EMP : 1. KOLOKVIJ

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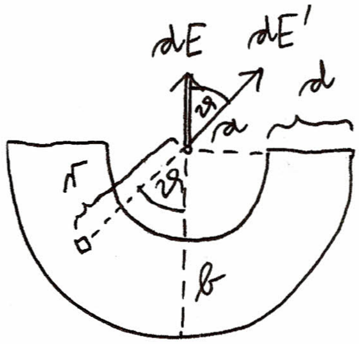

polje kaže NAVZGOR!

$+\left\{ dE' = \dfrac{de}{4\pi\varepsilon_0 r^2} \right.$

$\frac{1}{4}\left\{ de = \rho\, r^2 dr\, d\varphi\, d(\cos\vartheta) \right.$

$+\left\{ dE = \cos\vartheta\, dE' \right.$

$$dE = \frac{\rho\, dr\, d\varphi \cos\vartheta\, d(\cos\vartheta)}{4\pi\varepsilon_0}$$

$$\underbrace{E = \frac{\rho}{4\pi\varepsilon_0}\int_a^b dr \int_0^{2\pi} d\varphi \int_{\cos\vartheta = 0}^{1} \cos\vartheta\, d(\cos\vartheta)}_{\frac{1}{4}} = \underbrace{\frac{\rho}{4\pi\varepsilon_0}\underbrace{(b-a)}_{d}\, 2\pi\, \frac{1}{2}}_{+} = \underbrace{\boxed{\frac{\rho d}{4\varepsilon_0}}}_{+}$$

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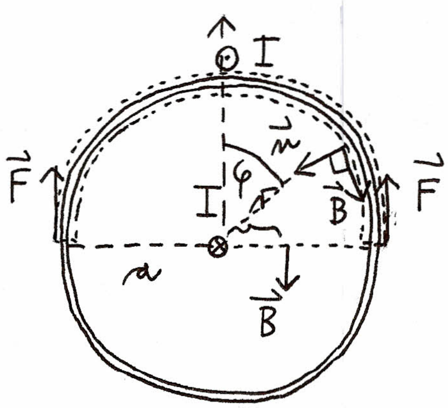

$+\left\{ B = \dfrac{\mu_0 I}{2\pi r} \right.$ znotraj cevi

$+\left\{ B = 0 \right.$ zunaj cevi

prispevek le znotraj

$$+\left\{ \vec{F}_m = \frac{1}{\mu_0}\oint\left[\vec{B}\underbrace{(\vec{B}\cdot\vec{n})}_{0} - \frac{1}{2}\underbrace{B^2}_{\text{konst}}\vec{n}\right] dS \right.$$

– integriramo po tanki polovici cevi, ki ravno objame cev koaksialnega kabla } +

$$\vec{F}_m = -\frac{B^2}{2\mu_0}\oint \vec{n}\, dS \quad \rightarrow \text{kaže v nasprotni smeri} \; \} +$$

$$F_m = +\frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi a}\right)^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underbrace{\cos\varphi}_{+} \cdot l\, a\, d\varphi$$

$F_m = 2F$

$+\left\{ F_m = \dfrac{\mu_0^2 I^2}{8\mu_0 \pi^2 a^2} \cdot 2\, l\, a = \dfrac{\mu_0^2 I^2}{4\pi^2 \mu_0 a}\, l \right.$

$\boxed{\dfrac{F}{l} = \dfrac{\mu_0 I^2}{8\pi^2 a}}$ } +

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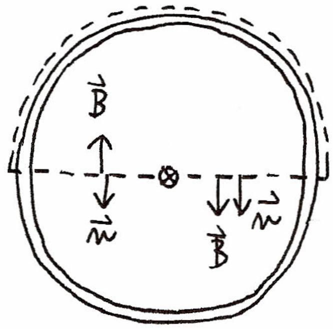
2 – napačna rešitev : integracija po ravnini SKOZI VODNIK



$$B = \frac{\mu_0 I}{2\pi r} \quad \text{znotraj cevi}$$

$$B = 0 \quad \text{zunaj cevi}$$

$$\vec{F}_m = \frac{1}{\mu_0} \oint \underbrace{\left[\vec{B}(\vec{B}\cdot\vec{n}) - \frac{1}{2}B^2\vec{n}\right]}_{\text{prispevek le znotraj, po ravnini}} dS$$

