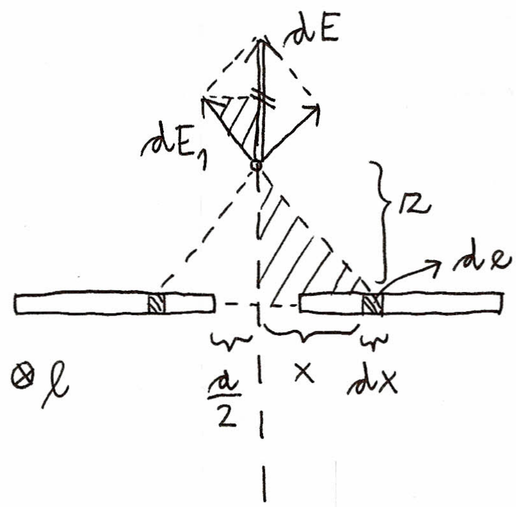
EMP: 3. PISNI IZPIT IZ VAJ

1



a) 1/4 $\begin{cases} de = \sigma l\, dx \\ dE_1 = \dfrac{de}{2\pi\varepsilon_0 l \sqrt{x^2+r^2}} = \dfrac{\sigma\, dx}{2\pi\varepsilon_0 \sqrt{x^2+r^2}} \end{cases}$

polje dolge nabite žice

+ $\begin{cases} \dfrac{\frac{dE}{2}}{dE_1} = \dfrac{r}{\sqrt{x^2+r^2}} \\ dE = \dfrac{\sigma r\, dx}{\pi\varepsilon_0 (x^2+r^2)} \end{cases}$

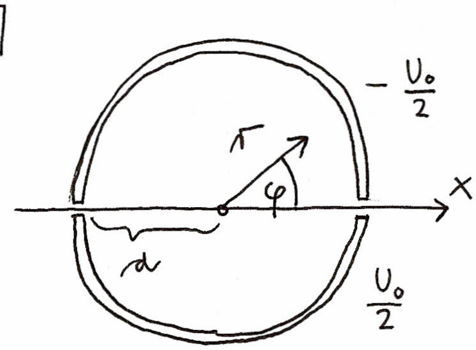
$$E = \frac{\sigma}{\pi\varepsilon_0}\int_{x=\frac{d}{r}}^{\infty} \frac{d\left(\frac{x}{r}\right)}{1+\left(\frac{x}{r}\right)^2} \leftarrow$$

+ $\left\{ E = \dfrac{\sigma}{\pi\varepsilon_0} \operatorname{arctg}\dfrac{x}{r}\Big|_{x=\frac{d}{2}}^{\infty} = \boxed{\dfrac{\sigma}{\pi\varepsilon_0}\left[\dfrac{\pi}{2} - \operatorname{arctg}\dfrac{d}{2r}\right]} \right.$ $\boxed{\frac{1}{2}}$

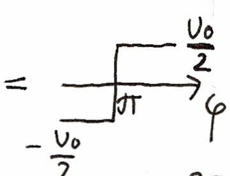
b) $\underbrace{|\vec{\nabla} E| = \left|\dfrac{\partial E}{\partial r}\right| = \dfrac{\sigma}{\pi\varepsilon_0}\, \dfrac{1}{1+\left(\frac{d}{2r}\right)^2} \cdot \dfrac{d}{2}\, \dfrac{1}{r^2} = \boxed{\dfrac{2\sigma}{\pi\varepsilon_0}\, \dfrac{1}{1+\left(\frac{2r}{d}\right)^2}}}_{\frac{1}{4}}$

gradient očitno pada z razdaljo $\Rightarrow$ max pri $\underline{r = 0}$! } $\frac{1}{4}$ $\boxed{\frac{1}{2}}$

2



a) + $\begin{cases} \underline{U(r,\varphi) = \sum_m C_m r^m \sin m\varphi} \;,\; \underline{r \le d} \\ \text{ostalih členov NI!} \\ U(d,\varphi) = \sum_m C_m d^m \sin m\varphi = \end{cases}$

