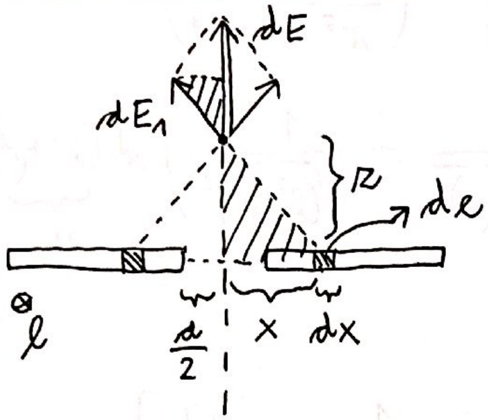
EMP: 1. POPRAVNI KOLOKVIJ

1



a) $\frac{1}{4}\begin{cases} de = \sigma l \, dx \\ dE_1 = \dfrac{de}{2\pi\varepsilon_0 l \sqrt{x^2+r^2}} = \dfrac{\sigma \, dx}{2\pi\varepsilon_0 \sqrt{x^2+r^2}} \end{cases}$

polje dolge nabite žice

$+\begin{cases} \dfrac{\frac{dE}{2}}{dE_1} = \dfrac{r}{\sqrt{x^2+r^2}} \\ dE = \dfrac{\sigma r \, dx}{\pi\varepsilon_0 (x^2+r^2)} \end{cases}$

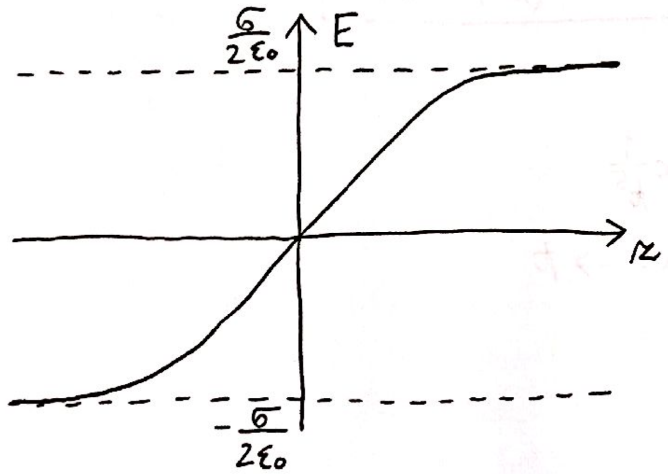
$$E = \frac{\sigma}{\pi\varepsilon_0} \int_{x=\frac{d}{2}}^{\infty} \frac{d\left(\frac{x}{r}\right)}{1+\left(\frac{x}{r}\right)^2} \leftarrow$$

$$+\left\{ E = \frac{\sigma}{\pi\varepsilon_0} \operatorname{arctg} \frac{x}{r} \Big|_{x=\frac{d}{2}}^{\infty} = \boxed{\frac{\sigma}{\pi\varepsilon_0}\left[\frac{\pi}{2} - \operatorname{arctg}\frac{d}{2r}\right]} \right.$$

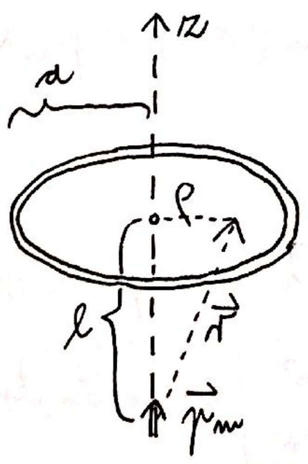
b) $+\left\{ \underbrace{r \gg d} : \quad \operatorname{arctg}\frac{d}{2r} \to 0, \quad \underbrace{E \to \frac{\sigma}{2\varepsilon_0}} \right.$ polje ravne plošče

$+\left\{ \underbrace{r \ll d} : \quad \operatorname{arctg}\frac{d}{2r} \to \frac{\pi}{2} - \frac{2r}{d}, \quad \underbrace{E \to \frac{\sigma}{\pi\varepsilon_0}\frac{2r}{d}} \right.$ linearno

