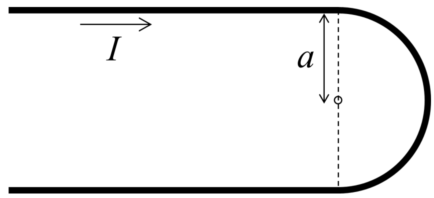
Elektromagnetno polje: 1. kolokvij

(18. 11. 2016 ob 15:00)

asistent: Martin Klanjšek (01 477 3866, *martin.klanjsek@ijs.si*)

1. naloga

Tanek vodnik v obliki črke "U" je sestavljen iz dveh dolgih vzporednih ravnih delov, ki sta povezana s polkrožnim delom polmera a, tako da celoten vodnik leži v ravnini. Po vodniku spustimo električni tok I, kakor prikazuje slika. Izračunaj gostoto magnetnega polja v središču polkrožnega dela.



2. naloga

Dolg vodoraven valj iz izolatorskega materiala enakomerno nabijemo z nabojem prostorninske gostote ρ. Polmer valja je a. Izračunaj silo na dolžinsko enoto valja, ki deluje na zgornjo polovico valja.

3. naloga

V razsežen kos kovine izdolbemo krogelno votlino polmera a, v središče votline pa postavimo točkast električni kvadrupol s tenzorjem kvadrupolnega momenta

$$\mathbf{Q} = \begin{bmatrix} -Q/2 & 0 & 0 \\ 0 & -Q/2 & 0 \\ 0 & 0 & Q \end{bmatrix}.$$

a) Določi potencial električnega polja povsod znotraj votline.

b) Izračunaj skupni naboj, ki se inducira na površini votline.

Opomnik: potencial električnega polja točkastega kvadrupola na mestu $\vec{r}$ glede na kvadrupol zapišemo kot

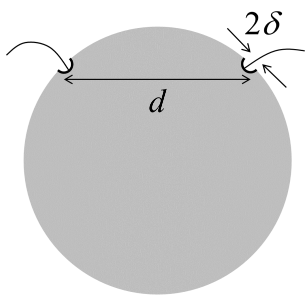
$$U_Q(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{2} \sum_{ij} \frac{Q_{ij} r_i r_j}{r^5},$$

kjer so r_i komponente vektorja $\vec{r}$, Q_{ij} pa komponente tenzorja $\mathbf{Q}$.

4. naloga (za dodatne točke)

V obod tanke okrogle ploščice polmera a in debeline D iz snovi s specifično prevodnostjo σ vrežemo majhni polkrožni vdolbini polmera δ, tako da je med njima razdalja $d \gg \delta$. Površini vdolbin prekrijemo z dobro prevodnima elektrodama, kakor prikazuje slika. Izračunaj upor, ki ga izmerimo med elektrodama.

Nasvet: ustrezen elektrostatski problem lahko rešiš tako, da vsako elektrodo nadomestiš z valjem polmera δ, katerega os je pravokotna na ploščico.



Matematični pripomoček:
Rešitve Laplaceove enačbe $\nabla^2 U(r,\vartheta) = 0$ v krogelnih koordinatah:

$$U(r,\vartheta) = \sum_{l=0}^{\infty} \left[A_l r^l + B_l r^{-(l+1)} \right] P_l(\cos\vartheta),$$

kjer so $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$, $P_3(x) = (5x^3 - 3x)/2$, ... Legendrovi polinomi.

Čas reševanja: 90 minut.
Dovoljeni pripomočki: podani spisek enačb, matematični priročnik, kalkulator.
Rešitve nalog, ocene ter kraj in čas ogleda kolokvija bodo objavljeni na spletni strani `http://www-f5.ijs.si/emp-2016-2017.html`.

Electromagnetic field: 1st written examination

(18th of November 2016 at 15:00)

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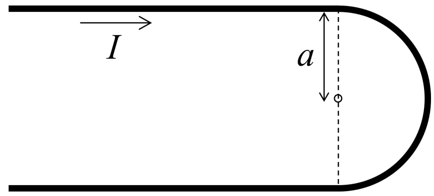
Problem 1

A thin U-shaped conductor consists of two long straight parallel parts connected with a semicircular part of radius a, so that the whole conductor lies in a plane. A conductor is carrying the electric current I, as shown in the figure. Determine the magnetic field in the center of the semicircular part.



Problem 2

A long horizontal cylinder of radius a made of isolating material is uniformly charged with the volume charge density ρ. Determine the force per unit length of the cylinder acting on the upper half of the cylinder.

Problem 3

A large piece of metal contains a spherical cavity of radius a. A point electric quadrupole with the quadrupole moment tensor

$$\mathbf{Q} = \begin{bmatrix} -Q/2 & 0 & 0 \\ 0 & -Q/2 & 0 \\ 0 & 0 & Q \end{bmatrix}$$

is placed in the center of the cavity.

a) Determine the electric field potential everywhere inside the cavity.

b) Determine the total charge induced on the surface of the cavity.

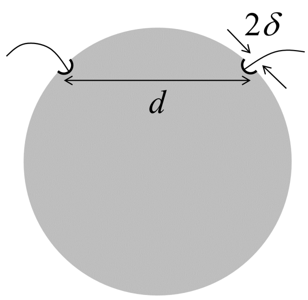
Reminder: the electric field potential of a point quadrupole at position $\vec{r}$ relative to the quadrupole is written as

$$U_Q(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{2} \sum_{ij} \frac{Q_{ij} r_i r_j}{r^5},$$

where r_i are the components of the vector $\vec{r}$, and Q_{ij} are the components of the tensor $\mathbf{Q}$.

Problem 4 (for bonus points)

A thin circular plate of radius a and thickness D made of the material with specific conductivity σ has two small semicircular indentations of radius δ drilled into its edge. The distance between the indentations is $d \gg \delta$. The surfaces of the indentations are covered with ideally conducting electrodes, as shown in the figure. Determine the resistivity measued between the electrodes.



Advice: the corresponding electrostatic problem can be solved by replacing each electrode with the cylinder of radius δ perpendicular to the plate.

Mathematical tool:
Solutions of the Laplace equation $\nabla^2 U(r,\vartheta) = 0$ in the spherical coordinates:

$$U(r,\vartheta) = \sum_{l=0}^{\infty} \left[A_l r^l + B_l r^{-(l+1)}\right] P_l(\cos\vartheta),$$

where $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$, $P_3(x) = (5x^3 - 3x)/2$, $\ldots$ are Legendre polynomials.

Duration of examination: 90 minutes.
Allowed accessories: given list of equations, mathematical handbook, calculator.
Solutions of the problems, scores, and place and time of the access to the assessed exams will be announced on the website
`http://www-f5.ijs.si/emp-2016-2017.html`.