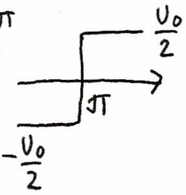


robni pogoj $\quad / \cdot \sin n\varphi \rightarrow \int_0^{2\pi}$

− leva stran:

+ $\left\{ \int_0^{2\pi} \sin m\varphi \sin n\varphi\, d\varphi = \delta_{mn} \underbrace{\int_0^{2\pi} \sin^2 m\varphi\, d\varphi}_{\frac{1}{2}\cdot 2\pi} = \underline{\pi\,\delta_{mn}} \right.$

– desna stran:



$$+\quad \int_0^{2\pi} \ldots \sin n\varphi\, d\varphi = -\frac{U_0}{2}\Big[\underbrace{\int_0^{\pi} \sin n\varphi\, d\varphi}_{\frac1n(-\cos n\varphi)\big|_0^{\pi}} - \underbrace{\int_{\pi}^{2\pi} \sin n\varphi\, d\varphi}_{\frac1n(-\cos n\varphi)\big|_{\pi}^{2\pi}}\Big] =$$

$$= -\frac{U_0}{2}\left[\frac1n(1-\cos n\pi) - \frac1n(\cos n\pi - 1)\right] = -\frac{U_0}{n}\Big[1-\underbrace{\cos n\pi}_{(-1)^n}\Big] = \underline{-\frac{U_0}{n}\left[1-(-1)^n\right]}$$

$\Downarrow$ izenačimo

$$+\quad C_n a^n \pi = -\frac{U_0}{n}\left[1-(-1)^n\right] \Rightarrow \underline{C_n = -\frac{U_0}{\pi n}\left[1-(-1)^n\right]\frac{1}{a^n}}$$

$$U(r,\varphi) = -U_0 \sum_n \frac{1}{\pi n}\left[1-(-1)^n\right]\left(\frac{r}{a}\right)^n \sin n\varphi = \boxed{-U_0 \sum_{n\ \text{lih}} \frac{2}{\pi n}\left(\frac{r}{a}\right)^n \sin n\varphi}$$

$r \leq a$

$\boxed{\frac12}$

$+$ – rezultat za zunaj $(r \geq a)$ je ENAK, le da nadomestimo $\underline{\frac{r}{a} \to \frac{a}{r}}$

b) $$de = \varepsilon_0 \underbrace{dS}_{l a\, d\varphi} \cdot \Big(\underbrace{E_{ZUN}}_{-\frac{\partial U_{ZUN}}{\partial r}\big|_{r=a}} - \underbrace{E_{NOT}}_{-\frac{\partial U_{NOT}}{\partial r}\big|_{r=a}}\Big)$$

$$+\quad de = \varepsilon_0 l a\, d\varphi \cdot \frac{2U_0}{\pi}\frac{1}{a}\sum_{n\ \text{lih}}\left[-n\left(\frac{r}{a}\right)^{-n-1}_{r=a}\frac{\sin n\varphi}{n} - n\left(\frac{r}{a}\right)^{n-1}_{r=a}\frac{\sin n\varphi}{n}\right] =$$

$$= -\varepsilon_0 l\, d\varphi \cdot \frac{2U_0}{\pi}\, 2 \underbrace{\sum_{n\ \text{lih}} \cancel{\frac{n}{n}} \sin n\varphi}_{\frac{1}{2\sin\varphi}} = -\frac{2U_0}{\pi}\varepsilon_0 l \frac{d\varphi}{\sin\varphi}$$