$\frac{1}{4}$



2



a) $$\vec{B} = \frac{\mu_0}{4\pi}\left[\frac{3\vec{r}(\vec{p}_m \cdot \vec{r})}{r^5} - \frac{\vec{p}_m}{r^3}\right]$$

$$\left.\begin{aligned} \vec{p}_m &= p_m \hat{e}_r \\ \vec{r} &= l\hat{e}_r + \rho\hat{e}_\rho \end{aligned}\right\} \Rightarrow \begin{aligned} \vec{p}_m \cdot \hat{e}_r &= p_m \\ \vec{p}_m \cdot \vec{r} &= p_m l \\ \vec{r} \cdot \hat{e}_r &= l \end{aligned}$$

$$B_r = \vec{B}\cdot\hat{e}_r = \frac{\mu_0}{4\pi}\left[\frac{3}{r^5}(\vec{r}\cdot\hat{e}_r)(\vec{p}_m\cdot\vec{r}) - \frac{1}{r^3}\vec{p}_m\cdot\hat{e}_r\right]$$

$+$ $$B_z = \frac{\mu_0}{4\pi}\left[\frac{3}{r^5} p_m l^2 - \frac{1}{r^3} p_m\right] = \frac{\mu_0 p_m}{4\pi}\left(\frac{3l^2}{r^5} - \frac{1}{r^3}\right)$$

$+$ $$d\Phi = B_z \cdot 2\pi\rho\, d\rho = \frac{\mu_0 p_m}{4}\left(\frac{3l^2}{\sqrt{(l^2+\rho^2)^5}} - \frac{1}{\sqrt{(l^2+\rho^2)^3}}\right) d\underbrace{(l^2+\rho^2)}_{y}$$

$\frac{1}{4}$ $$\Phi = \frac{\mu_0 p_m}{4}\left[3l^2\int_{l^2}^{l^2+a^2}\frac{dy}{y^{\frac{5}{2}}} - \int_{l^2}^{l^2+a^2}\frac{dy}{y^{\frac{3}{2}}}\right] =$$

$$= \frac{\mu_0 p_m}{4}\left[-2l^2\left\{\frac{1}{(l^2+a^2)^{\frac{3}{2}}} - \frac{1}{l^3}\right\} + 2\left\{\frac{1}{\sqrt{l^2+a^2}} - \frac{1}{l}\right\}\right] =$$

$$= \frac{\mu_0 p_m}{4}\, 2\left[\frac{l^2+a^2}{(l^2+a^2)^{\frac{3}{2}}} - \frac{l^2}{(l^2+a^2)^{\frac{3}{2}}}\right] = \boxed{\frac{\mu_0 p_m}{2}\frac{a^2}{(l^2+a^2)^{\frac{3}{2}}}}$$

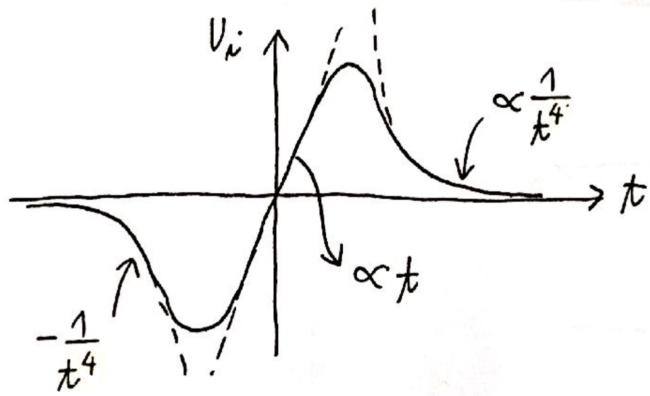
Do tega rezultata veliko hitreje pridemo z uporabo $\vec{A}$!

