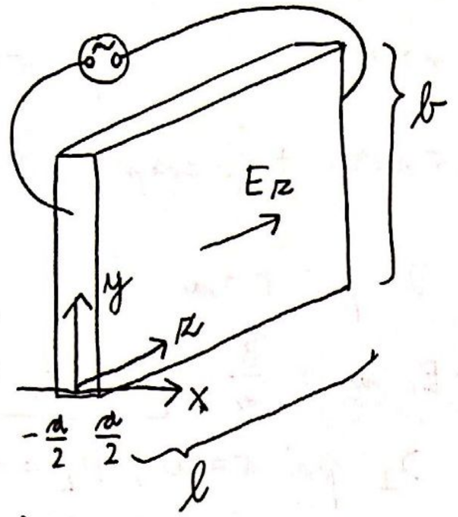
2. KOLOKVIJ

1 KOŽNI POJAV V TRAKU



$$\left.\begin{aligned}\vec\nabla\times\vec E &= -\frac{\partial\vec B}{\partial t}\\ \vec\nabla\times\vec B &= \mu_0\vec j = \mu_0\sigma\vec E\end{aligned}\right\}\Rightarrow \overbrace{\vec\nabla\times(\vec\nabla\times\vec E)}^{\vec\nabla(\vec\nabla\cdot\vec E)-\nabla^2\vec E} = -\mu_0\sigma\frac{\partial\vec E}{\partial t}$$

$$\nabla^2\vec E - \mu_0\sigma\frac{\partial\vec E}{\partial t} = 0$$

$$\vec E = \vec E(\vec r)\,e^{i\omega t} \Rightarrow \nabla^2\vec E - \underbrace{i\mu_0\sigma\omega}\,\vec E = 0$$

$$k = (1+i)\sqrt{\frac{\mu_0\sigma\omega}{2}} \Leftarrow k^2$$

a)

1/4 – zanima nas E_z, $E_z = E_z(x)$, saj sta dimenziji v y & z veliki

$$\Rightarrow \nabla^2 = \frac{\partial^2}{\partial x^2}$$

$$E_z'' - k^2E_z = 0 \quad\rightarrow\quad \underline{E_z(x) = A\,\mathrm{ch}\,kx},$$ saj mora biti SODA funkcija zaradi simetrije

\+ – robni pogoj: $E_z\left(\pm\frac{a}{2}\right) = A\,\mathrm{ch}\frac{ka}{2} = E_0 \Rightarrow A = \frac{E_0}{\mathrm{ch}\frac{ka}{2}}$

$$\underline{E_z(x) = E_0\frac{\mathrm{ch}\,kx}{\mathrm{ch}\frac{ka}{2}}}$$

– celoten električni tok:

1/4
$$I = \sigma\int E_z(x)\,dS = \sigma b\int_{-\frac{a}{2}}^{\frac{a}{2}}E_z(x)\,dx = \frac{\sigma bE_0}{\mathrm{ch}\frac{ka}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}}\mathrm{ch}\,kx\,dx =$$

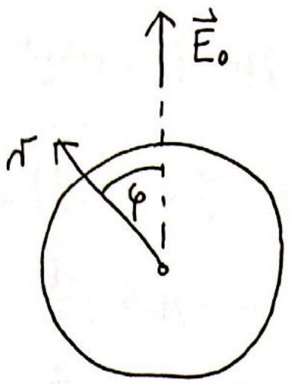
$$= \frac{\sigma bE_0}{k\,\mathrm{ch}\frac{ka}{2}}\,2\,\mathrm{sh}\frac{ka}{2} = \sigma abE_0\frac{2}{ka}\,\mathrm{th}\frac{ka}{2}$$

\+
$$Z = \frac{U}{I} = \frac{E_0 l}{I} = \underbrace{\frac{l}{\sigma ab}}_{R_0}\frac{\frac{ka}{2}}{\mathrm{th}\frac{ka}{2}} = \boxed{R_0\frac{\frac{ka}{2}}{\mathrm{th}\frac{ka}{2}}}$$

b) visoke $\omega \rightarrow \frac{ka}{2} \gg 1 \Rightarrow \overbrace{\mathrm{th}\frac{ka}{2} = 1}^{+} \Rightarrow Z = R_0\frac{ka}{2}$ 1

\+ $R = \mathrm{Re}(Z) = R_0\frac{a}{2}\sqrt{\frac{\mu_0\sigma\omega}{2}} \Rightarrow \boxed{\frac{R}{R_0} = \sqrt{\frac{\mu_0\sigma\omega}{2}}\,\frac{a}{2}}$

2 DIELEKTRIČNI VALJ V HOMOGENEM POLJU



$$\nabla^2 U(r,\varphi) = 0$$

$$\frac{1}{4}\left\{\quad U(r,\varphi) = \begin{cases} A r \cos\varphi & , \ r < a \\ -E_0 r \cos\varphi + \frac{B}{r}\cos\varphi & , \ r > a \end{cases}\right.$$

$+\{$ RP1: zveznost U pri $r=a$

$$A a = -E_0 a + \frac{B}{a} \quad , \quad \underbrace{A = -E_0 + \frac{B}{a^2}}$$