$\frac14$

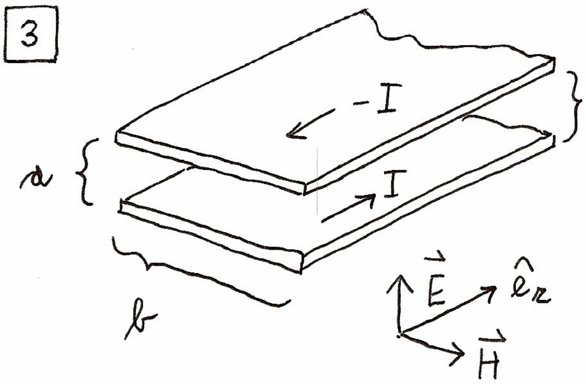
$$e = -\frac{2\varepsilon_0 U_0}{\pi} l \underbrace{\int_{\delta}^{\pi-\delta} \frac{d\varphi}{\sin\varphi}}_{\ln \mathrm{tg}\frac{\varphi}{2}\big|_{\delta}^{\pi-\delta}} \qquad \underbrace{\mathrm{ctg}\frac{\delta}{2} \approx \frac{2}{\delta}} \qquad \underbrace{\approx \frac{\delta}{2}}$$

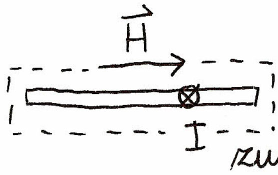
$$= \ln \mathrm{tg}\left(\frac{\pi}{2}-\frac{\delta}{2}\right) - \ln \mathrm{tg}\frac{\delta}{2} \approx -2\ln\frac{\delta}{2}$$

$$e = \frac{4\varepsilon_0 U_0}{\pi} l \ln\frac{\delta}{2} \Rightarrow \frac{C}{l} = \frac{1}{l}\frac{e}{U_0} = \boxed{\frac{4\varepsilon_0}{\pi}\ln\frac{\delta}{2}}$$

$\boxed{\frac12}$

3





zunaj ni polja!

a) $\left.\omega = kc = k\dfrac{c_0}{\sqrt{\varepsilon}}\right\}+$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\Downarrow$$

$$+\left\{\begin{array}{l} \omega = \dfrac{k c_0}{\sqrt{1-\dfrac{\omega_p^2}{\omega^2}}} \\[2ex] k^2 c_0^2 = \omega^2 - \omega_p^2 \rightarrow \boxed{\omega = \sqrt{\omega_p^2 + k^2 c_0^2}} \end{array}\right.$$

$\boxed{\frac{1}{4}}$

b) $\left.\vec{H} = \dfrac{1}{Z}\hat{e}_z \times \vec{E}\,, \quad Z = \sqrt{\dfrac{\mu_0}{\varepsilon\varepsilon_0}} = \dfrac{Z_0}{\sqrt{\varepsilon}}\right\}+$

$$H = \frac{\sqrt{\varepsilon}}{Z_0} E$$

– Ampere za posamezno ploščo:

$$\left.\mu_0 I = Bb \Rightarrow B = \frac{\mu_0 I}{b} \rightarrow \underline{H = \frac{I}{b}}\right\}+$$

– definicija napetosti:

$$\left.U = Ea \Rightarrow \underline{E = \frac{U}{a}}\right\}+$$

$$\left.\begin{array}{l} \dfrac{I}{b} = \dfrac{1}{Z_0}\sqrt{1-\dfrac{\omega_p^2}{\omega^2}}\,\dfrac{U}{a} \\[2ex] Z' = \dfrac{U}{I} = \boxed{Z_0 \dfrac{a}{b}\dfrac{1}{\sqrt{1-\dfrac{\omega_p^2}{\omega^2}}}} \end{array}\right\}\frac{1}{4}$$

– primer $Z' = Z_0 \Rightarrow \sqrt{1-\dfrac{\omega_p^2}{\omega^2}} = \dfrac{a}{b}$

$$\frac{\omega_p^2}{\omega^2} = 1 - \left(\frac{a}{b}\right)^2$$

$\boxed{\frac{3}{4}}$

$$+\left\{\boxed{\omega = \frac{\omega_p}{\sqrt{1-\left(\frac{a}{b}\right)^2}}}\right. > \omega_p \rightarrow$$

pri taki frekvenci je $\varepsilon > 0$, kakor mora biti!