$+$ $$l = vt \rightarrow \Phi(t) = \frac{\mu_0 p_m}{2}\frac{a^2}{(v^2t^2+a^2)^{\frac{3}{2}}}$$

$$U_i = -\frac{d\Phi}{dt} = \frac{\mu_0 p_m a^2}{2}\,\frac{3}{2}\,\frac{v^2 2t}{(v^2t^2+a^2)^{\frac{5}{2}}}$$

$$\boxed{U_i = \frac{3\mu_0 p_m a^2 v}{2}\,\frac{vt}{(v^2t^2+a^2)^{\frac{5}{2}}}}$$

$+$



b) $$\vec{p}_m = p_m \hat{e}_x \Rightarrow \vec{p}_m \cdot \vec{r} = p_m \rho\, \hat{e}_x \cdot \hat{e}_\rho = p_m \rho \cos\varphi = p_m x$$

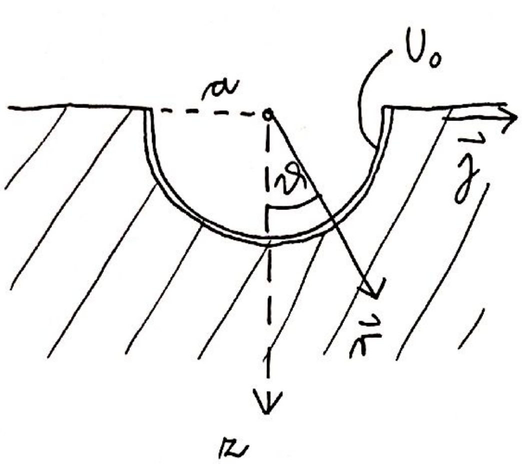
$\frac{1}{4}$ $$\vec{p}_m \cdot \hat{e}_z = 0$$

$$\Downarrow$$

$$B_z = \frac{\mu_0}{4\pi}\frac{3l}{r^5} p_m x \underbrace{\propto x}$$ $\Rightarrow$ prispevka pri x & $-x$ se vedno odštejeta!

$$\boxed{\begin{array}{c}\Phi = 0 \\ U_i = 0\end{array}}$$

3



$$U(r,\vartheta) = \sum_l \underbrace{\frac{B_l}{r^{l+1}}}_{\text{v poštev pridejo le padajoče rešitve}} P_l(\cos\vartheta)$$

$$\underline{U(a,\vartheta) = U_0} \rightarrow \text{robni pogoj}$$

- Ker je robni pogoj definiran le na intervalu $0 < \vartheta < \frac{\pi}{2}$ oziroma $0 < \cos\vartheta < 1$, ne moremo takoj uporabiti skalarnega produkta, ki ga izračunamo na intervalu $-1 < \cos\vartheta < 1$.

- $\vartheta = \frac{\pi}{2} \rightarrow \vec{j}$ ima radialno smer $\Rightarrow$ v poštev pridejo le SODI l

- za sode l je integral na $-1 < \cos\vartheta < 1$ kar 2-krat večji kot na $0 < \cos\vartheta < 1$ $\Rightarrow$ skalarni produkt lahko uporabimo

$\Downarrow$

$\frac{1}{4}$ + iz robnega pogoja sledi, da preživi LE člen z $l = 0$

$$U(r,\vartheta) = \frac{B_0}{r}, \quad U_0(a,\vartheta) = \frac{B_0}{a} = U_0 \Rightarrow B_0 = a U_0$$

$$\underline{U(r,\vartheta) = U_0 \frac{a}{r}}$$

$\frac{1}{4}$

$$\vec{j} = \sigma \vec{E} = -\sigma \vec{\nabla} U = -\sigma \frac{\partial V}{\partial r} \rightarrow \text{ima radialno smer}$$

$$j = -\sigma \frac{\partial U}{\partial r} = -\sigma U_0 a \left(-\frac{1}{r^2}\right) = \frac{\sigma U_0 a}{r^2}$$

$\frac{1}{4}$

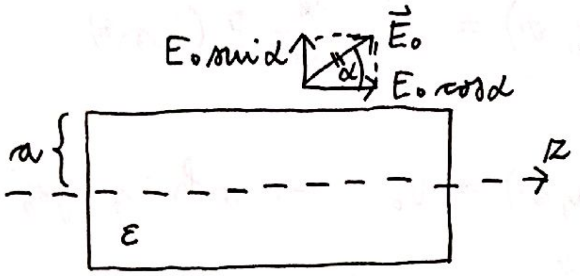
$$I = \int j \, dS = j \cdot 2\pi r^2 = 2\pi\sigma U_0 a$$

