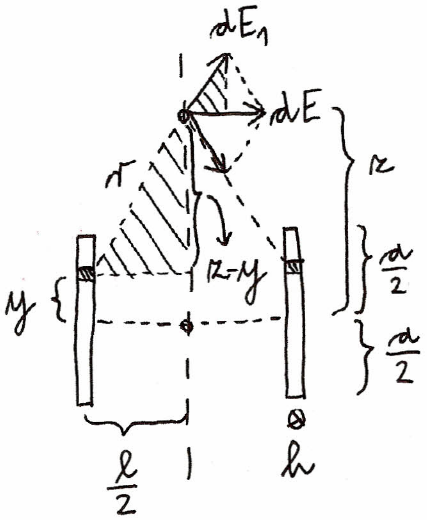
EMP: 1. KOLOKVIJ

(uganjena komponenta : en +)

1



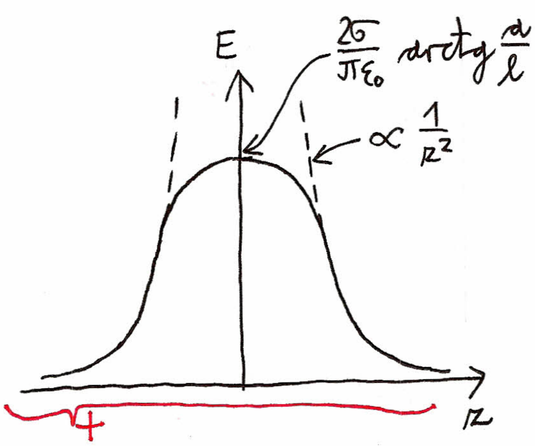
a) integriramo po PARIH dolgih ŽIC, iz katerih je sestavljena posamezna plošča

$\frac{1}{4}$

$$de = \sigma h\, dy$$

$$dE_1 = \frac{de}{2\pi\varepsilon_0 h r}, \qquad r = \sqrt{\left(\frac{l}{2}\right)^2 + (z-y)^2}$$

$$dE_1 = \frac{\sigma\, dy}{2\pi\varepsilon_0 \sqrt{\left(\frac{l}{2}\right)^2 + (z-y)^2}}$$



$\boxed{\frac{1}{4}+}$

$\frac{1}{4}$

$$\frac{\frac{1}{2}dE}{dE_1} = \frac{\frac{l}{2}}{\sqrt{\left(\frac{l}{2}\right)^2 + (z-y)^2}}$$ podobna trikotnika na sliki

$$dE = \frac{\sigma l}{2\pi\varepsilon_0} \frac{d(y-z)}{\left(\frac{l}{2}\right)^2 + (y-z)^2}$$

$\boxed{\frac{1}{2}+}$

$$E = \frac{\sigma l}{2\pi\varepsilon_0} \frac{2}{l}\, \mathrm{arctg}\, \frac{2(y-z)}{l} \Bigg|_{y=-\frac{a}{2}}^{y=\frac{a}{2}}$$

$+$ $$\boxed{E = \frac{\sigma}{\pi\varepsilon_0}\left[\mathrm{arctg}\,\frac{2z+a}{l} - \mathrm{arctg}\,\frac{2z-a}{l}\right]}$$

b) $\underline{z \gg a, l}$ $\Rightarrow$

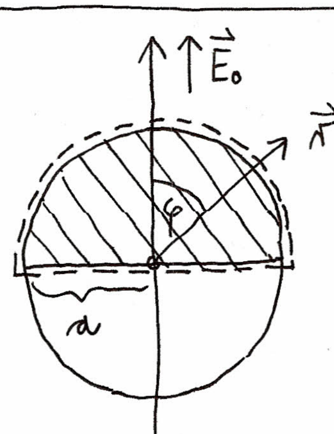
$+$ $$\left.\begin{aligned} \mathrm{arctg}\,\frac{2z+a}{l} &\approx \frac{\pi}{2} - \frac{l}{2z+a} \\ \mathrm{arctg}\,\frac{2z-a}{l} &\approx \frac{\pi}{2} - \frac{l}{2z-a} \end{aligned}\right\} \Leftarrow \mathrm{arctg}\,\frac{1}{x} \approx \frac{\pi}{2} - x$$