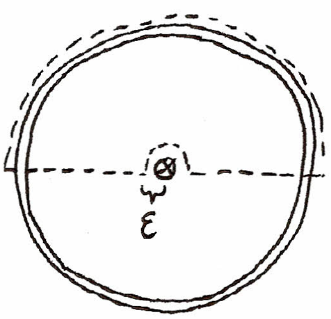
$$\left.\begin{array}{l} \text{levo: } \vec{B}(\vec{B}\cdot\vec{n}) = \vec{B}(-B) = B^2\vec{n} \\ \text{desno: } \vec{B}(\vec{B}\cdot\vec{n}) = \vec{B}\,B = B^2\vec{n} \end{array}\right\} \longrightarrow \vec{B}(\vec{B}\cdot\vec{n}) - \frac{1}{2}B^2\vec{n} = \frac{1}{2}B^2\vec{n}$$

$$\vec{F}_m = \frac{1}{2\mu_0}\vec{n} \int_{-a}^{a} \left(\frac{\mu_0 I}{2\pi r}\right)^2 \underbrace{l\,dr}_{dS} = \frac{\mu_0 I^2 l}{8\pi^2}\vec{n} \boxed{\int_{-a}^{a}\frac{dr}{r^2}} \longrightarrow \boxed{\neq -\frac{2}{a}} \; !!!$$

[integral je problematičen, saj gre preko SINGULARNOSTI

$$\int_{-a}^{a}\frac{dr}{r^2} = \left[\int_{-a}^{-\varepsilon}\frac{dr}{r^2} + \int_{\varepsilon}^{a}\frac{dr}{r^2}\right]_{\varepsilon\to 0} = \left(-\frac{2}{a} + \frac{2}{\varepsilon}\right)\Big|_{\varepsilon\to 0} \rightarrow \text{DIVERGIRA}$$

V resnici moramo SINGULARNOST OBITI!



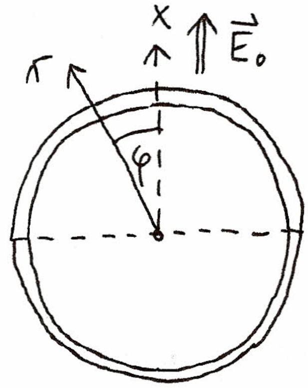
Integracija po mali polovici valja s polmerom ε vodi ravno do prispevka $\boxed{-\frac{2}{\varepsilon}}$ (na povsem enak način kot na prejšnji strani predstavljena pravilna rešitev), ki se odšteje s prej dobljenim $\frac{2}{\varepsilon}$!

– napačna rešitev 2 : [izračun sile, s katero deluje osrednji vodnik na polovico cevi

Ta rešitev spregleda prispevek ene polovice cevi na drugo!

Za vsako od obeh rešitev sem dodelil $\boxed{\frac{1}{2}}$ točke.

3



a)

+ $V_\infty(r,\varphi) = -E_0 r\cos\varphi \rightarrow$ robni pogoj v neskončnosti

$V(r,\varphi) = -E_0 r\cos\varphi + \frac{B}{r}\cos\varphi$

samo člena s $\cos\varphi$ preživita

+ $V(a,\varphi) = 0 = \left(-E_0 a + \frac{B}{a}\right)\cos\varphi \rightarrow$ robni pogoj na obodu

$$\Downarrow$$

$$\underline{B = E_0 a^2}$$

$$\underline{V(r,\varphi) = \left(\frac{a^2}{r} - r\right) E_0 \cos\varphi}$$ zunaj

(znotraj NI polja)

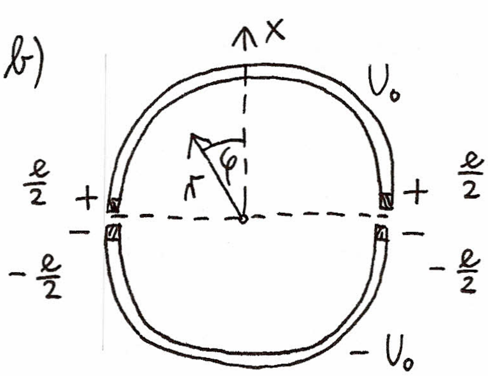
$\frac{1}{2}$

+ $\sigma = -\varepsilon_0 \vec{E}\big|_{obod} = -\varepsilon_0 \vec{\nabla} V\big|_{r=a} = -\varepsilon_0 E_0 \cos\varphi\left(-\frac{a^2}{r^2} - 1\right)\Big|_{r=a} =$

