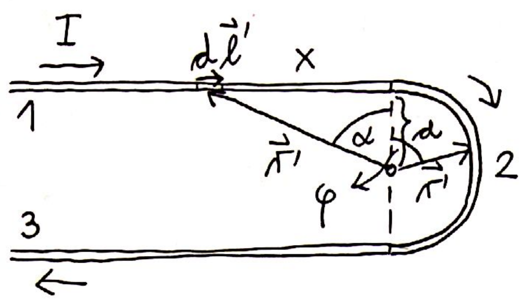
1. KOLOKVIJ

1 VODNIK V OBLIKI ČRKE „U"



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}')\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\, d^3\vec{r}'$$

$$\vec{j}(\vec{r}')\, d^3\vec{r}' = I\, d\vec{l}'$$

$\vec{r} = 0$ (izhodišče = središče)

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{\vec{r}'\times d\vec{l}'}{r'^3}$$

– segment 1

$\frac{1}{4}$

$$\vec{r}' = \begin{bmatrix} x \\ d \\ 0 \end{bmatrix}, \quad d\vec{l}' = \begin{bmatrix} dx \\ 0 \\ 0 \end{bmatrix} \Rightarrow \vec{r}'\times d\vec{l}' = \begin{bmatrix} 0 \\ 0 \\ -d\,dx \end{bmatrix}$$

$$r' = \sqrt{x^2+d^2}$$

$$B_1 = -\frac{\mu_0 I d}{4\pi}\int_{-\infty}^{0} \frac{dx}{(x^2+d^2)^{\frac{3}{2}}}$$

$\frac{1}{4}$

$$x = d\,\mathrm{tg}\,\alpha \Rightarrow dx = \frac{d}{\cos^2\alpha}\, d\alpha, \quad x^2+d^2 = d^2(1+\mathrm{tg}^2\alpha) = \frac{d^2}{\cos^2\alpha}$$

$$\int_{-\infty}^{0}\frac{dx}{(x^2+d^2)^{\frac{3}{2}}} = \frac{d}{d^3}\int_{-\frac{\pi}{2}}^{0}\frac{d\alpha}{\cos^2\alpha}\cos^3\alpha = \frac{1}{d^2}\sin\alpha\Big|_{-\frac{\pi}{2}}^{0} = \frac{1}{d^2}$$

$$B_1 = -\frac{\mu_0 I}{4\pi d} \quad \left(-\frac{1}{2}\cdot\frac{\mu_0 I}{2\pi d}\right)$$

→ rezultat lahko dobimo tudi kot POLOVICO rezultata za ravni vodnik, saj gre za simetrijsko točko

+ – segment 3 : $B_3 = B_1 = -\frac{\mu_0 I}{4\pi d}$

– segment 2

1

$\frac{1}{4}$

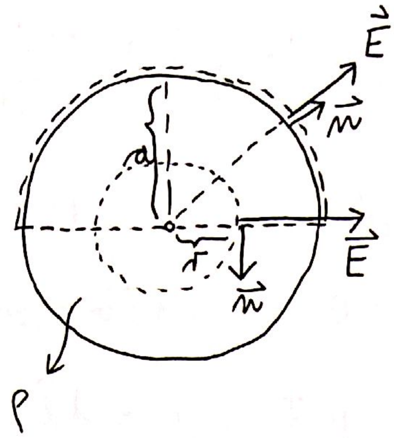
$$\vec{r}'\times d\vec{l}' = -\hat{e}_z d\cdot d\,d\varphi = -d^2\, d\varphi\, \hat{e}_z, \quad r' = d$$

$$B_2 = -\frac{\mu_0 I}{4\pi}\int_0^{\pi}\frac{d^2\,d\varphi}{d^3} = -\frac{\mu_0 I}{4\pi d}\pi$$

+ – skupaj : $B = B_1 + B_2 + B_3 = \boxed{-\frac{\mu_0 I}{4\pi d}(\pi+2)}$ kaže NAVZDOL!

↳ predznaki

2 SILA NA POLOVICO NABITEGA VALJA



$$+\left\{\begin{aligned} &\varepsilon_0 E\cdot \underbrace{2\pi r l}_{S} = \rho\cdot\underbrace{\pi r^2 l}_{V} \\ &\underline{E = \frac{\rho r}{2\varepsilon_0}} \end{aligned}\right.$$

→ Gauss za valj s polmerom r, ZNOTRAJ, $r \leq a$!