(en + že za trud v zvezi s tem)

$+$ $$E \approx \frac{\sigma}{\pi\varepsilon_0} \frac{l}{2z}\left[\frac{1}{1-\frac{a}{2z}} - \frac{1}{1+\frac{a}{2z}}\right] \approx \frac{\sigma}{\pi\varepsilon_0}\frac{l}{2z}\frac{2a}{2z} = \boxed{\frac{\sigma l a}{2\pi\varepsilon_0}\cdot\frac{1}{z^2}}$$

$\underline{z = 0}$ $\Rightarrow$ $$E = \frac{2\sigma}{\pi\varepsilon_0}\,\mathrm{arctg}\,\frac{a}{l}$$

2



v razvoju potenciala preživita le člena s $\cos\varphi$ zaradi robnega pogoja pri $r \to \infty$

$\frac{1}{4}$ $$U(r) = -E_0 r \cos\varphi + \frac{A}{r}\cos\varphi$$

$+$ $$U(a) = 0 = \left(-E_0 a + \frac{A}{a}\right)\cos\varphi \Rightarrow A = E_0 a^2$$

$\boxed{\frac{1}{4}+}$

$$\boxed{U(r) = E_0\left(\frac{a^2}{r} - r\right)\cos\varphi}$$

(en + samo za zunanje polje v rešitvi)

$$\vec F = \varepsilon_0 \oint \left[\vec E(\vec E\cdot\vec n) - \tfrac{1}{2}E^2 \vec n\right] dS$$

računamo silo na zgornjo (osenčeno) polovico valja:

- prerez : $E = 0$, ni prispevka
- obod : $\vec E \parallel \vec n \Rightarrow \vec E(\vec E\cdot\vec n) - \frac{1}{2}E^2\vec n = \frac{1}{2}E^2 \vec n$

$$E = -\frac{\partial U}{\partial r}\Big|_{r=a} = \underline{2E_0\cos\varphi}$$

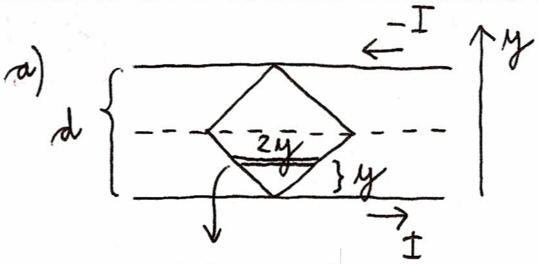
$$\vec F = \varepsilon_0 \int 2E_0^2\cos^2\varphi\, \boxed{\vec n}\, l\, a\, d\varphi \quad\Rightarrow\quad F = 2\varepsilon_0 a l E_0^2 \underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\varphi\, d\varphi}_{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1-\sin^2\varphi)\, d(\sin\varphi) = \frac{4}{3}}$$

$\boxed{\cos\varphi}$ v velikosti sile

sila kaže NAVZGOR, torej se polovici valja ODBIJATA!

$$\boxed{\frac{F}{l} = \frac{8}{3}\varepsilon_0 E_0^2 a}$$

$\boxed{3}$ a)



$dS = \underline{2y}\, dy$

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right)$$

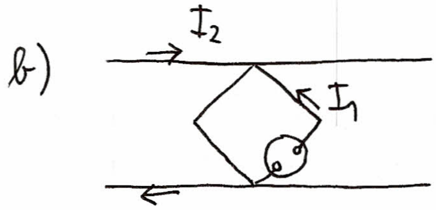
pretok skozi spodnjo POLOVICO:

$$\frac{\Phi}{2} = \frac{\mu_0 I}{2\pi}\int_0^{\frac{d}{2}}\left(\frac{1}{y}+\frac{1}{d-y}\right) 2y\, dy = \frac{\mu_0 I}{2\pi}\, 2\int_0^{\frac{d}{2}}\underbrace{\left(1 + \frac{y}{d-y}\right)}_{\frac{d}{d-y}} dy =$$