$+\{$ RP2: zveznost $D_\perp$ pri $r=a$, $D_\perp = -\varepsilon \frac{\partial U}{\partial r}$

$$\underbrace{-\varepsilon A = E_0 + \frac{B}{a^2}}$$

$\frac{1}{4}\{$ (RP1) − (RP2) : $A(1+\varepsilon) = -2E_0$

$$\underbrace{A = -\frac{2}{\varepsilon+1}E_0}$$

– znotraj valja: $U(r,\varphi) = -\frac{2}{\varepsilon+1}E_0 \overbrace{r\cos\varphi}^{z}$

$$\frac{1}{4}\left\{\begin{array}{l} E_1 = -\frac{\partial U}{\partial z} = \frac{2}{\varepsilon+1}E_0 \rightarrow \text{homogeno polje} \\ P = \varepsilon_0(\varepsilon-1)E_1 = \boxed{\varepsilon_0 \frac{2(\varepsilon-1)}{\varepsilon+1}E_0} \rightarrow \text{homogena polarizacija} \end{array}\right.$$

$\boxed{1}$

– alternativna pot:

(RP1) + (RP2) : $(1-\varepsilon)A = \frac{2B}{a^2} \rightarrow B = -\frac{\varepsilon-1}{2}a^2\left(-\frac{2}{\varepsilon+1}\right)E_0$

$$\underbrace{B = \frac{\varepsilon-1}{\varepsilon+1}a^2 E_0}$$

$$\left.\begin{array}{l} \sigma_v = \varepsilon_0\left(E_\perp^{ZUN} - E_\perp^{NOT}\right) = \varepsilon_0\left(E_0 + \frac{B}{a^2} + A\right)\cos\varphi = \\ = \varepsilon_0 \frac{\varepsilon-1-2+\varepsilon+1}{\varepsilon+1}E_0\cos\varphi = \varepsilon_0\frac{2(\varepsilon-1)}{\varepsilon+1}E_0\cos\varphi \\ \sigma_v = \vec{P}\cdot\vec{n} = P\cos\varphi \end{array}\right\} \Rightarrow \boxed{P = \varepsilon_0\frac{2(\varepsilon-1)}{\varepsilon+1}E_0}$$

$\boxed{3}$ KRIŽNA ANTENA

+

a) ena prečka: $p_e = e\,l$, $\quad \dot{p}_e = \dot{e}\,l = I\,l$, $\quad \ddot{p}_e = \dot{I}\,l$

cela antena: $I_1 = I_0 \cos\omega t$, $\quad I_2 = I_0 \sin\omega t$

$\downarrow$

$$\ddot{\vec{p}}_e = I_0 l\omega \left[\hat{e}_y \cos\omega t - \hat{e}_x \sin\omega t\right] \Leftarrow \begin{cases} \ddot{\vec{p}}_{e1} = \dot{I}_1 l\,\hat{e}_x = -I_0 l\omega \sin\omega t\,\hat{e}_x \\ \ddot{\vec{p}}_{e2} = \dot{I}_2 l\,\hat{e}_y = I_0 l\omega \cos\omega t\,\hat{e}_y \end{cases}$$

+

$$\vec{B} = -\frac{\mu_0}{4\pi c_0 r}\,\hat{e}_r \times \ddot{\vec{p}}_e(t_r) =$$

$$= \underline{-\frac{\mu_0 I_0 l\omega}{4\pi c_0 r}\left[\hat{e}_r\times\hat{e}_y \cos\omega t_r - \hat{e}_r\times\hat{e}_x \sin\omega t_r\right]}$$

$$\hat{e}_r = \begin{bmatrix}\cos\varphi\sin\vartheta \\ \sin\varphi\sin\vartheta \\ \cos\vartheta\end{bmatrix}, \quad \hat{e}_r\times\hat{e}_y = \begin{bmatrix}-\cos\vartheta \\ 0 \\ \cos\varphi\sin\vartheta\end{bmatrix}, \quad \hat{e}_r\times\hat{e}_x = \begin{bmatrix}0 \\ \cos\vartheta \\ -\sin\varphi\sin\vartheta\end{bmatrix}$$

+

$$\vec{E} = -\hat{e}_r \times c_0\vec{B}$$

$$\vec{P} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = -\frac{c_0}{\mu_0}\underbrace{(\hat{e}_r\times\vec{B})\times\vec{B}}_{\vec{B}\underbrace{(\hat{e}_r\cdot\vec{B})}_{0} - \hat{e}_r B^2} = \underline{\frac{c_0}{\mu_0}B^2\hat{e}_r}$$

$\frac{1}{4}$

$$B^2 = \frac{\mu_0^2 I_0^2 l^2\omega^2}{16\pi^2 c_0^2 r^2}\Big[(\cos^2\vartheta + \cos^2\varphi\sin^2\vartheta)\overbrace{\cos^2\omega t_r}^{\langle\rangle \to \frac{1}{2}} + (\cos^2\vartheta + \sin^2\varphi\sin^2\vartheta)\overbrace{\sin^2\omega t_r}^{\langle\rangle \to \frac{1}{2}} + \underbrace{\quad}\overbrace{\cos\omega t_r\sin\omega t_r}^{\langle\rangle \to 0}\Big]$$