$= \underline{2\varepsilon_0 E_0 \cos\varphi} \rightarrow$ zgoraj pozitiven

+ $e = \int \sigma\, dS = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\varepsilon_0 E_0 \cos\varphi \cdot l a\, d\varphi = 2\varepsilon_0 E_0 l a \cdot 2 \Rightarrow \boxed{\frac{e}{l} = 4\varepsilon_0 E_0 a}$

b)



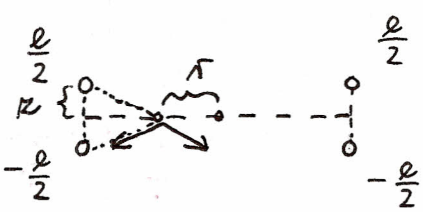
— $\underline{\text{intuitivna rešitev}}$: naboj se nabere vzdolž štirih PREMIC na robovih polovic cevi

$\frac{1}{4}$

↓

$\frac{1}{4}$ ker sta špranji majhni, pozitivna in negativna naboja sovpadeta, kar pomeni, da ni električnega polja in NI gradienta

$$\boxed{\frac{\partial E_x}{\partial x} = 0}$$



v to se lahko prepričamo tudi s seštevanjem štirih prispevkov in pošiljanjem $z \rightarrow 0$

$\frac{1}{2}$

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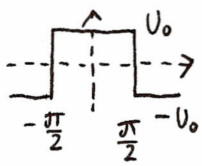
3 b) – formalna rešitev

rešujemo robni problem, kjer sta potenciala zgornje in spodnje polovice cevi V_0 in $-V_0$, V_0 pa moramo določiti iz skupnega naboja na vsaki polovici cevi

+

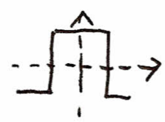
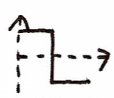
$$U = \sum_m A_m \cos m\varphi \, r^m \qquad \text{notranjost cevi} \qquad / \cdot \cos n\varphi,\ \int_{-\pi}^{\pi} d\varphi$$

$$U(a,\varphi) =$$  robni pogoj

+

enako kot naloga z vaj

desna stran: $$\int_{-\pi}^{\pi} d\varphi \cos m\varphi \cos n\varphi = \delta_{mn} \cdot \frac{1}{2} 2\pi = \underline{\pi \delta_{mn}}$$

leva stran: $\int_{-\pi}^{\pi} d\varphi$  $\cos n\varphi = 2 \int_0^{\pi}$  $\cos n\varphi \, d\varphi =$

$$= 2V_0 \Big[\underbrace{\int_0^{\pi/2} \cos n\varphi \, d\varphi}_{\frac{1}{n} \sin \frac{n\pi}{2}} - \underbrace{\int_{\pi/2}^{\pi} \cos n\varphi \, d\varphi}_{-\frac{1}{n} \sin \frac{n\pi}{2}} \Big] = \underline{\frac{4V_0}{n} \sin \frac{n\pi}{2}}$$

$\Downarrow$

$$A_n \pi a^n = \frac{4V_0}{n} \sin \frac{n\pi}{2} \Rightarrow \underline{A_n = \frac{4V_0}{\pi n} \frac{1}{a^n} \sin \frac{n\pi}{2}}$$

$\downarrow$

$$\underline{U(r,\varphi) = \frac{4V_0}{\pi} \sum_m \frac{1}{m} \left(\frac{r}{a}\right)^m \sin \frac{m\pi}{2} \cos m\varphi}$$

$\frac{1}{4}$

$$E_x\left(\varphi = \frac{\pi}{2}\right) = \frac{1}{r} \frac{\partial U}{\partial \varphi}\Big|_{\varphi = \frac{\pi}{2}}$$

$$\frac{\partial E_x}{\partial x}\Big|_{\varphi=\frac{\pi}{2}} = \frac{1}{r^2} \frac{\partial^2 U}{\partial \varphi^2}\Big|_{\varphi = \frac{\pi}{2}} = -\frac{4V_0}{\pi} \sum_m m \left(\frac{r}{a}\right)^m \underbrace{\sin \frac{m\pi}{2} \cos \frac{m\pi}{2}}_{\frac{1}{2} \sin m\pi = \boxed{0}}$$

