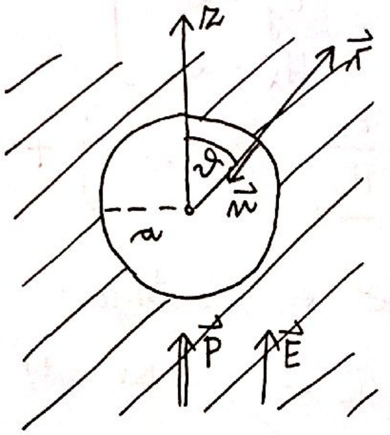
EMP : 2. KOLOKVIJ

1



$\sigma_v = \vec{P}\cdot\vec{n} = \ominus P\cos\vartheta$ → normala navznoter

↳ le členi s $\cos\vartheta$

$$U(r,\vartheta) = \begin{cases} A r\cos\vartheta, & r<a \\ \frac{B}{r^2}\cos\vartheta - \underbrace{Er\cos\vartheta}_{\text{homogeno polje daleč od votline}}, & r>a \end{cases}$$

robna pogoja:

1) zveznost U ⇒ $\underbrace{Aa = \frac{B}{a^2} - Ea}$ ⇒ $A+E = \frac{B}{a^3}$

2) Gauss za $E_\perp$ ⇒ $\frac{\sigma_v}{\varepsilon_0} = E_\perp(r=a^+) - E_\perp(r=a^-)$

$$\underbrace{-\frac{P}{\varepsilon_0} = \frac{2B}{a^3} + E + A} \Rightarrow -\frac{P}{\varepsilon_0} = \frac{3B}{a^3}, \quad \underbrace{B = -\frac{Pa^3}{3\varepsilon_0}}, \quad \underbrace{A+E = -\frac{P}{3\varepsilon_0}}$$

$U(r<a) = Ar\cos\vartheta = Az$ ⇒ homogeno polje

$$E_A = -\frac{\partial U}{\partial z} = -A = \boxed{E + \frac{P}{3\varepsilon_0}}$$

$$U(r>a) = -\underbrace{\frac{Pa^3\cos\vartheta}{3\varepsilon_0 r^2}}_{\text{dipolni člen}} - Er\cos\vartheta$$

$$\| \quad \frac{p\cos\vartheta}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{p = -\frac{4\pi a^3}{3}P}$$

kaže v smeri NASPROTI $\vec{P}$

– morebitno reševanje z dielektričnostjo (alternativa, se zdi)

robna pogoja:

1) $Aa = \frac{B}{a^2} - Ea$

2) $-A = \varepsilon\left(\frac{2B}{a^3} + E\right)$

primerjava s prejšnjim 2) vodi do

$$\frac{P}{\varepsilon_0} + \left(\frac{2B}{a^3}+E\right) = \varepsilon\left(\frac{2B}{a^3}+E\right)$$

$$\Downarrow$$

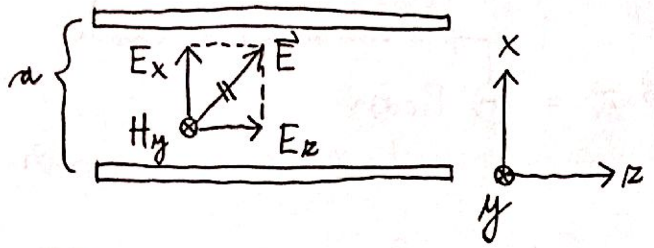
$$\underbrace{P = \varepsilon_0(\varepsilon-1)\left(E + \frac{2B}{a^3}\right)}$$

↓

[ne velja torej zveza $P = \varepsilon_0(\varepsilon-1)E$, saj P v nalogi NI POSLEDICA polja E, ampak obratno!

– [vpeljati HKRATI σ_v in ε NE pride v poštev!

