1. PISNI IZPIT

1 a)

$$V(\vartheta) = V_0 \cos^3\vartheta = V_0 \cdot \frac{2}{5}\cdot\frac{1}{2}\left(5\cos^3\vartheta - 3\cos\vartheta\right) + V_0\cdot\frac{3}{5}\cos\vartheta =$$

$$= \frac{2}{5}V_0 \, P_3(\cos\vartheta) + \frac{3}{5}V_0 \, P_1(\cos\vartheta)$$

$$\Downarrow$$

nastavek za potencial:

+

$$U(\vartheta) = \begin{cases} \dfrac{B_3}{r^4} P_3(\cos\vartheta) + \dfrac{B_1}{r^2} P_1(\cos\vartheta), & r > a \\ A_3 r^3 P_3(\cos\vartheta) + A_1 r \, P_1(\cos\vartheta), & r < a \end{cases}$$

primerjava s potencialom na površini: $B_3 = \frac{2}{5}V_0 a^4$, $A_3 = \frac{2}{5}\frac{V_0}{a^3}$

+

$B_1 = \frac{3}{5}V_0 a^2$, $A_1 = \frac{3}{5}\frac{V_0}{a}$

$$\boxed{U(\vartheta) = \begin{cases} \dfrac{2}{5}V_0 \dfrac{a^4}{r^4} P_3(\cos\vartheta) + \dfrac{3}{5}V_0 \dfrac{a^2}{r^2} P_1(\cos\vartheta), & r > a \\ \dfrac{2}{5}V_0 \dfrac{r^3}{a^3} P_3(\cos\vartheta) + \dfrac{3}{5}V_0 \dfrac{r}{a} P_1(\cos\vartheta), & r < a \end{cases}}$$

b)

$\frac{1}{4}$

$$\frac{\sigma}{\varepsilon_0} = E^r_{ZUN} - E^r_{NOT} = \frac{2}{5}V_0 \cdot 4\frac{1}{a} P_3(\cos\vartheta) + \frac{3}{5}V_0\cdot 2\frac{1}{a} P_1(\cos\vartheta) +$$

$$+ \frac{2}{5}V_0\cdot 3\frac{1}{a} P_3(\cos\vartheta) + \frac{3}{5}V_0\cdot\frac{1}{a} P_1(\cos\vartheta) =$$

$$= \frac{14}{5}\frac{V_0}{a}\left(\frac{5}{2}\cos^3\vartheta - \frac{3}{2}\cos\vartheta\right) + \frac{9}{5}\frac{V_0}{a}\cos\vartheta =$$

$$= \left(7\cos^3\vartheta - \frac{12}{5}\cos\vartheta\right)\frac{V_0}{a}$$

$$e = \int_{-1}^{1} \sigma \cdot 2\pi a^2 \, d(\cos\vartheta) = \varepsilon_0 \frac{V_0}{a} 2\pi a^2 \int_{-1}^{1}\left(7x^3 - \frac{12}{5}x\right)dx = \boxed{0}$$

še lažje: - monopolni člen je $\frac{e}{4\pi\varepsilon_0 r} P_0(\cos\vartheta) \Rightarrow e = 0$

$\frac{1}{4}$

- dipolni člen je $\frac{p_e}{4\pi\varepsilon_0 r^2} P_1(\cos\vartheta) \Rightarrow$

$$\Rightarrow \frac{3}{5}V_0 a^2 = \frac{p_e}{4\pi\varepsilon_0} \Rightarrow \boxed{p_e = \frac{12}{5}\pi\varepsilon_0 V_0 a^2}$$

c) kvadrupolni člen bi bil $\propto P_2(\cos\vartheta) \Rightarrow \boxed{Q = 0}$

+

To je mogoče videti tudi iz integrala, saj je σ LIHA funkcija $\cos\vartheta$, Q_{ij} pa so drugega reda.