+

$$R = \frac{U_0}{I} = \boxed{\frac{1}{2\pi\sigma a}}$$

EMP : 2. POPRAVNI KOLOKVIJ

1



– razstavimo na komponenti

$E_{x0} = E_0 \sin\alpha \;\rightarrow\; xy$ ravnina

$E_{z0} = E_0 \cos\alpha \;\rightarrow\; z$ os

in vsako obravnavamo posebej

– xy ravnina

1/4

$$U(r\to\infty) = -E_{x0}\,x = -E_{x0}\, r\cos\varphi$$

rešitev vsebuje le člene s $\cos\varphi$!

$$U(r,\varphi) = \begin{cases} A\, r\cos\varphi & , \; r<a \\ \frac{B}{r}\cos\varphi - E_{x0}\, r\cos\varphi & , \; r>a \end{cases}$$

– robna pogoja:

+ 1) zveznost U : $\underline{A a = \frac{B}{a} - E_{x0} a} \;\Rightarrow\; \frac{B}{a^2} = A + E_{x0}$

+ 2) zveznost D_r : $\varepsilon\left(-\frac{\partial U}{\partial r}\right)\Big|_{r=a^-} = \left(-\frac{\partial U}{\partial r}\right)\Big|_{r=a^+}$

$$\underline{-\varepsilon A = \frac{B}{a^2} + E_{x0}} = A + 2E_{x0}$$

$$-(\varepsilon+1)A = 2E_{x0}$$

$$A = -\frac{2}{\varepsilon+1}E_{x0}$$

$$U(r,\varphi) = -\frac{2}{\varepsilon+1}E_{x0}\underbrace{r\cos\varphi}_{x} \quad , \; r<a$$

+ $$E_x = -\frac{\partial U}{\partial x} = \underline{\frac{2}{\varepsilon+1}E_{x0}} \quad , \; r<a$$

$$\underline{\frac{2}{\varepsilon+1}E_0\sin\alpha}$$

+ – z os

3) zveznost E_z : $E_{z0} = E_z$ znotraj

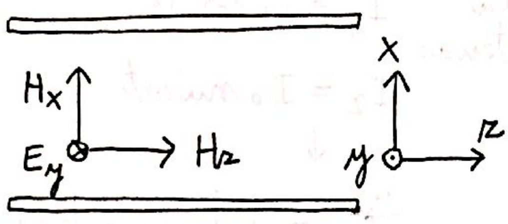
$$E_z = E_{z0} = \underline{E_0\cos\alpha}$$

1/4 – polje znotraj

$$E = \sqrt{E_x^2+E_z^2} = \boxed{E_0\sqrt{\cos^2\alpha + \left(\frac{2}{\varepsilon+1}\right)^2\sin^2\alpha}}$$

$$\operatorname{tg}\alpha' = \frac{E_x}{E_z} = \frac{2}{\varepsilon+1}\operatorname{tg}\alpha \;\Rightarrow\; \boxed{\alpha' = \operatorname{arctg}\left(\frac{2}{\varepsilon+1}\operatorname{tg}\alpha\right)}$$

2



TE način: $E_z = 0$, $H_y = 0$, $E_x = 0$ } +

$$\Big(\nabla_\perp^2 + \underbrace{\frac{\omega^2}{c_0^2} - k^2}_{\varkappa^2}\Big) H_z = 0$$