$\boxed{2}$



TM način: $\left.\begin{array}{l} H_z = 0 \\ E_y = 0 \\ H_x = 0 \end{array}\right\} +$

$Z = \dfrac{E_\perp}{H_\parallel} = \dfrac{E_x}{H_y}$ iščemo! $\}+$

$$\Big(\nabla_\perp^2 + \underbrace{\frac{\omega^2}{c_0^2} - k^2}_{\varkappa^2}\Big) E_z = 0$$

$\varkappa^2 \Rightarrow \left.\begin{array}{l} \omega^2 = c_0^2 (k^2 + \varkappa^2) \\ \omega_{min} = c_0 \varkappa = \omega_0 \end{array}\right\} +$

$\dfrac{\partial}{\partial y} = 0 \leftarrow$ simetrija

$\dfrac{\partial}{\partial z} = ik \leftarrow e^{i(kz-\omega t)}$

$\dfrac{\partial}{\partial t} = -i\omega$

$$+\left\{\begin{array}{l} \vec\nabla \times \vec H = \dfrac{\partial \vec D}{\partial t} = -i\omega\varepsilon_0 \vec E \\[2ex] \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z}\end{bmatrix} \times \begin{bmatrix} 0 \\ H_y \\ 0 \end{bmatrix} = -i\omega\varepsilon_0 \begin{bmatrix} E_x \\ 0 \\ E_z \end{bmatrix} \end{array}\right.$$

$\xrightarrow{\text{prva komponenta}}$

$$\left.\begin{array}{l} -\dfrac{\partial}{\partial z} H_y = -i\omega\varepsilon_0 E_x \\ ik H_y = i\omega\varepsilon_0 E_x \\ Z = \dfrac{E_x}{H_y} = \dfrac{k}{\omega\varepsilon_0} \end{array}\right\} \frac{1}{4}$$

$$\frac{1}{4}\left\{\begin{array}{l} Z = \dfrac{\sqrt{\frac{\omega^2}{c_0^2} - \varkappa^2}}{\varepsilon_0 \omega} = \dfrac{\sqrt{\omega^2 - c_0^2\varkappa^2}}{c_0 \varepsilon_0 \omega} = \\[2ex] = \dfrac{\sqrt{\omega^2 - \omega_0^2}}{\omega} \underbrace{\dfrac{\sqrt{\varepsilon_0\mu_0}}{\varepsilon_0}}_{Z_0} = \boxed{Z_0 \sqrt{1 - \frac{\omega_0^2}{\omega^2}}} \end{array}\right.$$

QED

$\boxed{1}$

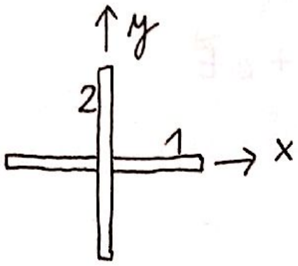
$\boxed{4}$ do konca

$$\underbrace{\begin{array}{l} (k^2 c_0^2 - \omega^2) E_\perp = i \dfrac{ne}{\varepsilon_0} \omega v_\perp \\ -\omega^2 E_\parallel = i \dfrac{ne}{\varepsilon_0} \omega v_\parallel \end{array}}_{\text{novi enačbi}} \begin{array}{l} \Rightarrow \\ \Rightarrow \end{array} \overbrace{\left[\begin{array}{l} i\omega v_\perp = \omega_c v_\parallel - i \dfrac{\omega_p^2 \omega}{k^2 c_0^2 - \omega^2} v_\perp \\ i\omega v_\parallel = -\omega_c v_\perp - i \dfrac{\omega_p^2 \omega}{-\omega^2} v_\parallel \end{array}\right.}^{\text{iz starih dveh enačb}}$$

$$\left.\begin{array}{l} i\omega v_\perp \left(1 + \dfrac{\omega_p^2}{k^2 c_0^2 - \omega^2}\right) = \omega_c v_\parallel \\ i\omega v_\parallel \left(1 - \dfrac{\omega_p^2}{\omega^2}\right) = -\omega_c v_\perp \end{array}\right\} \Rightarrow \boxed{\omega^2 \left(1 + \frac{\omega_p^2}{k^2 c_0^2 - \omega^2}\right)\left(1 - \frac{\omega_p^2}{\omega^2}\right) = \omega_c^2}$$

disperzijska relacija v zapleteni obliki

3



$S = \pi a^2$

$$I_1 = I_0 \cos\omega t \,, \quad \vec{p}_{m1} = I_1 \pi a^2 \hat{e}_y$$
$$I_2 = I_0 \sin\omega t \,, \quad \vec{p}_{m2} = I_2 \pi a^2 \hat{e}_x$$