$$= \frac{\mu_0 I}{\pi} d\left[-\ln(d-y)\right]\Big|_0^{\frac{d}{2}} = \underline{\frac{\mu_0 I}{\pi} d \ln 2}$$

$$\Phi = \boxed{\underbrace{\frac{2\ln 2}{\pi}\mu_0 d}_{L_{12}}}\cdot I$$

b)



samoinduktivnost para vzporednih vodnikov:

$$\Phi_2 = \int B\underbrace{dS}_{l\,dy} = \frac{\mu_0 I_2 l}{2\pi}\underbrace{\int_a^d\left(\frac{1}{y}+\frac{1}{d-y}\right)dy}_{\ln\frac{d}{a} - \ln\frac{a}{d}} = \frac{\mu_0 l}{\pi}\ln\frac{d}{a}\cdot I_2 \Rightarrow \boxed{L_2 = \frac{\mu_0 l}{\pi}\ln\frac{d}{a}}$$

$U_{i2} = -\frac{d\Phi}{dt} = -L_{12}\dot I_1$

$U_{i2} = L_2 \dot I_2$ za drugi tokovni krog

$$\boxed{\left|\frac{I_2}{I_1}\right| = \frac{L_{12}}{L_2} = \frac{2\ln 2}{\frac{l}{d}\ln\frac{d}{a}} = 0.06}$$

$\boxed{4}$ – gre za nalogo iz ZRCALJENJA, magnetna različica

– kakšno smer ima zrcalni tok glede na originalnega?

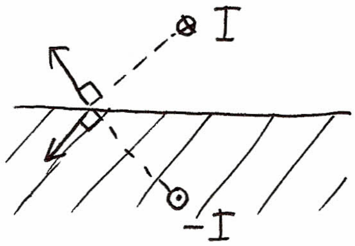
$\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{S} = 0$ okoli površine superprevodnika

$\Downarrow$

$\boxed{B_{\perp 1} = B_{\perp 2}}$ robni pogoj na površini

$B_{\perp 1} = 0$ ZNOTRAJ $\Rightarrow$ $\boxed{B_{\perp 2} = 0 \quad \text{ZUNAJ}}$

$\Downarrow$



originalni in zrcalni tok skupaj proizvedeta nasprotni pravokotni komponenti

$\Downarrow$

+ $\boxed{\text{zrcalni tok je NASPROTEN originalnemu}}$

$\downarrow$

$\boxed{\text{sila je ODBOJNA}}$

$\boxed{+}$

$p_m = I a^2$

$\vec{p}_m = I a^2 \hat{e}_z$

a) $b \gg a$ → sila med nasprotno obrnjenima dipoloma

+ $$F = \frac{\partial(\vec{p}_m\cdot\vec{B})}{\partial z} = p_m \frac{\partial B}{\partial z}\Big|_{z=b}$$

+ $$B(z) = \frac{\mu_0}{4\pi}\,\frac{3(-p_m)-(-p_m)}{(z+b)^3} = -\frac{\mu_0}{4\pi}\,\frac{2p_m}{(z+b)^3}$$

$\boxed{\frac{1}{2}}$

+ $$\frac{\partial B}{\partial z}\Big|_{z=b} = -\frac{\mu_0}{4\pi}\,2p_m(-3)\,\frac{1}{16b^4} = \frac{\mu_0}{4\pi}\,\frac{3p_m}{8b^4}$$

+ $$F = \frac{\mu_0}{4\pi}\,\frac{3p_m^2}{8b^4} = \boxed{\frac{\mu_0}{4\pi}\,\frac{3I^2a^4}{8b^4}}$$

b) $b \ll a$ → sila med vzporednima vodnikoma z nasprotnim tokom

+

+ $$B = \frac{\mu_0 I}{2\pi}\cdot\frac{1}{2b}$$ → razdalja med vodnikoma

+ $$F = IB\cdot 4a = \boxed{\frac{\mu_0}{4\pi}\cdot\frac{4I^2a}{b}}$$

$\boxed{\frac{1}{4}+}$