$\varkappa^2 \Rightarrow \omega^2 = c_0^2\left(k^2+\varkappa^2\right)$

$\omega_{min} = c_0 \varkappa = \omega_0$

$\omega^2 = c_0^2 k^2 + \omega_0^2$ } +

$\underbrace{Z = \frac{E_\parallel}{H_\perp} = \frac{E_y}{H_x}}$ iščemo! } +

$\frac{\partial}{\partial y} = 0$ ← simetrija

$\frac{\partial}{\partial z} = ik$ ← $e^{i(kz-\omega t)}$

$\frac{\partial}{\partial t} = -i\omega$

+ {

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = i\omega\mu_0 \vec{H}$$

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \times \begin{bmatrix} 0 \\ E_y \\ 0 \end{bmatrix} = i\omega\mu_0 \begin{bmatrix} H_x \\ 0 \\ H_z \end{bmatrix} \xrightarrow{\text{prva komponenta}}$$

$-\frac{\partial}{\partial z} E_y = i\omega\mu_0 H_x$

$-ik E_y = i\omega\mu_0 H_x$

$Z = \frac{E_y}{H_x} = \underbrace{-\frac{\omega\mu_0}{k}}$ } $\frac{1}{4}$

$\frac{1}{4}$ {

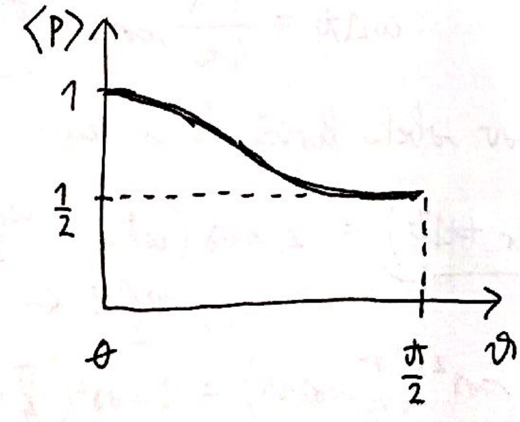
$$Z = -\omega\mu_0 \frac{c_0}{\sqrt{\omega^2-\omega_0^2}} = -\underbrace{\mu_0 \frac{1}{\sqrt{\mu_0\varepsilon_0}}}_{Z_0} \frac{\omega}{\sqrt{\omega^2-\omega_0^2}} = \boxed{-Z_0 \frac{1}{\sqrt{1-\frac{\omega_0^2}{\omega^2}}}}$$

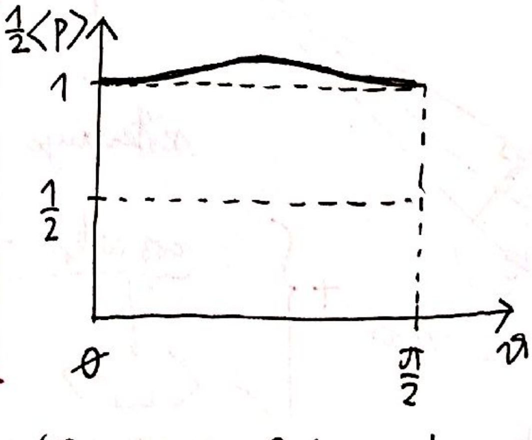
(rešitev na naslednji strani)

3 skici:

a) $\langle P \rangle \propto \frac{1}{2}\left(1+\cos^2\vartheta\right)$
ena križna antena

b) $\langle P \rangle \propto \frac{1}{2}\left(1+\cos^2\vartheta\right) \cdot 2\cos^2\left(\frac{\pi}{4}\cos\vartheta\right)$
dve križni anteni

+ 

+ 

Skoraj NI kotne odvisnosti!

3 a) ena prečka: $\quad p_e = l e$

$$\dot{p}_e = l \dot{e} = l I$$

$$\ddot{p}_e = l \dot{I}$$

cela antena: $\quad I_1 = I_0 \cos\omega t$

$$I_2 = I_0 \sin\omega t$$

$$\downarrow$$

$$\ddot{\vec{p}}_{e1} = l \dot{I}_1 \hat{e}_x = -l I_0 \omega \sin\omega t \, \hat{e}_x$$

$$\ddot{\vec{p}}_{e2} = l \dot{I}_2 \hat{e}_y = l I_0 \omega \cos\omega t \, \hat{e}_y$$

