EMP : 3. PISNI IZPIT IZ VAJ

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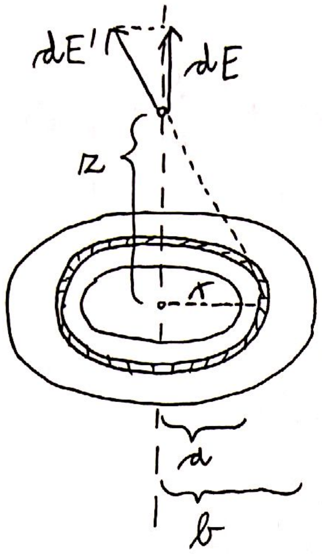


$$
+\left\{\begin{aligned}
dE' &= \frac{de}{4\pi\varepsilon_0 (r^2+z^2)} \\
de &= \sigma \cdot 2\pi r\, dr \\
dE &= \frac{z}{\sqrt{z^2+r^2}}\, dE'
\end{aligned}\right.
$$

$$
\frac{1}{4}\left\{\begin{aligned}
dE &= \frac{\sigma \cdot 2\pi r\, dr}{4\pi\varepsilon_0}\,\frac{z}{(z^2+r^2)^{\frac{3}{2}}} = \frac{\sigma z}{4\varepsilon_0}\,\frac{d(r^2+z^2)}{(r^2+z^2)^{\frac{3}{2}}} \\
E &= \frac{\sigma z}{4\varepsilon_0}\int_{r=a}^{r=b}\frac{d(r^2+z^2)}{(r^2+z^2)^{\frac{3}{2}}} = \boxed{\frac{\sigma z}{2\varepsilon_0}\left[\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\right]}
\end{aligned}\right.
$$

a) $\underbrace{z \ll a, b}$ $\Rightarrow$

$$
+\left\{\begin{aligned}
\frac{z}{\sqrt{a^2+z^2}} &\approx \frac{z}{a} \\
\frac{z}{\sqrt{b^2+z^2}} &\approx \frac{z}{b}
\end{aligned}\right\}\quad \boxed{E(z) \approx \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) z} \;\Big\}+
$$

linearno z z

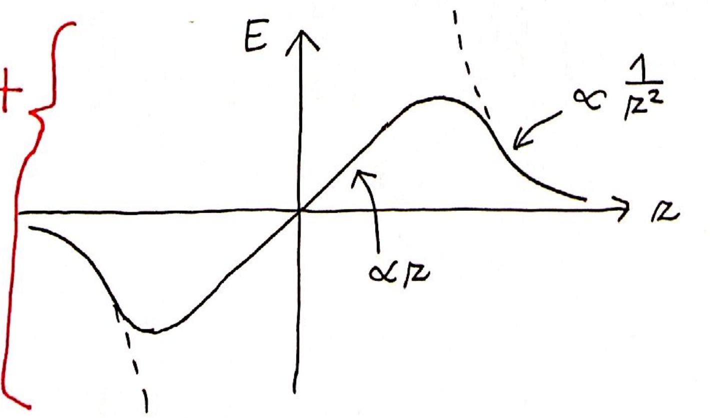
b) $\underbrace{z \gg a, b}$ $\Rightarrow$

$$
\frac{z}{\sqrt{a^2+z^2}} = \frac{1}{\sqrt{1+\left(\frac{a}{z}\right)^2}} \approx 1 - \frac{1}{2}\left(\frac{a}{z}\right)^2
$$

$$
\frac{z}{\sqrt{b^2+z^2}} \approx 1 - \frac{1}{2}\left(\frac{b}{z}\right)^2
$$

$$
\overbrace{E(z) \approx \frac{\sigma}{2\varepsilon_0}\,\frac{1}{2z^2}\,(b^2-a^2)}^{+} = \underbrace{\frac{\sigma\,\overbrace{\pi(b^2-a^2)}^{\text{ploščina kolobarja}}}{4\pi\varepsilon_0 z^2} = \boxed{\frac{e}{4\pi\varepsilon_0 z^2}}}_{+}
$$

c) +



pomembno:
- liha funkcija
- linearen začetek
- sorazmernost z $\frac{1}{z^2}$ pri velikih z