$\Downarrow$

$$\boxed{\frac{\partial E_x}{\partial x}\Big|_{\varphi = \frac{\pi}{2}} = 0}$$

$\boxed{3}$ b) – ekvivalenca obeh rešitev (informativno)

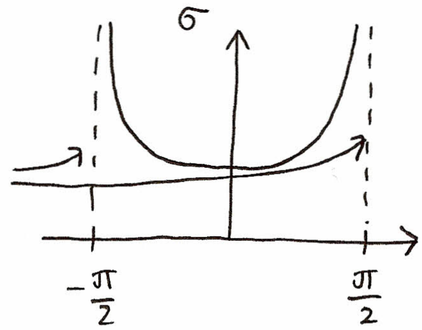
$$\sigma = +\varepsilon_0 \Bigg[\underbrace{\frac{\partial U}{\partial r}\Big|_{r=a^+}} - \underbrace{\frac{\partial U}{\partial r}\Big|_{r=a^-}} \Bigg] = -2\varepsilon_0 \frac{\partial U}{\partial r}\Big|_{r=a^-}$$

izkaže se, da sta samo nasprotna $\left[\left(\frac{r}{a}\right)^n \text{ vs } \left(\frac{a}{r}\right)^n \right]$

$$\sigma = -2\varepsilon_0 \frac{4U_0}{\pi a} \sum_n \left(\frac{a}{a}\right)^{n-1} \sin\frac{n\pi}{2} \cos n\varphi =$$

$$= -\frac{8\varepsilon_0 U_0}{\pi a} \underbrace{\sum_n \sin\frac{n\pi}{2} \cos n\varphi}_{-\sum\limits_{n \text{ lih}} (-1)^n \cos n\varphi \, = \, \frac{1}{2\cos\varphi}}$$

$$\underline{\sigma = -\frac{4\varepsilon_0 U_0}{\pi a} \frac{1}{\cos\varphi}}$$



zgornja polovica cevi

divergenci na robovih polovice cevi

funkcija NI integrabilna, integral je neskončen

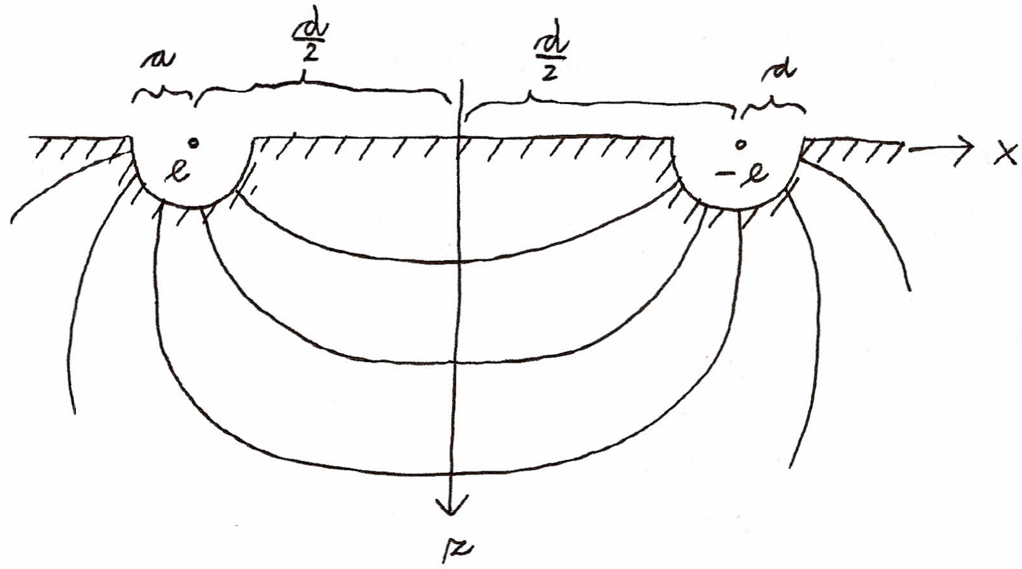
$\Downarrow$

pri KONČNEM naboju na polovici cevi se ves naboj nabere na robovih polovice cevi !

