dr}{r\left[r^2+\left(\frac{l}{2}\right)^2\right]}$$

$\boxed{+}$ integral

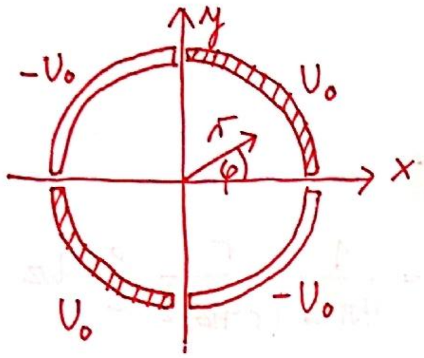
$$\int\frac{dr}{r\left[r^2+\left(\frac{l}{2}\right)^2\right]} = \int dr\left[\frac{1}{r} - \frac{r}{r^2+\left(\frac{l}{2}\right)^2}\right]\frac{1}{\left(\frac{l}{2}\right)^2} = \frac{4}{l^2}\left[\ln r - \frac{1}{2}\ln\left\{r^2+\left(\frac{l}{2}\right)^2\right\}\right] =$$

$$= \frac{4}{l^2}\ln\frac{r}{\sqrt{r^2+\left(\frac{l}{2}\right)^2}}$$, lahko pa integral preberemo iz priročnika

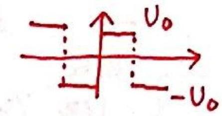
$\boxed{+}$ $$F = \frac{e^2}{16\pi\varepsilon_0}\frac{4}{l^2}\ln\frac{r}{\sqrt{r^2+\left(\frac{l}{2}\right)^2}}\Bigg|_a^{\infty} = \frac{e^2}{4\pi\varepsilon_0 l^2}\left[\overbrace{\ln 1}^{0} - \ln\frac{a}{\sqrt{a^2+\left(\frac{l}{2}\right)^2}}\right] =$$

$$= \frac{e^2}{4\pi\varepsilon_0 l^2}\ln\frac{\sqrt{a^2+\left(\frac{l}{2}\right)^2}}{a} \approx \boxed{\frac{e^2}{4\pi\varepsilon_0 l^2}\ln\frac{l}{2a}}$$ $\boxed{+}$

$\boxed{2}$

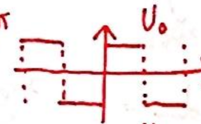


a) $\boxed{+}$ $U(r,\varphi) = \sum_m C_m r^m \sin m\varphi$, ostalih členov ni!

$U(a,\varphi) = \sum_m C_m a^m \sin m\varphi =$ 

na obeh straneh $\int_0^{2\pi} \sin n\varphi \, d\varphi$

levo: $\int_0^{2\pi} \sin n\varphi \sin m\varphi \, d\varphi = \delta_{mn} \underbrace{\int_0^{2\pi} \sin^2 m\varphi \, d\varphi}_{\frac{1}{2}\cdot 2\pi} = \pi \delta_{mn}$ $\boxed{+}$

desno: $\int_0^{2\pi}$  $\sin n\varphi \, d\varphi = U_0 \left[\int_0^{\frac{\pi}{2}} \sin n\varphi \, d\varphi - \int_{\frac{\pi}{2}}^{\pi} \sin n\varphi \, d\varphi + \int_{\pi}^{\frac{3\pi}{2}} \sin n\varphi \, d\varphi - \int_{\frac{3\pi}{2}}^{2\pi} \sin n\varphi \, d\varphi \right] =$

$\boxed{\frac{1}{2}+}$ $\int \sin n\varphi \, d\varphi = -\frac{1}{n} \cos n\varphi$

$= \frac{U_0}{n} \left[\left(1 - \cos\frac{n\pi}{2}\right) + \left(\cos n\pi - \cos\frac{n\pi}{2}\right) + \left(\cos n\pi - \cos\frac{3n\pi}{2}\right) + \left(\cos 2\pi n - \cos\frac{3n\pi}{2}\right) \right] =$

$= \frac{2U_0}{n} \left[1 \underbrace{- \cos\frac{n\pi}{2}} + \underbrace{\cos n\pi} - \underbrace{\cos\frac{3n\pi}{2}} \right] = \frac{2U_0}{n} \{0, 4, 0, 0\}$

$n = 1,2,3,4 \Rightarrow$ $0,1,0,-1$ $\quad -1,1,-1,1$ $\quad 0,1,0,-1$ $\quad$ za $n = 1,2,3,4$ $\quad \boxed{\frac{1}{4}}$

izenačimo levo = desno

$C_n a^n \pi = \frac{2U_0}{n} \{0,4,0,0\} \Rightarrow \boxed{C_n = \frac{2U_0}{\pi n a^n} \{0,4,0,0\}}$

za $n = 1, \boxed{2}, 3, 4$

$\boxed{+}$ $\boxed{U(r,\varphi) = \sum_{m=2,6,10,\dots} \frac{8U_0}{\pi m} \left(\frac{r}{a}\right)^m \sin m\varphi}$ $\rightarrow$ samo vsake četrti vodi do neničelnega člena!

b) $\varphi = 45°$ $\rightarrow$ $\boxed{+}$ zaradi simetrije polje nima azimutalne komponente!