1

2 a) $\vec{P} = P_0 \left(\frac{r}{a}\right)^2 \hat{e}_r = \frac{P_0}{a^2} r \vec{r}$

+ $\rho_v = -\vec{\nabla}\cdot\vec{P}$, $\quad \vec{\nabla}\cdot(r\vec{r}) = \vec{\nabla} r\cdot\vec{r} + r\vec{\nabla}\cdot\vec{r} = \frac{\vec{r}}{r}\cdot\vec{r} + 3r = 4r$

$\rho_v = -\frac{4P_0}{a^2} r$

+ $\sigma_v = \vec{P}\cdot\vec{n} = P_0\left(\frac{a}{a}\right)^2 = P_0$

+ – naboj do polmera r v krogli:

$e_r = \int_0^r \rho_v 4\pi r'^2 \, dr' = -\frac{16\pi P_0}{a^2}\int_0^r r'^3 \, dr' = -\frac{4\pi P_0 r^4}{a^2}$

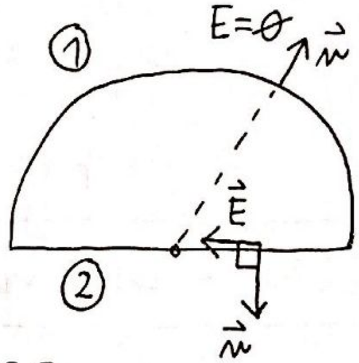
+ – Gaussov zakon:

$e_r = \varepsilon_0 \cdot 4\pi r^2 \cdot E \Rightarrow \boxed{E = -\frac{P_0}{\varepsilon_0}\left(\frac{r}{a}\right)^2}$ v radialni smeri, $r < a$

+ – Gaussov zakon za $r > a$:

$e = e_a + e_\sigma = -\frac{4\pi P_0 a^4}{a^2} + P_0 \cdot 4\pi a^2 = 0 \Rightarrow \boxed{E = 0}$ za $r \geq a$

b) $\vec{F}_e = \varepsilon_0 \oint \left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS$



+ ① → $\vec{F}_{e1} = 0$ zaradi $E = 0$

+ ② → $\vec{E}\cdot\vec{n} = 0$

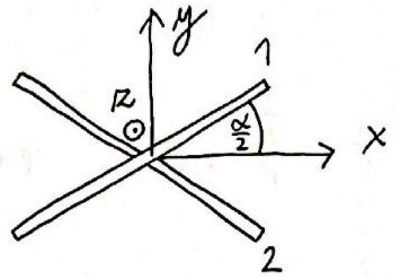
$\vec{F}_{e2} = -\frac{\varepsilon_0}{2}(-\hat{e}_z)\int_0^a \frac{P_0^2}{\varepsilon_0^2}\frac{r^4}{a^4} 2\pi r \, dr =$

+ $= \frac{\varepsilon_0}{2}\hat{e}_z \frac{P_0^2}{\varepsilon_0^2}\frac{2\pi}{a^4}\frac{a^6}{6} = \boxed{\frac{P_0^2 \pi a^2}{6\varepsilon_0}\hat{e}_z}$

↳ kaže NAVZGOR, kroglo vleče narazen

1

$\boxed{3}$



$$I_1 = I_0 \cos \omega t$$

$$I_2 = I_0 \sin \omega t$$

$$+\left\{ \hat{e}_{1,2} = \hat{e}_x \cos\frac{\alpha}{2} \pm \hat{e}_y \sin\frac{\alpha}{2} = \begin{bmatrix} \cos\frac{\alpha}{2} \\ \pm\sin\frac{\alpha}{2} \\ 0 \end{bmatrix} \right.$$

$$\Downarrow$$

$$\ddot{\vec{p}}_e = \ddot{\vec{p}}_{e1} + \ddot{\vec{p}}_{e2} \propto \begin{bmatrix} \cos\frac{\alpha}{2}\left(\cos\omega t_r + \sin\omega t_r\right) \\ \sin\frac{\alpha}{2}\left(\cos\omega t_r - \sin\omega t_r\right) \\ 0 \end{bmatrix}$$

$$\vec{B} = -\frac{\mu_0}{4\pi c_0 r}\, \hat{e}_r \times \ddot{\vec{p}}_e$$

a) <u>ravnina antene</u> : $\hat{e}_r = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix}$