$\leftarrow$

z drugimi besedami, pri končnem naboju zaradi močnih divergenc $\sigma(\varphi)$ preide v par δ funkcij

4



+ $d \gg a \rightarrow$ silnice opisujejo polje DIPOLA

+ $$U(x,y,z) = \frac{e}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{(x+\frac{d}{2})^2+y^2+z^2}} - \frac{1}{\sqrt{(x-\frac{d}{2})^2+y^2+z^2}}\right]$$

$\frac{1}{4}$ $$U(-\tfrac{d}{2}+a,0,0) - U(\tfrac{d}{2}-a,0,0) = U_0 \rightarrow \text{napetost med kapicama}$$

$$\frac{e}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{d-a} - \frac{1}{d-a} + \frac{1}{a}\right] = U_0 \Rightarrow \underbrace{\frac{e}{4\pi\varepsilon_0} = \frac{U_0}{2\left[\frac{1}{a}-\frac{1}{d-a}\right]} \approx \frac{U_0 a}{2}}_{\text{predfaktor}}$$

– izračunajmo električni tok skozi ravnino $x=0$

+ $$j = -\sigma \frac{\partial U}{\partial x}\Big|_{x=0} = -\sigma \frac{U_0 a}{2}\left(-\frac{1}{2}\right)\left[\frac{2\frac{d}{2}}{\big[(\frac{d}{2})^2+\underbrace{y^2+z^2}_{\rho^2}\big]^{\frac{3}{2}}} - \frac{2(-\frac{d}{2})}{\big[(\frac{d}{2})^2+\underbrace{y^2+z^2}_{\rho^2}\big]^{\frac{3}{2}}}\right]$$

+ $$j = \frac{\sigma U_0 a d}{2}\,\frac{1}{\left[(\frac{d}{2})^2+\rho^2\right]^{\frac{3}{2}}} \rightarrow I = \int j\,dS,\quad dS = \pi\rho\,d\rho$$

+ $$I = \frac{\pi\sigma U_0 a d}{4}\int_0^\infty \frac{d\left[(\frac{d}{2})^2+\rho^2\right]}{\left[(\frac{d}{2})^2+\rho^2\right]^{\frac{3}{2}}} = \frac{\pi\sigma U_0 a d}{4}(-2)\frac{1}{\sqrt{(\frac{d}{2})^2+\rho^2}}\Bigg|_0^\infty = \pi\sigma U_0 a$$

+ $$I = \pi\sigma U_0 a = \frac{U_0}{R} \Rightarrow \boxed{R = \frac{1}{\pi\sigma a}}$$

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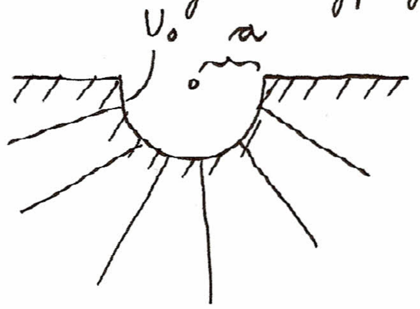
4 - alternativna pot (lažja, a bolj tricky)

- izračunajmo najprej upor za primer ENE KAPICE



silnice so tokrat radialne → MONOPOL

$+\left\{ U = \frac{e}{4\pi\varepsilon_0 r} \right.$

$+\left\{ U(r=a) = \frac{e}{4\pi\varepsilon_0 a} = U_0 \Rightarrow \frac{e}{4\pi\varepsilon_0} = a U_0 \right.$

$\downarrow$

$U(r) = \frac{U_0 a}{r}$

$+\left\{ j = -\sigma \frac{\partial U}{\partial r} = +\sigma U_0 a \frac{1}{r^2} \right.$

$I = \int j\, dS = \int \sigma U_0 a \frac{1}{r^2} \cdot r^2\, d\Omega = \sigma U_0 a \cdot \underbrace{2\pi}_{\text{polovica prostorskega kota}}$

$+\left\{ I = 2\pi\sigma a U_0 = \frac{U_0}{R_1} \Rightarrow R_1 = \frac{1}{2\pi\sigma a} \right.$

$\frac{1}{4}$ - primer DVEH KAPIC ≡ ZAPOREDNO „vezani" kapici (prva do neskončnosti, nato od neskončnosti do druge)

$\frac{1}{4}$ $R = R_1 + R_1 = 2R_1$

$\boxed{R = \frac{1}{\pi\sigma a}}$ ENAK rezultat!