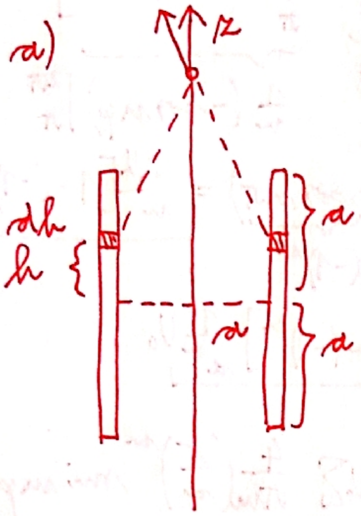
1. POPRAVNI KOLOKVIJ

1 a)



$\frac{1}{2}+$

$$dE = \frac{1}{4\pi\varepsilon_0}\underbrace{\frac{\sigma\cdot 2\pi a\cdot dh}{a^2+(z-h)^2}}_{\text{deli obročka}}\cdot\underbrace{\frac{z-h}{\sqrt{a^2+(z-h)^2}}}_{\text{ustrezna komponenta}}\ \boxed{+}$$

$\boxed{+}$

$$dE = \frac{\sigma}{2\varepsilon_0}\, a\,(-1)\frac{1}{2}\,\frac{d(z-h)^2}{\left[a^2+(z-h)^2\right]^{\frac{3}{2}}} = \quad d(z-h)^2 \to d\left[a^2+(z-h)^2\right] \quad \text{substitucija}\ \boxed{+}$$

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{a}{2}\right)\frac{1}{\left(-\frac{1}{2}\right)}\,\frac{1}{\sqrt{a^2+(z-h)^2}}\Bigg|_{-a}^{a} = \ \boxed{+}$$

$$= \boxed{\frac{\sigma a}{2\varepsilon_0}\left[\frac{1}{\sqrt{a^2+(z-a)^2}} - \frac{1}{\sqrt{a^2+(z+a)^2}}\right]}\ \boxed{+}$$

b) $E(z=0) = 0$

$$\frac{\sigma}{2\varepsilon_0}\left[\frac{1}{\sqrt{1+\left(1-\frac{z}{a}\right)^2}} - \frac{1}{\sqrt{1+\left(1+\frac{z}{a}\right)^2}}\right]$$

$z \ll a$:
$$\underline{E} = \frac{\sigma}{2\varepsilon_0}\frac{1}{\sqrt{2}}\left[\frac{1}{\left(1-\frac{z}{a}\right)^{\frac{1}{2}}} - \frac{1}{\left(1+\frac{z}{a}\right)^{\frac{1}{2}}}\right] = \frac{\sigma}{2\varepsilon_0}\sqrt{2}\,\underline{\frac{z}{a}}$$

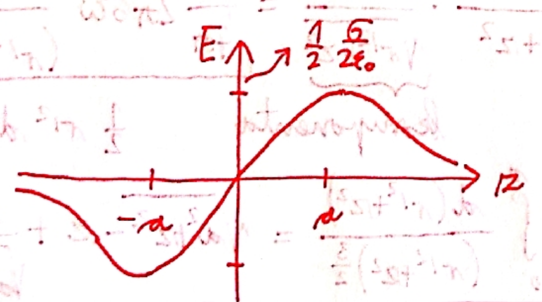
linearno

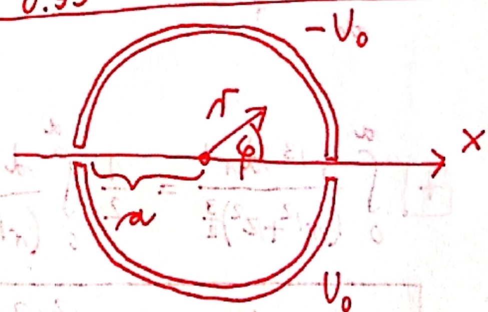
$\frac{1}{4}+$

$z \gg a$:
$$\underline{E} = \frac{\sigma}{2\varepsilon_0}\left[\frac{1}{\sqrt{z^2-2az}} - \frac{1}{\sqrt{z^2+2az}}\right] = \frac{\sigma}{2\varepsilon_0}\,\underline{\frac{2a}{z^2}}$$

$$z\sqrt{1-2\tfrac{a}{z}} \qquad z\sqrt{1+2\tfrac{a}{z}}$$

$$E(z=\pm a) = \frac{\sigma a}{2\varepsilon_0}\left(\frac{1}{a} - \frac{1}{\sqrt{5}}\frac{1}{a}\right) = \frac{\sigma}{2\varepsilon_0}\underbrace{\left(1-\frac{1}{\sqrt{5}}\right)}_{0.55}$$





2 a) $\boxed{+}\ V(r,\varphi) = \sum_m C_m r^m \sin m\varphi$, ostalih členov ni!