– obod valja

$$\vec{F}_e = \varepsilon_0 \oint\left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS$$

$\frac{1}{4}+$

$$\vec{E} \parallel \vec{n} \Rightarrow \vec{E}(\vec{E}\cdot\vec{n}) = E\vec{n}E = E^2\vec{n}$$

$$\vec{F}_{e1} = \frac{\varepsilon_0}{2}\int E^2\vec{n}\, dS$$

$$E = \frac{\rho a}{2\varepsilon_0}, \quad \vec{n} = \begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix}, \quad dS = a\, d\varphi\cdot l$$

$$\vec{F}_{e1} = \frac{\varepsilon_0}{2}\frac{\rho^2 a^2}{4\varepsilon_0^2} a l \int_0^{\pi}\begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix} d\varphi = \underline{\frac{\rho^2 a^3 l}{4\varepsilon_0}\hat{e}_y}$$

– presek valja (ploskev skozi središče)

$\frac{1}{4}+$

$$\vec{E} \perp \vec{n} \Rightarrow \vec{E}\cdot\vec{n} = 0$$

$$\vec{F}_{e2} = -\frac{\varepsilon_0}{2}\int E^2\vec{n}\, dS$$

$$E = \frac{\rho r}{2\varepsilon_0}, \quad \vec{n} = -\hat{e}_y, \quad dS = l\, dr$$

$$\vec{F}_{e2} = \frac{\varepsilon_0}{2}\frac{\rho^2 l}{4\varepsilon_0^2}\hat{e}_y\int_{-a}^{a} r^2\, dr = \frac{\rho^2 l}{8\varepsilon_0} 2\frac{a^3}{3}\hat{e}_y = \underline{\frac{\rho^2 a^3 l}{12\varepsilon_0}\hat{e}_y}$$

– skupaj

$$+\left\{\ \vec{F}_e = \vec{F}_{e1} + \vec{F}_{e2} = \frac{\rho^2 a^3 l}{3\varepsilon_0}\hat{e}_y, \quad \boxed{\frac{\vec{F}_e}{l} = \frac{\rho^2 a^3}{3\varepsilon_0}\hat{e}_y}\right.$$

kaže NAVZGOR za zgornjo polovico!

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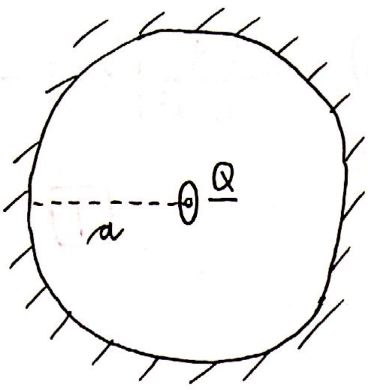
3 TOČKASTI KVADRUPOL V OKROGLI VOTLINI PREVODNIKA



$$\underline{Q} = Q \begin{bmatrix} -\frac{1}{2} & & \\ & -\frac{1}{2} & \\ & & 1 \end{bmatrix}$$

$$U_Q(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{2} \sum_{ij} \frac{Q_{ij} r_i r_j}{r^5}$$

1/4

$$U_Q(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{2} Q \frac{1}{r^5} \Big(\underbrace{-\frac{1}{2}x^2 - \frac{1}{2}y^2}_{-\frac{1}{2} r^2 \sin^2\vartheta} + \underbrace{z^2}_{r^2\cos^2\vartheta} \Big) = \frac{Q}{16\pi\varepsilon_0 r^3} \big(\underbrace{2\cos^2\vartheta - \sin^2\vartheta}_{3\cos^2\vartheta - 1} \big)$$

$$U_Q(\vec{r}) = \frac{Q}{8\pi\varepsilon_0 r^3} \cdot \frac{3\cos^2\vartheta - 1}{2} = \underline{\frac{Q}{8\pi\varepsilon_0 r^3} P_2(\cos\vartheta)}$$

– rešitev v votlini

$$U(r, \vartheta) = \sum_l \left[A_l r^l + B_l r^{-(l+1)} \right] P_l(\cos\vartheta)$$

1/4

1. RP : $U(r \to 0, \vartheta) = U_Q(r, \vartheta) \Rightarrow$ imamo le $\underline{l = 2}$

$$\underline{B_2 = \frac{Q}{8\pi\varepsilon_0}}$$

$$U(r, \vartheta) = \left[A_2 r^2 + \frac{Q}{8\pi\varepsilon_0 r^3} \right] P_2(\cos\vartheta)$$