$+$

$$\vec{B} = -\frac{\mu_0}{4\pi c_0^2 r}\, \hat{e}_r \times \left(\hat{e}_r \times \ddot{\vec{p}}_m\underbrace{\left(t - \frac{r}{c_0}\right)}_{t_r}\right)$$

$$\vec{p}_m = \vec{p}_{m1} + \vec{p}_{m2}$$

$$\ddot{\vec{p}}_m = -I_0 \pi a^2 \omega^2 \left(\hat{e}_y \cos\omega t_r + \hat{e}_x \sin\omega t_r\right)$$

$$+\left\{\ \vec{B} = \frac{\mu_0 I_0 \pi a^2 \omega^2}{4\pi c_0^2 r}\, \underbrace{\hat{e}_r \times \left[\hat{e}_r \times \left(\hat{e}_y \cos\omega t_r + \hat{e}_x \sin\omega t_r\right)\right]}_{\text{vektorski del}}\right.$$

$\frac{1}{4}$

a1) $\hat{e}_r = \hat{e}_z$

$$\hat{e}_z \times \left[\hat{e}_z \times \left(\hat{e}_y \cos\omega t_r + \hat{e}_x \sin\omega t_r\right)\right] = -\underbrace{\left(\hat{e}_x \sin\omega t_r + \hat{e}_y \cos\omega t_r\right)}_{\substack{\text{fazni zamik } \frac{\pi}{2} \\ \text{v pravokotnih smereh}}}$$

$$\hat{e}_z \times \hat{e}_y = -\hat{e}_x \qquad \hat{e}_z \times \hat{e}_x = \hat{e}_y$$
$$\hat{e}_z \times (-\hat{e}_x) = -\hat{e}_y \qquad \hat{e}_z \times \hat{e}_y = -\hat{e}_x$$

↓

KROŽNA polarizacija

$\frac{1}{4}$

a2) $\hat{e}_r = \hat{e}_x \cos\varphi + \hat{e}_y \sin\varphi$

$$\begin{bmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{bmatrix} \times \underbrace{\left(\begin{bmatrix}\cos\varphi\\ \sin\varphi\\ 0\end{bmatrix} \times \begin{bmatrix}\sin\omega t_r\\ \cos\omega t_r\\ 0\end{bmatrix}\right)}_{\begin{bmatrix}0\\ 0\\ \underbrace{\cos\varphi\cos\omega t_r - \sin\varphi\sin\omega t_r}_{\cos(\omega t_r + \varphi)}\end{bmatrix}} = \begin{bmatrix}\sin\varphi\cos(\omega t_r+\varphi)\\ -\cos\varphi\cos(\omega t_r+\varphi)\\ 0\end{bmatrix} = \cos(\omega t_r+\varphi)\underbrace{\begin{bmatrix}\sin\varphi\\ -\cos\varphi\\ 0\end{bmatrix}}$$

LINEARNA polarizacija

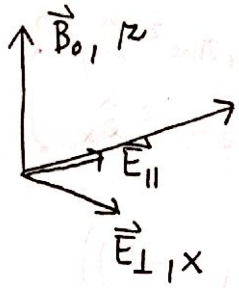
$\frac{1}{4}$

b) $$\frac{\langle P_{a1}\rangle}{\langle P_{a2}\rangle} = \frac{\langle B_{a1}^2\rangle}{\langle B_{a2}^2\rangle} = \frac{\langle \overbrace{\sin^2\omega t_r + \cos^2\omega t_r}^{1}\rangle}{\langle \cos^2(\omega t_r+\varphi)\underbrace{(\sin^2\varphi + \cos^2\varphi)}_{1}\rangle} =$$

$$= \frac{1}{\langle \cos^2(\omega t_r+\varphi)\rangle} = \frac{1}{\frac{1}{2}} = \boxed{2}$$