$\downarrow$

$$\langle P\rangle = \frac{\mu_0 I_0^2 l^2\, 4\pi^2 c_0^2}{16\pi^2 c_0 r^2\lambda^2}\left[(\cos^2\vartheta + \cos^2\varphi\sin^2\vartheta)\frac{1}{2} + (\cos^2\vartheta + \sin^2\varphi\sin^2\vartheta)\frac{1}{2} + 0\right] =$$

$\mu_0 c_0 \downarrow$

$$= \frac{Z_0 I_0^2 l^2}{4r^2\lambda^2}\cdot\frac{1}{2}\left(2\cos^2\vartheta + \sin^2\vartheta\right) = \underline{\frac{Z_0 I_0^2}{4r^2}\left(\frac{l}{\lambda}\right)^2\cdot\frac{1}{2}(1+\cos^2\vartheta)}$$

$$+\left\{ \langle \int P\,dS \rangle = \frac{Z_0 I_0^2}{4}\left(\frac{\ell}{\lambda}\right)^2 2\pi \underbrace{\int d(\cos\vartheta)\,\frac{1}{2}\left(1+\cos^2\vartheta\right)}_{\frac{1}{2}\cdot\left(2+\frac{2}{3}\right)=\frac{4}{3}} = \boxed{\frac{2\pi}{3} Z_0^2 I_0^2 \left(\frac{\ell}{\lambda}\right)^2} \right.$$

b) primer Hertzovega dipola → le eden izmed členov v izrazu za $\vec{B}$ → recimo z $\hat{e}_y$

$$+\left\{ \begin{aligned} B^2 &= \frac{\mu_0^2 I_0^2 \ell^2 \omega^2}{16\pi^2 c_0^2 r^2}\left(\cos^2\vartheta + \cos^2\varphi \sin^2\vartheta\right)\cos^2\omega t_r \\ &\downarrow \\ \langle P \rangle &= \frac{Z_0 I_0^2 \ell^2}{4 r^2 \lambda^2}\cdot \frac{1}{2}\left(\cos^2\vartheta + \cos^2\varphi \sin^2\vartheta\right) \end{aligned} \right.$$

$$+\left\{ \langle \int P\,dS \rangle = \frac{Z_0 I_0^2}{4}\left(\frac{\ell}{\lambda}\right)^2 \frac{1}{2} \underbrace{\int d\varphi\, d(\cos\vartheta)\left(\cos^2\vartheta + \cos^2\varphi \overbrace{\sin^2\vartheta}^{1-\cos^2\vartheta}\right)}_{2\pi\cdot\frac{2}{3} + \frac{1}{2}2\pi\left(2-\frac{2}{3}\right) = \frac{4\pi}{3}+\frac{4\pi}{3} = \frac{8\pi}{3}} \right.$$

$$\boxed{\langle \int P\,dS \rangle = \frac{\pi}{3} Z_0^2 I_0^2 \left(\frac{\ell}{\lambda}\right)^2} \rightarrow$$ DVAKRAT manj kot za križno anteno! $\boxed{1}$

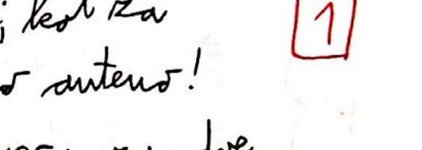
Rezultat se zdi intuitiven, a ni trivialen. Primer: za dve enaki vzporedni anteni je moč 4-krat večja, če sta v fazi, in enaka 0, če sta v nasprotni fazi.

$\boxed{4}$ KOAKSIALNI KABEL → TM način

$E_z(r,\varphi) = R(r)\,\Phi(\varphi)$, Besslova enačba za $R(r)$

$$+\left\{ R(r) = A_m J_m(\varkappa r) + B_m N_m(\varkappa r) \right.$$ → Neumannove funkcije so v koaksialnem kablu mogoče!

RP: $$+\left\{ \begin{aligned} R(a) &= A_m J_m(\varkappa a) + B_m N_m(\varkappa a) = 0 \\ R(2a) &= A_m J_m(2\varkappa a) + B_m N_m(2\varkappa a) = 0 \end{aligned} \right.$$



$\frac{1}{4}$ $$\left\{ f(x) \qquad \begin{bmatrix} J_m(\varkappa a) & N_m(\varkappa a) \\ J_m(2\varkappa a) & N_m(2\varkappa a) \end{bmatrix}\begin{bmatrix} A_m \\ B_m \end{bmatrix} = 0 \Rightarrow \underline{\det = 0} \right.$$ $\boxed{1}$

$$\underline{J_m(\varkappa a) N_m(2\varkappa a) - J_m(2\varkappa a) N_m(\varkappa a) = 0} \rightarrow m = 0, \text{ min ničla } \boxed{\varkappa a = \frac{\omega_{min}}{c_0} a = 3.12} \Big\} \frac{1}{2}$$