Elektromagnetno polje: 2. kolokvij

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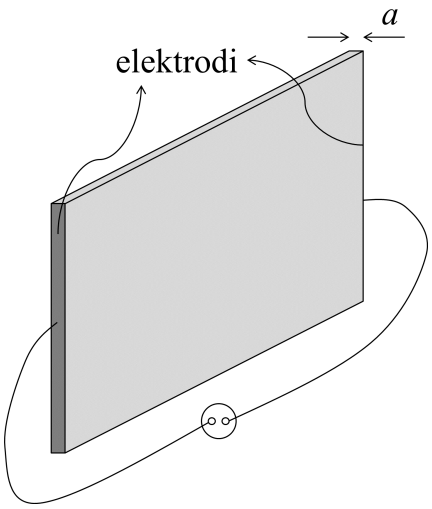
1. naloga

Skrajni ploskvi dolgega in širokega kovinskega traku debeline a premažemo z idealno prevodnima elektrodama in mednju priključimo vir izmenične napetosti krožne frekvence ω, kakor prikazuje slika. Debelina traku je precej manjša od preostalih dveh razsežnosti traku, specifična prevodnost kovine pa je σ.



a) Pokaži, da impedanco traku v opisani postavitvi lahko zapišemo kot

$$Z = R_0 \frac{ka/2}{\mathrm{th}\,(ka/2)},$$

kjer je $k = (1+i)\sqrt{\mu_0\sigma\omega/2}$ kompleksni valovni vektor, R_0 statični upor traku, th pa označuje hiperbolični tangens.

b) Izračunaj faktor, za katerega se pri *visokih* frekvencah (zaradi kožnega pojava) upor traku poveča glede na njegov statični upor.

2. naloga

Dolg vodoraven valj iz snovi z dielektrično konstanto ε postavimo v navpično homogeno električno polje jakosti E_0, zaradi česar se valj električno polarizira. Izračunaj polarizacijo valja.

3. naloga

Križna antena je sestavljena iz dveh enakih vodoravnih prečk, ki sta pravokotni druga na drugo, njuni središči pa sovpadata. Prečki sta izolirani druga od druge, napajanje ene prečke pa je za četrt nihaja zamaknjeno za napajanjem druge prečke, tako da tokova v prečkah lahko zapišemo kot $I_1 = I_0 \cos\omega t$ in $I_2 = I_0 \sin\omega t$, kjer je ω krožna frekvenca napajanja. Dolžina posamezne prečke l je majhna glede na valovno dolžino λ valovanja, ki ga antena oddaja, tako da prečki lahko obravnavaš kot Hertzova dipola. Slika prikazuje takšno anteno pritrjeno na navpični nosilec.

a) Izračunaj časovno povprečje celotnega izsevanega energijskega toka za takšno anteno.

b) Dobljeni rezultat primerjaj z ustreznim rezultatom za anteno, ki bi vsebovala le eno izmed obeh prečk, se pravi za običajni Hertzov dipol.

4. naloga (za dodatne točke)

Izračunaj najnižjo frekvenco transverzalnih magnetnih (TM) valov, ki se lahko širijo po praznem koaksialnem kablu s polmerom žile a in notranjim polmerom plašča $2a$. Rezultat izrazi s c_0/a, kjer je c_0 hitrost svetlobe v vakuumu.

Predfaktorje v rezultatu določi numerično. Vse potrebne matematične pripomočke poišči v matematičnem priročniku.

Matematični pripomoček:
Rešitve Laplaceove enačbe $\nabla^2 U(r,\varphi) = 0$ v polarnih koordinatah:

$$U(r,\varphi) = A_0 + B_0 \ln r + \sum_{m=1}^{\infty} \left(A_m r^m + B_m r^{-m}\right) \cos(m\varphi) + \sum_{m=1}^{\infty} \left(C_m r^m + D_m r^{-m}\right) \sin(m\varphi).$$

Čas reševanja: 90 minut.
Dovoljeni pripomočki: podani spisek enačb, matematični priročnik, kalkulator.
Rešitve nalog, ocene ter kraj in čas ogleda kolokvija bodo objavljeni na spletni strani `http://www-f5.ijs.si/emp-2016-2017.html`.