$\boxed{1}$

$\boxed{4}$



$$\underbrace{e^{i(kz-\omega t)}} \quad \vec\nabla = i\vec k, \qquad \frac{\partial}{\partial t} = -i\omega$$

$$\omega_c = -\frac{eB_0}{m}$$

$$\omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$$

$\underbrace{\vec E, \vec B}$ → polji v valovanju

$$m\dot{\vec v} = e\vec v\times\vec B_0 + e\vec E$$

$$-i\omega\vec v \qquad \vec v = \begin{bmatrix} v_\perp \\ v_\parallel \\ 0 \end{bmatrix}$$

$$e\vec v\times\vec B_0 = eB_0\begin{bmatrix} v_\perp \\ v_\parallel \\ 0 \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = eB_0\begin{bmatrix} v_\parallel \\ -v_\perp \\ 0 \end{bmatrix}$$

$$\begin{cases} i\omega v_\perp = \omega_c v_\parallel - \frac{e}{m}E_\perp \\ i\omega v_\parallel = -\omega_c v_\perp - \frac{e}{m}E_\parallel \end{cases}$$

→ sklopljeni enačbi, treba je najti še zvezo med $\vec E$ in $\vec v$!

1) $$\vec\nabla\times\vec B = \mu_0\vec j + \mu_0\varepsilon_0\frac{\partial\vec E}{\partial t}$$

$$\underbrace{\vec\nabla\times(\vec\nabla\times\vec B)}_{\vec\nabla(\vec\nabla\cdot\vec B)-\nabla^2\vec B} = \mu_0\underbrace{\vec\nabla\times}_{i\vec k\times}\vec j - i\mu_0\varepsilon_0\omega\,\vec\nabla\times\vec E, \qquad \vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t} = i\omega\vec B$$

$$\underbrace{\vec\nabla(\vec\nabla\cdot\vec B)}_{0} - \underbrace{\nabla^2\vec B}_{k^2\vec B}$$

$$\vec k = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \vec j = \begin{bmatrix} j_\perp \\ j_\parallel \\ 0 \end{bmatrix}$$

$$k^2\vec B = i\mu_0\vec k\times\vec j + \overbrace{\mu_0\varepsilon_0\omega^2}^{\frac{\omega^2}{c_0^2}}\vec B \Rightarrow \underbrace{\left(k^2 - \frac{\omega^2}{c_0^2}\right)\vec B = i\mu_0\vec k\times\vec j = -i\mu_0 k\begin{bmatrix} 0 \\ 0 \\ j_\perp \end{bmatrix}}$$

obstaja le B_z, kar sledi že iz $\vec\nabla\cdot\vec B = i\vec k\cdot\vec B = 0$

2) $$\vec\nabla\times\vec E = -\frac{\partial\vec B}{\partial t}$$

$$\vec k\times\vec E = \omega\vec B$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\times\begin{bmatrix} E_\perp \\ E_\parallel \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -E_\perp \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \omega B_z \end{bmatrix} \Rightarrow kE_\perp = -\omega B_z$$

$$\left(k^2 - \frac{\omega^2}{c_0^2}\right)\frac{E_\perp}{\omega} = i\mu_0 j_\perp = i\mu_0 n e v_\perp$$

$$\underbrace{\left(k^2 - \frac{\omega^2}{c_0^2}\right)E_\perp = i\mu_0\omega n e v_\perp}$$

3) $$\vec\nabla\cdot\vec E = \frac{\rho}{\varepsilon_0}, \quad \dot\rho + \vec\nabla\cdot\vec j = 0$$

$$\vec\nabla\cdot\left(\dot{\vec E} + \frac{1}{\varepsilon_0}\vec j\right) = 0$$

$$\vec k\cdot\left(-i\omega\vec E + \frac{1}{\varepsilon_0}\vec j\right) = 0 \Rightarrow +i\varepsilon_0\omega E_\parallel = j_\parallel = n e v_\parallel$$

$$\underbrace{-\frac{\omega^2}{c_0^2}E_\parallel = i\mu_0\omega n e v_\parallel}$$

željeni dve enačbi!