+

2. RP : $U(r = a, \vartheta) = 0 \Rightarrow \underline{A_2 = -\frac{Q}{8\pi\varepsilon_0 a^5}}$

$$\boxed{U(r, \vartheta) = \frac{Q}{8\pi\varepsilon_0} \left(\frac{1}{r^3} - \frac{r^2}{a^5} \right) P_2(\cos\vartheta)}$$

– inducirani naboj na površini

+

$$\sigma_{IND} = -\varepsilon_0 \left(-\frac{\partial U}{\partial r} \right)_{r=a} = \frac{Q}{8\pi\varepsilon_0} \left(-\frac{3}{a^4} - \frac{2a}{a^5} \right) P_2(\cos\vartheta) = \boxed{-\frac{5Q}{8\pi\varepsilon_0 a^4} P_2(\cos\vartheta)}$$

$\frac{1}{4}$

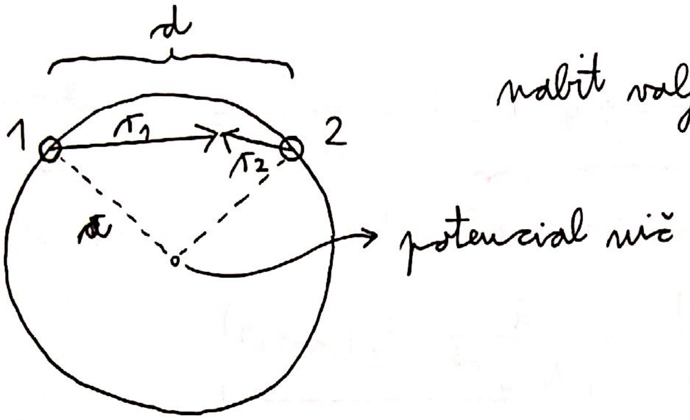
$$e_{IND} = \int \sigma_{IND}\, dS = -\frac{5Q}{8\pi\varepsilon_0 a^4} \int_{\cos\vartheta=-1}^{1} P_2(\cos\vartheta)\, 2\pi a^2\, d(\cos\vartheta) =$$

$$= -\frac{5Q}{4\varepsilon_0 a^2} \int_{\cos\vartheta=-1}^{1} \frac{1}{2}\left(3\cos^2\vartheta - 1\right) d(\cos\vartheta) = -\frac{5Q}{8\varepsilon_0 a^2}\left[3\cdot\frac{2}{3} - 2\right] = 0$$

$$\boxed{e_{IND} = 0}$$

$\boxed{1}$

$\boxed{4}$ UPOR PREVODNE PLOŠČICE



nabit valj: $e = 2\pi\varepsilon_0 E l r$

$$E = \frac{e}{2\pi\varepsilon_0 l r} = \frac{\lambda}{2\pi\varepsilon_0 r} = -\frac{\partial U}{\partial r}$$

$$+\begin{cases} U_1 = -\dfrac{\lambda}{2\pi\varepsilon_0} \ln\dfrac{r_1}{a} \\[2ex] U_2 = \dfrac{\lambda}{2\pi\varepsilon_0} \ln\dfrac{r_2}{a} \end{cases}$$

$$U(r_1, r_2) = U_1 + U_2 = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{r_2}{r_1}$$

$\frac{1}{4}+$

elektroda 1: $U(\delta, d-\delta) = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{d-\delta}{\delta} \approx \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{d}{\delta}$

elektroda 2: $U(d-\delta, \delta) = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{\delta}{d-\delta} \approx -\frac{\lambda}{2\pi\varepsilon_0} \ln\frac{d}{\delta}$

razlika: $U_0 = +\frac{\lambda}{2\pi\varepsilon_0} 2\ln\frac{d}{\delta}$ (tok teče od 1 do 2)

$\frac{1}{4}$

rob elektrode 1: $E_\delta = \frac{\lambda}{2\pi\varepsilon_0 \delta} = \frac{U_0}{2\delta \ln\frac{d}{\delta}}$

$$I = \sigma E_\delta \cdot \pi\delta D = \frac{\pi\sigma U_0 D}{2\ln\frac{d}{\delta}} \Rightarrow \boxed{R = \frac{2\ln\frac{d}{\delta}}{\pi\sigma D}}$$

$\frac{1}{4}$

Analogija s primerom dveh valjev je možna, ker so ekvipotencialne ploskve krožnice, prav tako pa so krožnice tudi silnice, tako da je na robu ploščice robni pogoj izpolnjen! $\boxed{1}$