EMP : 2. KOLOKVIJ

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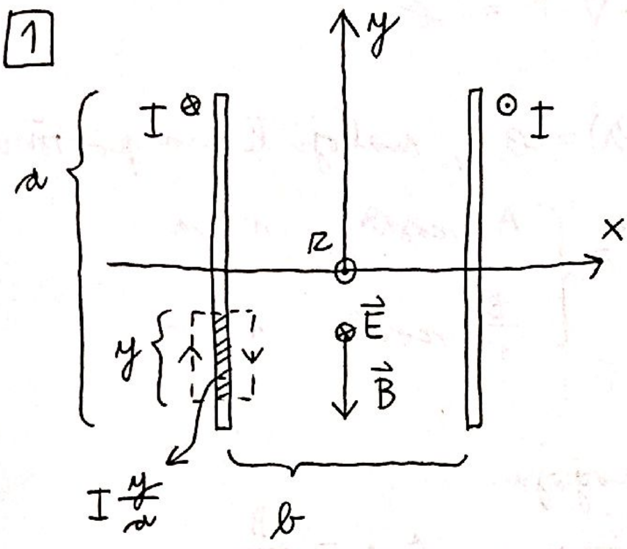

$$I(t) = I\left(1-\frac{t}{t_0}\right) \Rightarrow \dot{I} = -\frac{I}{t_0}$$

a) $\vec{B} = 0$ zunaj, $\vec{B} \parallel -y$ znotraj

1/4 Ampere po zanki:

$$\mu_0 I \frac{y}{a} = -B_y y \Rightarrow \underline{B_y = -\frac{\mu_0 I}{a}}$$

1/4

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\downarrow$ y komponenta (le $B_y \neq 0$)

$$\underbrace{\frac{\partial E_x}{\partial z}}_{0 \text{ (vzdolž } z \text{ homogeno)}} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} = \frac{\mu_0}{a}\dot{I}$$

$$\boxed{E_z = -\frac{\mu_0 \dot{I}}{a} x}$$

b) 1/4

$$\vec{P} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$P_x = -\frac{1}{\mu_0} E_z B_y = -\frac{1}{\mu_0}\frac{\mu_0 \dot{I} x}{a}\frac{\mu_0 I}{a} = -\frac{\mu_0 \dot{I} I}{a^2} x$$

$$\int \vec{P} \cdot d\vec{S} = \left[P_x\left(\frac{b}{2}\right) - P_x\left(-\frac{b}{2}\right)\right] a l = -\frac{\mu_0 \dot{I} I}{a^2} b a l = \boxed{-\mu_0 \dot{I} I \frac{bl}{a}}$$

> 0, ker $\dot{I} < 0$

c) 1/4

$$W_m = \frac{B_y^2}{2\mu_0} \cdot b a l = \frac{\mu_0 I^2}{2}\frac{bl}{a}$$

$$\boxed{\frac{dW_m}{dt} = \mu_0 \dot{I} I \frac{bl}{a}} = -\int \vec{P} \cdot d\vec{S} \Rightarrow$$ zmanjševanje magnetne energije vodi do energijskega toka

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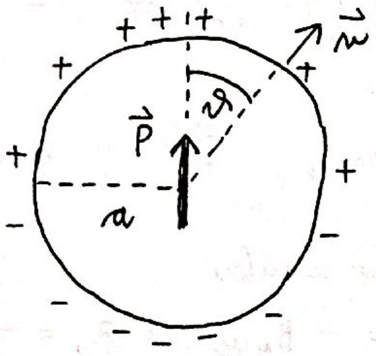
2 – *neprerezana krogla*



$$+\left\{\ \sigma_v = \vec{P}\cdot\vec{n} = P\cos\vartheta\right.$$

$$\rho_v = -\vec{\nabla}\cdot\vec{P} = 0$$

$\nabla^2 U(r,\vartheta) = 0$, naboji le na površini

$$+\left\{\ U(r,\vartheta) = \begin{cases} A r\cos\vartheta, & r<a \\ \dfrac{B}{r^2}\cos\vartheta, & r>a \end{cases}\right.$$

robna pogoja:

$$+\left\{\ 1)\ U \text{ zvezen}:\quad A a = \frac{B}{a^2}\right.$$

$$+\left\{\ 2)\ E(r=a^+) - E(r=a^-) = \frac{\sigma_v}{\varepsilon_0}\right.$$

$$\left(\frac{2B}{a^3} + A\right)\cos\vartheta = \frac{P\cos\vartheta}{\varepsilon_0}$$

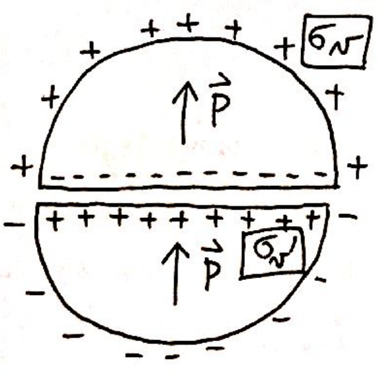
3/4

$$+\left\{\ \frac{2B}{a^3} + A = 3A = \frac{P}{\varepsilon_0} \Rightarrow \underline{A = \frac{P}{3\varepsilon_0}}\right.$$

$$U(r<a,\vartheta) = \frac{P}{3\varepsilon_0} r\cos\vartheta = \frac{P}{3\varepsilon_0} z$$

$$+\left\{\ \vec{E}(r<a) = -\frac{\partial U}{\partial z}\hat{e}_z = \boxed{-\frac{\vec{P}}{3\varepsilon_0}}\right.$$