Electromagnetic field: 2nd written examination

(20th of November 2017 at 15:00)

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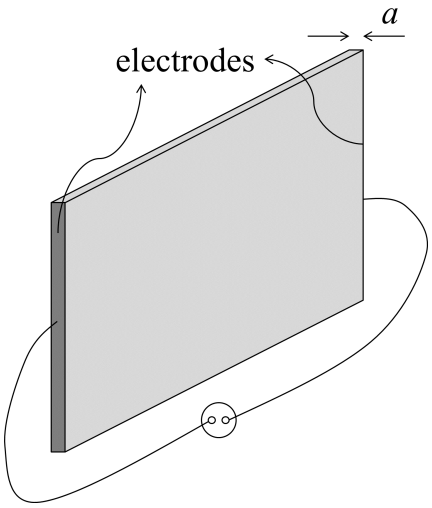
Problem 1

Ideally conducting electrodes cover the boundary surfaces of a long and wide metallic ribbon of thickness a, as shown in the figure. An alternating voltage of angular frequency ω is applied between the electrodes. The thickness of the ribbon is much smaller than the remaining two dimensions of the ribbon, while the specific conductivity of the metal is σ.



a) Show that the impedance of the ribbon in the described situation can be expressed as

$$Z = R_0 \frac{ka/2}{\mathrm{th}\,(ka/2)},$$

where $k = (1+i)\sqrt{\mu_0\sigma\omega/2}$ is the complex wave vector, R_0 is the static resistivity of the ribbon, and th denotes the hyperbolic tangens.

b) Calculate the factor, by which the resistance of the ribbon increases at *high* frequencies (due to the skin effect) with respect to its static resistance.

Problem 2

When a long horizontal cylinder made of the material with the dielectric constant ε is placed in the vertical uniform electric field E_0, it gets electrically polarized. Determine the polarization of the cylinder.

Problem 3

A "cross" antenna is made from two equal horizontal rods perpendicular to each other, so that their centers coincide. The rods are isolated from each other. They are fed with electric currents shifted by a quarter of the cycle with respect to each other, so that the currents in the rods are written as $I_1 = I_0 \cos\omega t$ and $I_2 = I_0 \sin\omega t$, where ω is the angular frequency. The length l of each rod is short with respect to the wave length λ of the transmitted waves, i.e., the rods can be treated as Hertzian dipoles. The figure shows such an antenna mounted on the vertical bar.

a) Determine the temporal average of the total transmitted energy flux for such an antenna.

b) Compare the obtained result to the corresponding result for the antenna, which would contain only a single rod, i.e., to the usual Hertzian dipole.

Problem 4 (for bonus points)

Determine the lowest frequency of the transverse magnetic (TM) waves that can propagate along the empty coaxial cable with the inner and outer radiuses a and $2a$, respectively. Express the result in terms of c_0/a, where c_0 is the speed of light in vacuum.

Evaluate the prefactors in the result numerically. Find all the necessary mathematical tools in the mathematical handbook.

Mathematical tool:
Rešitve Laplaceove enačbe $\nabla^2 U(r,\varphi) = 0$ v polarnih koordinatah:

$$U(r,\varphi) = A_0 + B_0 \ln r + \sum_{m=1}^{\infty} \left(A_m r^m + B_m r^{-m}\right) \cos(m\varphi) + \sum_{m=1}^{\infty} \left(C_m r^m + D_m r^{-m}\right) \sin(m\varphi).$$

Duration of examination: 90 minutes.
Allowed accessories: given list of equations, mathematical handbook, calculator.
Solutions of the problems, scores, and place and time of the access to the assessed exams will be announced on the website
`http://www-f5.ijs.si/emp-2016-2017.html`.