$\boxed{+}$

$$V(a,\varphi) = \sum_m C_m a^m \sin m\varphi = \quad \Big/ \cdot \sin n\varphi \to \int_0^{2\pi}$$

$$\boxed{+}\ \int_0^{2\pi}\sin m\varphi\,\sin n\varphi\,d\varphi = \delta_{mn}\frac{1}{\cancel{\pi}}\underbrace{\int_0^{2\pi}\sin^2 n\varphi\,d\varphi}_{\frac{1}{2}\cdot 2\pi} = \underline{\frac{\pi}{\cancel{\pi}}\delta_{mn}}$$

$\boxed{\frac14}$ $\int_0^{2\pi} \begin{cases} U_0 \\ -U_0 \end{cases} \sin m\varphi \, d\varphi = -U_0\left[\underbrace{\int_0^{\pi} \sin m\varphi\, d\varphi}_{\frac{1}{m}(-\cos m\varphi)\big|_0^{\pi}} - \underbrace{\int_{\pi}^{2\pi} \sin m\varphi \, d\varphi}_{\frac{1}{m}(-\cos m\varphi)\big|_{\pi}^{2\pi}}\right] =$

$\boxed{\frac34}$ $= U_0\left[\frac{1}{m}(1-\cos m\pi) - \frac{1}{m}(\cos m\pi - 1)\right] = -\frac{2U_0}{m}(1-\underbrace{\cos m\pi}_{(-1)^m}) = \underline{\frac{2U_0}{m}\left[1-(-1)^m\right]}$

$\boxed{+}$ $C_m a^m \frac{\pi}{\cancel{\ }} = -\frac{2U_0}{m}\left[1-(-1)^m\right] \Rightarrow \underline{C_m = -\frac{2}{\pi m}\left[1-(-1)^m\right]\frac{1}{a^m}U_0}$

$$\boxed{U(r,\varphi) = -U_0 \sum_m \frac{2}{\pi m}\left[1-(-1)^m\right]\left(\frac{r}{a}\right)^m \sin m\varphi} = U_0 \sum_{m \text{ lih}} \frac{4}{\pi m}\left(\frac{r}{a}\right)^m \sin m\varphi$$

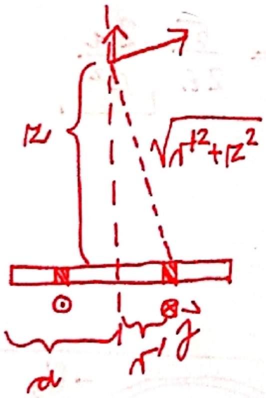
b) navpična ravnina: $\underline{\varphi = +\frac{\pi}{2}}$ $\boxed{+}$ $\boxed{+}$

$\boxed{\frac14}$ $E = -\frac{\partial U}{\partial r}\left(\varphi = \frac{\pi}{2}\right) = \sum_{m \text{ lih}} \frac{4U_0}{\pi m} \sin\left(\frac{m\pi}{2}\right) \cdot \frac{m}{a}\left(\frac{r}{a}\right)^{m-1} = \frac{4U_0}{\pi a} \cdot \frac{1}{1+\left(\frac{r}{a}\right)^2}$

$$\boxed{E = \frac{4U_0 a}{\pi(r^2+a^2)}}$$

$\boxed{3}$ a)

$\boxed{\frac12}$



$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{j}(\vec{r}\,') \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} d^3\vec{r}\,'$, $\vec{j}(\vec{r}\,')d^3\vec{r}\,' = \underline{dI\, dl'}$

$\boxed{+}$ $de = \sigma 2\pi r' dr' \Rightarrow \underline{dI} = \frac{de}{t_0} = \frac{de}{2\pi}\omega = \underline{\sigma\omega r' dr'}$

$\boxed{+}$ $dB = \frac{\mu_0}{4\pi}\frac{dI\, dl'}{r'^2+z^2} \cdot \underbrace{\frac{r'}{\sqrt{r'^2+z^2}}}_{\text{komponenta}} = \underline{\frac{\mu_0}{4\pi} 2\pi\sigma\omega \frac{r'^3 dr'}{(r'^2+z^2)^{\frac32}}}$

$\frac12 r'^2 d(r'^2)$

$\boxed{+}$ $\int_0^a \frac{r'^3 dr'}{(r'^2+z^2)^{\frac32}} = \frac12 \int_0^a \frac{d(r'^2+z^2)}{(r'^2+z^2)^{\frac12}} - \frac{z^2}{2}\int_0^a \frac{d(r'^2+z^2)}{(r'^2+z^2)^{\frac32}} = \sqrt{a^2+z^2} - z + \frac{z^2}{\sqrt{a^2+z^2}} - z =$