$$\vec{B} = -\frac{\mu_0}{4\pi c_0 r} \hat{e}_r \times \ddot{\vec{p}}_e \underbrace{\left(t - \frac{r}{c_0}\right)}_{t_r}$$

$$\ddot{\vec{p}}_e = \ddot{\vec{p}}_{e1} + \ddot{\vec{p}}_{e2} = l I_0 \omega \left(\hat{e}_y \cos\omega t_r - \hat{e}_x \sin\omega t_r\right)$$

$$\vec{B} = -\frac{\mu_0 l I_0 \omega}{4\pi c_0 r} \left(\underbrace{\hat{e}_r \times \hat{e}_y} \cos\omega t_r - \underbrace{\hat{e}_r \times \hat{e}_x} \sin\omega t_r\right)$$

$$\hat{e}_r = \begin{bmatrix} \cos\varphi \sin\vartheta \\ \sin\varphi \sin\vartheta \\ \cos\vartheta \end{bmatrix}, \quad \hat{e}_r \times \hat{e}_x = \begin{bmatrix} 0 \\ \cos\vartheta \\ -\sin\varphi \sin\vartheta \end{bmatrix}, \quad \hat{e}_r \times \hat{e}_y = \begin{bmatrix} -\cos\vartheta \\ 0 \\ \cos\varphi \sin\vartheta \end{bmatrix}$$

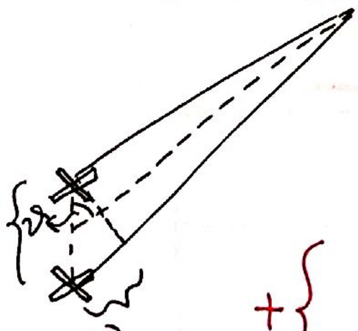
$$\vec{E} = c_0 \vec{B} \times \hat{e}_r, \quad \vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \Rightarrow \vec{P} = -\frac{c_0}{\mu_0} \underbrace{\left(\hat{e}_r \times \vec{B}\right) \times \vec{B}}_{\vec{B}\,\underbrace{(\hat{e}_r \cdot \vec{B})}_{0} - \hat{e}_r B^2} = \frac{c_0}{\mu_0} B^2 \hat{e}_r$$

$$\langle P \rangle \propto \langle B^2 \rangle \propto \left(\cos^2\vartheta + \sin^2\varphi \sin^2\vartheta\right) \underbrace{\langle \sin^2\omega t_r \rangle}_{\frac{1}{2}} + \left(\cos^2\vartheta + \cos^2\varphi \sin^2\vartheta\right) \underbrace{\langle \cos^2\omega t_r \rangle}_{\frac{1}{2}} = \boxed{\frac{1}{2}\left(1 + \cos^2\vartheta\right)}$$

b) dve križni anteni

$$\Delta r = \frac{\lambda}{4} \cos\vartheta \Rightarrow \Delta t = \frac{\Delta r}{c_0} = \frac{\lambda}{4 c_0} \cos\vartheta$$

$$\omega \Delta t = \frac{\omega \lambda}{4 c_0} \cos\vartheta = \underbrace{\frac{\pi}{2} \cos\vartheta}$$



$\frac{\lambda}{4}$

$\Delta r = \frac{\lambda}{4} \cos\vartheta$

seštevanje prispevkov obeh križnih anten:

$$\underbrace{\cos\omega t_r + \cos\left(\omega t_r + \omega \Delta t\right)} = 2 \cos\left(\omega t_r + \frac{\omega \Delta t}{2}\right) \cdot \cos\frac{\omega \Delta t}{2}$$

$$\langle \sqcup^2 \rangle = 4 \cdot \frac{1}{2} \cdot \cos^2\left(\frac{\pi}{4} \cos\vartheta\right) = 2 \cos^2\left(\frac{\pi}{4} \cos\vartheta\right)$$

Skici sta na prejšnji strani!

$$\boxed{\langle P \rangle \propto \frac{1}{2}\left(1 + \cos^2\vartheta\right) \cdot 2 \cos^2\left(\frac{\pi}{4} \cos\vartheta\right)}$$