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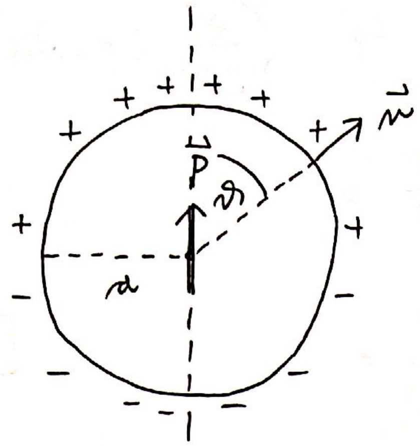


+ $\left\{ \begin{array}{l} \sigma_v = \vec{P}\cdot\vec{n} = P\cos\vartheta \\ \rho_v = -\vec{\nabla}\cdot\vec{P} = 0 \end{array} \right.$

$\nabla^2 U(r,\vartheta) = 0$, naboji so le na površini

+ $$U(r,\vartheta) = \begin{cases} A r\cos\vartheta, & r < a \\ \dfrac{B}{r^2}\cos\vartheta, & r > a \end{cases}$$

robna pogoja:

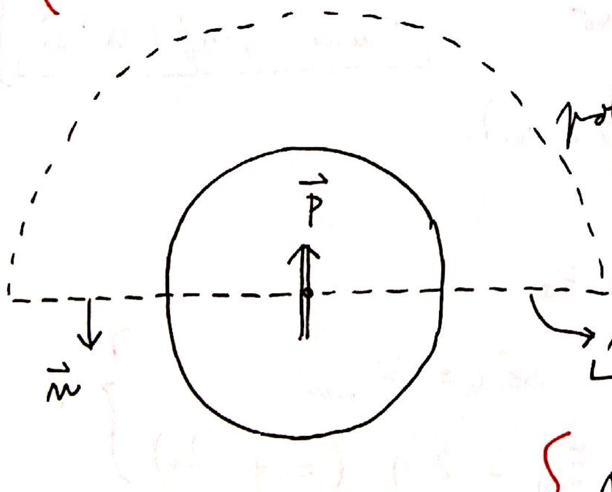
+ 1) U zvezen: $Aa = \dfrac{B}{a^2}$

+ 2) $E(r=a^+) - E(r=a^-) = \dfrac{\sigma_v}{\varepsilon_0}$

$$\left(\frac{2B}{a^3} + A\right)\cos\vartheta = \frac{P\cos\vartheta}{\varepsilon_0}$$

+ $$U(r,\vartheta) = \begin{cases} \dfrac{P}{3\varepsilon_0}\overbrace{r\cos\vartheta}^{z}, & r<a \\ \dfrac{Pa^3}{3\varepsilon_0}\dfrac{\cos\vartheta}{r^2}, & r>a \end{cases} \Leftarrow \qquad \frac{2B}{a^3} + A = 3A = \frac{P}{\varepsilon_0} \Rightarrow \underline{A = \frac{P}{3\varepsilon_0}}$$

$$\underline{B = \frac{Pa^3}{3\varepsilon_0}}$$



polsfera v NESKONČNOSTI!
↳ integral da nič, saj polje pada z $\frac{1}{r^3}$

→ ploskev s $\vartheta = \frac{\pi}{2}$! → $\vec{E}$ ima le komponento v smeri ϑ!
↳ v smeri z torej

+ $\left\{ \begin{array}{l} r<a: \quad E_z = -\dfrac{\partial U}{\partial z} = \underline{-\dfrac{P}{3\varepsilon_0}} \\ r>a: \quad E_z = -\dfrac{1}{r}\dfrac{\partial U}{\partial(\cos\vartheta)}\Big|_{\vartheta=\frac{\pi}{2}} = \underline{-\dfrac{Pa^3}{3\varepsilon_0 r^3}} \end{array} \right.$

$$\vec{F} = \varepsilon_0 \oint \left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS$$