$= \frac{a^2+2z^2}{\sqrt{a^2+z^2}} - 2z$

$\boxed{+}$ $$\boxed{B(z) = \frac12 \mu_0 \sigma\omega\left(\frac{a^2+2z^2}{\sqrt{a^2+z^2}} - 2z\right)}$$

$\boxed{\frac14}$ b) $p_m = \int S\, dI = \int_0^a \pi r'^2 \cdot \sigma\omega r' dr' = \pi\sigma\omega \int_0^a r'^3 dr' = \boxed{\frac{\pi}{4}\sigma\omega a^4}$ $\boxed{+}$ $\underline{\frac{\mu_0}{4\pi}\frac{p_m}{2z^3}}$ $\boxed{+}$

$\boxed{\frac14}$ c) $z \gg a$: $\frac{a^2+2z^2}{\sqrt{a^2+z^2}} - 2z \approx \frac{1}{z}(a^2+2z^2)\left(1-\frac12\frac{a^2}{z^2}+\frac38\frac{a^4}{z^4}\right) - 2z =$

$= \frac{a^2}{z} + 2z - \frac12\frac{a^4}{z^3} - \frac{a^2}{z} + \frac34\frac{a^6}{z^5} + \frac34\frac{a^4}{z^3} - 2z \approx \frac14\frac{a^4}{z^3}$ $\boxed{+}$ $\rightarrow \underline{B(z) \approx \frac{\mu_0}{4\pi}\frac{\pi}{2}\sigma\omega\frac{a^4}{z^3}}$

$\Downarrow$ $\boxed{+}$

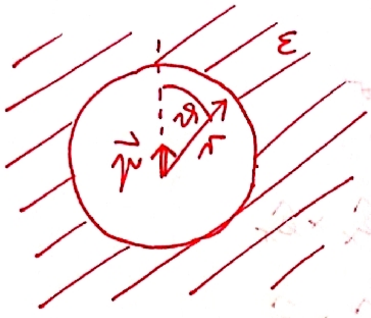
$\underline{p_m = \frac{\pi}{4}\sigma\omega a^4}$ ✓

1. PISNI IZPIT

c

[1] lest na popravnem kolokviju $\to \frac{p}{4\pi\varepsilon_0 r^2}\cos\vartheta$

[2]



a) [+] $U = \begin{cases} U_d + A\frac{r}{a}\cos\vartheta, & r<a \\ B\left(\frac{a}{r}\right)^2\cos\vartheta, & r>a \end{cases}$

robna pogoja, $r = a$:

[+] 1) U zvezen: $\frac{p}{4\pi\varepsilon_0 a^2} + A = B$

[+] 2) $D_\perp$ zvezen: $\frac{2p}{4\pi\varepsilon_0 a^3} - \frac{A}{a} = \varepsilon\frac{2B}{a}$

$\downarrow$ $-\varepsilon\frac{\partial}{\partial r}$

$\frac{2p}{4\pi\varepsilon_0 a^2} - A = 2\varepsilon B$

$\frac{1}{2}+$

[+] $A = \frac{2(1-\varepsilon)}{2\varepsilon+1}\frac{p}{4\pi\varepsilon_0 a^2}$, $\quad B = \frac{3}{2\varepsilon+1}\frac{p}{4\pi\varepsilon_0 a^2}$

$$\boxed{U = \begin{cases} \frac{p}{4\pi\varepsilon_0}\cos\vartheta\left[\frac{1}{r^2} - \frac{2(\varepsilon-1)}{2\varepsilon+1}\frac{r}{a^3}\right], & r<a \\ \frac{p}{4\pi\varepsilon_0}\cos\vartheta\,\frac{3}{2\varepsilon+1}\frac{1}{r^2}, & r>a \end{cases}} \Rightarrow [+]\ \boxed{p' = \frac{3p}{2\varepsilon+1}}$$

b) $\sigma_v = \underbrace{\vec{P}(r=a)\cdot\vec{n}}_{P_\perp(r=a)}$, $\quad \vec{P} = \varepsilon_0(\varepsilon-1)\vec{E}_{ZUN}$

$\vec{E}_{ZUN} = -\frac{\partial U_{ZUN}}{\partial r}$, $\quad U_{ZUN} = \frac{\vec{p}'\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = \frac{p'\cos\vartheta}{4\pi\varepsilon_0 r^2}$