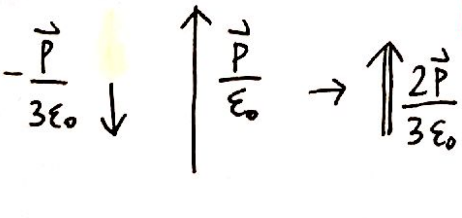
– *prerezana krogla*



σ_v povzroči v špranji polje $\vec{E}(r<a)$

$$+\left\{\ \sigma_v' = P \Rightarrow \underline{\vec{E}' = \frac{\vec{P}}{\varepsilon_0}}\ \text{(ploščati kondenzator)}\right.$$

$$+\left\{\ \vec{E}_{\text{ŠPRANJA}} = \vec{E} + \vec{E}' = -\frac{\vec{P}}{3\varepsilon_0} + \frac{\vec{P}}{\varepsilon_0} = \boxed{\frac{2\vec{P}}{3\varepsilon_0}}\right.$$



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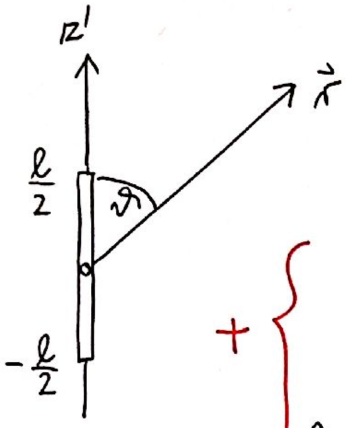
$\boxed{3}$ $\underline{\vec{B}(\vec{r},t) = -\frac{\mu_0}{4\pi r c_0}\,\hat{e}_r \times \frac{\partial}{\partial t}\int \vec{j}\left(\vec{r}', t_r + \hat{e}_r\cdot\frac{\vec{r}'}{c_0}\right) d^3\vec{r}'}$ s seznama enačb



$$d^3\vec{r}' = S'\,dz'$$

$$\vec{j}\,d^3\vec{r}' = I\,dz'\,\hat{e}_{z'}$$

$$\hat{e}_r\cdot\frac{\vec{r}'}{c_0} = \cos\vartheta\cdot\frac{z'}{c_0}$$

+

$$\hat{e}_r\times\frac{\partial}{\partial t}\int\vec{j}\,d^3\vec{r}' = \underbrace{\hat{e}_r\times\hat{e}_{z'}}_{-\hat{e}_\varphi\boxed{\sin\vartheta}}\;\frac{\partial}{\partial t}\int_{-\frac{l}{2}}^{\frac{l}{2}} I\left(z', t_r + \frac{z'}{c_0}\cos\vartheta\right)dz'$$

a) $I(z',t) = I_0\,e^{i(kz'-\omega t)}$

+

$$\int_{-\frac{l}{2}}^{\frac{l}{2}} I\left(z', t_r+\frac{z'}{c_0}\cos\vartheta\right)dz' = I_0\int_{-\frac{l}{2}}^{\frac{l}{2}} e^{i\left[kz'-\omega t_r + \underbrace{\frac{\omega}{c_0}}_{k} z'\cos\vartheta\right]}dz' =$$

3/4

1/4

$$= I_0\int_{-\frac{l}{2}}^{\frac{l}{2}} e^{i[kz'(1-\cos\vartheta)-\omega t_r]}dz' = I_0 e^{-i\omega t_r}\,\frac{e^{ikz'(1-\cos\vartheta)}}{ik(1-\cos\vartheta)}\Bigg|_{-\frac{l}{2}}^{\frac{l}{2}} =$$

$$= \frac{2I_0}{k}e^{-i\omega t_r}\,\frac{e^{i\frac{kl}{2}(1-\cos\vartheta)} - e^{-i\frac{kl}{2}(1-\cos\vartheta)}}{2i\,(1-\cos\vartheta)} = \frac{2I_0}{k}e^{-i\omega t_r}\boxed{\frac{\sin\left[\frac{kl}{2}(1-\cos\vartheta)\right]}{1-\cos\vartheta}}$$

kotni del

$\Downarrow$

+

$$P^2 \propto B^2 \propto \boxed{\left(\frac{\sin\vartheta}{1-\cos\vartheta}\right)^2\sin^2\left[\frac{kl}{2}(1-\cos\vartheta)\right]}\,,\quad \frac{kl}{2} = \frac{2\pi}{\lambda}\frac{l}{2} = \underline{\pi\frac{l}{\lambda}}$$

b) $\frac{\sin\vartheta}{1-\cos\vartheta} = \frac{2\sin\frac{\vartheta}{2}\cos\frac{\vartheta}{2}}{2\sin^2\frac{\vartheta}{2}} = \mathrm{ctg}\,\frac{\vartheta}{2}$

$$\frac{kl}{2}(1-\cos\vartheta) = \pi\frac{l}{\lambda}\,2\sin^2\frac{\vartheta}{2} \xrightarrow{l=2\lambda} 4\pi\sin^2\frac{\vartheta}{2}$$

$$\underline{P^2\propto\mathrm{ctg}^2\frac{\vartheta}{2}\,\sin^2\left[4\pi\sin^2\frac{\vartheta}{2}\right]} \xrightarrow{n=2} \boxed{\vartheta_{max} = 35.5^\circ}$$

1/4

1/4 +

antena V : maksimuma morata SOVPADATI

↳ kot med palicama je $\boxed{2\vartheta_{max} = 71^\circ}$ +

$\boxed{1}$