$\vec{E} \parallel \vec{n}$ na celi spodnji ploskvi $\Rightarrow$ $\vec{E}(\vec{E}\cdot\vec{n}) = E^2\vec{n}$

$$\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n} = \frac{1}{2}E^2\vec{n}$$

$$+\left\{\ F = \varepsilon_0 \frac{1}{2}\int_0^\infty E^2\, dS = \frac{\varepsilon_0}{2}\int_0^a \frac{p^2}{9\varepsilon_0^2}\, dS + \frac{\varepsilon_0}{2}\int_a^\infty \frac{p^2 a^6}{9\varepsilon_0^2 r^6}\, 2\pi r\, dr = \right.$$

$$= \frac{\varepsilon_0}{2}\,\frac{p^2}{9\varepsilon_0^2}\,\pi a^2 + \frac{\varepsilon_0}{2}\,\frac{p^2 a^6}{9\varepsilon_0^2}\, 2\pi \int_a^\infty \frac{dr}{r^5} =$$

$$= \frac{\pi}{18\varepsilon_0} p^2 a^2 + \frac{\pi}{9\varepsilon_0} p^2 a^6\, \frac{1}{4}\,\frac{1}{r^4}\Bigg|_\infty^a = \qquad \boxed{1}$$

$$+\left\{\ = \frac{\pi}{18\varepsilon_0} p^2 a^2 + \frac{\pi}{36\varepsilon_0} p^2 a^2 = \boxed{\frac{\pi}{12\varepsilon_0} p^2 a^2} \right. \rightarrow$$ sila kaže v smeri $\vec{n}$, torej NAVZDOL

$\boxed{3}$ $$\Big(\underbrace{\nabla_\perp^2 - k^2 + \frac{\omega^2}{c^2}}_{+\varkappa^2}\Big)\begin{Bmatrix} E_z \\ H_z \end{Bmatrix} = 0$$ valovna enačba

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \Rightarrow c = \frac{c_0}{\sqrt{\varepsilon}}\,,\quad \frac{\omega}{c} = \frac{\omega}{c_0}\sqrt{\varepsilon} = \frac{1}{c_0}\sqrt{\omega^2-\omega_p^2}\ \Big\}+$$

$$+\left\{\ \underline{\varkappa^2 = \frac{\omega^2-\omega_p^2}{c_0^2} - k^2}\right. \rightarrow$$ spremenjeni $\varkappa$ zaradi plazme

$$\Downarrow \quad \underline{\omega_{min} = \sqrt{\omega_p^2 + \varkappa^2 c_0^2}}\ \Big\}+$$

rešitve v valjni geometriji poznamo:

$$\begin{Bmatrix} E_z \\ H_z \end{Bmatrix}(r,\varphi) = \sum_m A_m J_m(\varkappa r)\cos(m\varphi + \varphi_m)$$

- $\underline{\text{TM način}}$: $H_z = 0$, enačba za E_z

$$E_z(r=a,\varphi) = 0 \Rightarrow J_m(\varkappa a) = 0\,,\quad \varkappa a = \xi_{mn}$$

$$(\xi_{mn})_{min} = \xi_{01} = 2.4 \ \text{(z grafa)}\ \Big\}+$$

$\boxed{1}$ $$(\omega_{TM})_{min} = \sqrt{2.1^2\frac{c_0^2}{a^2} + 2.4^2\frac{c_0^2}{a^2}} = \underline{3.19\,\frac{c_0}{a}}\ \Big\}+$$

- $\underline{\text{TE način}}$: $E_z = 0$, enačba za H_z

$$\frac{\partial H_z}{\partial r}(r=a,\varphi) = 0 \Rightarrow J_m'(\varkappa a) = 0\,,\quad \varkappa a = \xi_{mn}'$$

$$(\xi_{mn}')_{min} = \xi_{11}' = 1.8 \ \text{(z grafa)}\ \Big\}+$$

$$+\left\{\ \boxed{\frac{(\omega_{TE})_{min}}{(\omega_{TM})_{min}} = 0.87}\right. \Leftarrow (\omega_{TE})_{min} = \sqrt{2.1^2\frac{c_0^2}{a^2} + 1.8^2\frac{c_0^2}{a^2}} = \underline{2.77\,\frac{c_0}{a}}\ \Big\}+$$