$\frac{1}{4}$

$P_\perp = \varepsilon_0(\varepsilon-1)E_{ZUN,\perp} = -\varepsilon_0(\varepsilon-1)\frac{p'\cos\vartheta}{4\pi\varepsilon_0}(-2)\frac{1}{r^3}$

$$\sigma_v = \frac{p'}{4\pi a^3}2(\varepsilon-1)\cos\vartheta = \boxed{\frac{p}{4\pi a^3}\frac{6(\varepsilon-1)}{2\varepsilon+1}\cos\vartheta}$$

[+] c) $\rho_v = -\vec{\nabla}\cdot\vec{P} = -\varepsilon_0(\varepsilon-1)\underbrace{\vec{\nabla}\cdot\vec{E}_{ZUN}}_{0} = \boxed{0}$ $\Leftarrow$ polje dipola

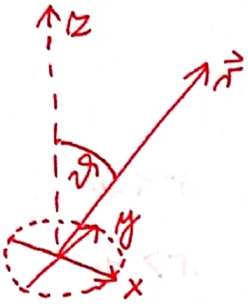
b) alternativa

$\sigma_v = \varepsilon_0(E_{NOT,\perp} - E_{ZUN,\perp})$ $\to$ samo zamenjamo, saj je dielektrik zunaj

$\vec{E}$ namreč določajo tako prosti $(=0)$ kot vezani naboji

$\boxed{3}$

a)



$$\left.\begin{aligned} p_x &= p\cos\omega t \\ p_y &= p\sin\omega t \end{aligned}\right\}\ \text{vrteči se dipol} \quad +$$

$$\vec B = \frac{\mu_0}{4\pi c r}\ddot{\vec p}\times\hat e_r \quad \text{podano}$$

$$\vec B_x = \frac{\mu_0}{4\pi c r}(-\omega^2)\,p\cos\omega t\ \ \hat e_x\times\hat e_r$$

$$\vec B_y = \frac{\mu_0}{4\pi c r}(-\omega^2)\,p\sin\omega t\ \ \hat e_y\times\hat e_r$$

$\boxed{\frac{1}{4}+}$

$$\hat e_x = \begin{bmatrix}1\\0\\0\end{bmatrix},\quad \hat e_y = \begin{bmatrix}0\\1\\0\end{bmatrix},\quad \hat e_r = \begin{bmatrix}\cos\varphi\sin\vartheta\\ \sin\varphi\sin\vartheta\\ \cos\vartheta\end{bmatrix}$$

$$\left.\begin{aligned} \hat e_x\times\hat e_r &= \begin{bmatrix}0\\ -\cos\vartheta\\ \sin\varphi\sin\vartheta\end{bmatrix} \\ \hat e_y\times\hat e_r &= \begin{bmatrix}\cos\vartheta\\ 0\\ -\cos\varphi\sin\vartheta\end{bmatrix}\end{aligned}\right\} \Rightarrow \boxed{\vec B = -\frac{\mu_0\omega^2 p}{4\pi c r}\begin{bmatrix}\cos\vartheta\sin\omega t_r\\ -\cos\vartheta\cos\omega t_r\\ \sin\vartheta\sin(\varphi-\omega t_r)\end{bmatrix}} \quad +$$

+

$$t_r = t - \frac{r}{c}$$

b) ravnina vrtenja, $\vartheta = \frac{\pi}{2} \Rightarrow \vec B \propto \begin{bmatrix}0\\0\\ \sin(\varphi-\omega t_r)\end{bmatrix} = \underbrace{\hat e_z}_{\text{linearna polarizacija}}\sin(\varphi-\omega t_r)$ +

$\boxed{\frac{1}{4}}$

+ os vrtenja, $\vartheta = 0 \Rightarrow \vec B \propto \begin{bmatrix}\sin\omega t_r\\ -\cos\omega t_r\\ 0\end{bmatrix} \rightarrow$ krožna polarizacija

c) $\vec E = -\hat e_r\times c\vec B$, $\vec P = \frac{1}{\mu_0}\vec E\times\vec B = -\frac{c}{\mu_0}(\hat e_r\times\vec B)\times\vec B = \underline{\frac{c}{\mu_0}B^2\hat e_r}$ +

$\boxed{\frac{1}{4}+}$

$$B^2 = \cos^2\vartheta\,(\overbrace{\sin^2\omega t_r+\cos^2\omega t_r}^{1}) + \sin^2\vartheta\sin^2(\varphi-\omega t_r) =$$
$$= \cos^2\vartheta + \sin^2\vartheta\sin^2(\varphi-\omega t_r)$$

$$\boxed{\langle B^2\rangle = \cos^2\vartheta + \frac{1}{2}\sin^2\vartheta} \quad +$$

$\vartheta = 0 \rightarrow 1$

$\vartheta = \frac{\pi}{2} \rightarrow \frac{1}{2}$