$E_r = -\frac{\partial U}{\partial r} (\varphi = 45°) = - \sum_{m=2,6,10,\dots} \frac{8U_0}{\pi} \frac{1}{a} \left(\frac{r}{a}\right)^{m-1} \underbrace{\sin\frac{m\pi}{4}}_{1,-1,1,-1,\dots} =$ $\quad$ za $m = 2,6,10,\dots$

$\boxed{\frac{1}{2}}$ $\boxed{+}$ $= -\frac{8U_0}{\pi a} \left[\frac{r}{a} - \left(\frac{r}{a}\right)^5 + \left(\frac{r}{a}\right)^9 - \dots \right] =$

$\boxed{+}$ $= -\frac{8U_0}{\pi a} \cdot \frac{r}{a} \left[\underbrace{1 - \left(\frac{r}{a}\right)^4 + \left(\frac{r}{a}\right)^8 - \dots}_{\text{geometrijska vrsta}} \right] = \boxed{-\frac{8U_0}{\pi a} \frac{\frac{r}{a}}{1 + \left(\frac{r}{a}\right)^4}}$

3 a) TEM način $\Rightarrow$ $k_z = k$, $\quad k = \frac{\omega}{c}$, $\quad c = \frac{c_0}{\sqrt{\varepsilon}}$

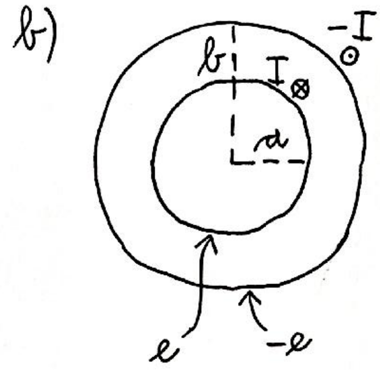
$$+ \quad k_z = \frac{\omega}{c_0}\sqrt{\varepsilon}, \quad \varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

$\frac{1}{4}$

$$k_z = \frac{\omega}{c_0}\sqrt{1-\frac{\omega_p^2}{\omega^2}} = \frac{1}{c_0}\sqrt{\omega^2-\omega_p^2}$$

$$+ \quad \boxed{\omega = \sqrt{\omega_p^2 + c_0^2 k_z^2}}$$

b)



splošna zveza med obema poljema:

$$\vec{H} = \frac{1}{Z}\hat{z}\times\vec{E}, \quad Z = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon}} = \frac{Z_0}{\sqrt{\varepsilon}} \quad +$$

$$H = \frac{1}{Z_0}\sqrt{\varepsilon}\,E \quad \Rightarrow \quad \begin{cases} H(a) = \frac{1}{Z_0}\sqrt{\varepsilon}\,E(a) \\ \frac{I}{2\pi a} = \frac{1}{Z_0}\sqrt{\varepsilon}\,\underbrace{E(a)}_{\text{iščemo}} \end{cases}$$

izračun polja E: $\quad e = \varepsilon_0\varepsilon E \cdot 2\pi r l$

$$E = \frac{e}{2\pi\varepsilon_0\varepsilon l}\cdot\frac{1}{r} \quad +$$

napetost med cevema: $\quad U = \int_a^b E\,dr = \frac{e}{2\pi\varepsilon_0\varepsilon l}\cdot\ln\frac{b}{a}$

$$E = \frac{U}{\ln\frac{b}{a}}\cdot\frac{1}{r} \Rightarrow E(a) = \frac{U}{a\ln\frac{b}{a}} \quad +$$

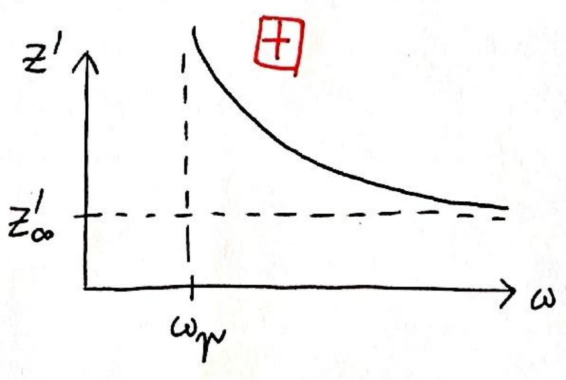
$\frac{3}{4}$

vstavimo v zgornjo enačbo

$$\frac{I}{2\pi a} = \frac{\sqrt{\varepsilon}}{Z_0}\,\frac{U}{a\ln\frac{b}{a}}$$

$$Z' = \frac{U}{I} = \frac{\ln\frac{b}{a}}{2\pi}\,\frac{Z_0}{\sqrt{\varepsilon}} \leftarrow \text{impedanca vodnika}$$

$$+ \quad \boxed{Z' = \frac{\ln\frac{b}{a}}{2\pi}Z_0\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}}}}$$



$$\left.\begin{aligned} \frac{b}{a} &= 2 \\ Z_0 &= 376\,\Omega \\ \omega &\to \infty \end{aligned}\right\} \Rightarrow \boxed{Z'_\infty = 41\,\Omega} \quad +$$