$$+\left\{ \vec{B} \propto \hat{e}_r \times \ddot{\vec{p}}_e \propto \begin{bmatrix} 0 \\ 0 \\ \cos\varphi \sin\frac{\alpha}{2}\left(\overbrace{\cos\omega t_r}^{c} - \overbrace{\sin\omega t_r}^{s}\right) - \sin\varphi\cos\frac{\alpha}{2}\left(\overbrace{\cos\omega t_r}^{c} + \overbrace{\sin\omega t_r}^{s}\right) \end{bmatrix} \right.$$

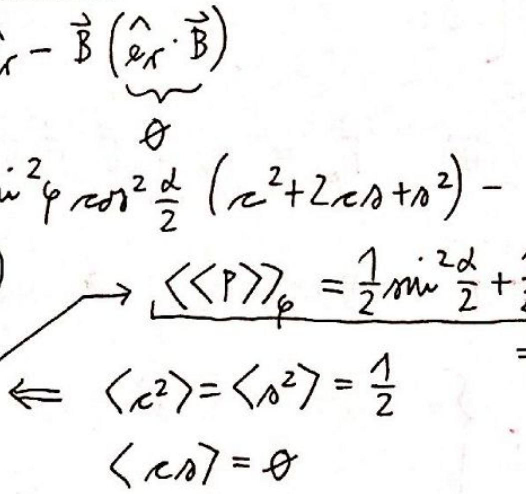
$$\vec{E} = c_0 \vec{B} \times \hat{e}_r \Rightarrow \vec{P} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{c_0}{\mu_0}\underbrace{\left(\vec{B}\times\hat{e}_r\right)\times\vec{B}}_{B^2\hat{e}_r - \vec{B}\underbrace{\left(\hat{e}_r\cdot\vec{B}\right)}_{0}} = \frac{c_0}{\mu_0}B^2\hat{e}_r \Rightarrow \underline{P \propto B^2}$$

$$+\left\{ \rightarrow P \propto \cos^2\varphi \sin^2\frac{\alpha}{2}\left(c^2 - 2cs + s^2\right) + \sin^2\varphi\cos^2\frac{\alpha}{2}\left(c^2 + 2cs + s^2\right) - 2\cos\varphi\sin\varphi\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}\left(c^2 - s^2\right) \right.$$

$$\rightarrow \underline{\langle\langle P\rangle\rangle_\varphi = \frac{1}{2}\sin^2\frac{\alpha}{2} + \frac{1}{2}\cos^2\frac{\alpha}{2}} = \boxed{\frac{1}{2}}$$

$$+\left\{ \boxed{\langle P\rangle \propto \cos^2\varphi\sin^2\frac{\alpha}{2} + \sin^2\varphi\cos^2\frac{\alpha}{2}} \right. \Leftarrow \langle c^2\rangle = \langle s^2\rangle = \frac{1}{2}$$

$$\langle cs\rangle = 0$$



$\}+$

b) <u>pravokotno na anteno</u> : $\hat{e}_r = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$+\left\{ \vec{B} \propto \hat{e}_r \times \ddot{\vec{p}}_e \propto \begin{bmatrix} -\sin\frac{\alpha}{2}(c-s) \\ \cos\frac{\alpha}{2}(c+s) \\ 0 \end{bmatrix} \right.$$

$$+\left\{ P \propto B^2 \propto \sin^2\frac{\alpha}{2}\left(c^2 - 2cs + s^2\right) + \cos^2\frac{\alpha}{2}\left(c^2 + 2cs + s^2\right) \right.$$

$$+\left\{ \langle P\rangle = \sin^2\frac{\alpha}{2}\cdot\left(\frac{1}{2}+\frac{1}{2}\right) + \cos^2\frac{\alpha}{2}\cdot\left(\frac{1}{2}+\frac{1}{2}\right) = \boxed{1} \right. \rightarrow \underline{\text{2-KRAT VEČJA}}$$

$\boxed{1}$