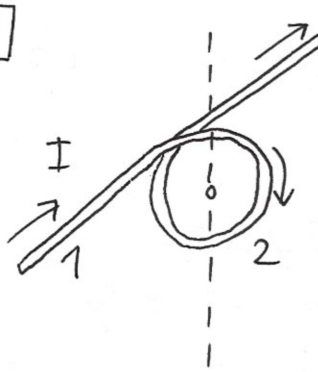
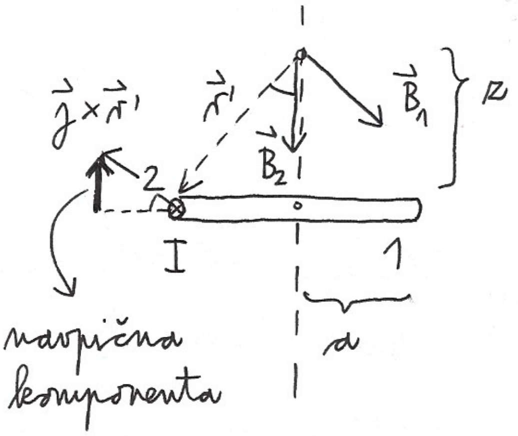
EMP – 2. PISNI IZPIT

1





Biot-Savart:

$$\vec B(\vec r) = \frac{\mu_0}{4\pi}\int \frac{\vec j(\vec r')\times(\vec r-\vec r')}{|\vec r-\vec r'|^3}\, d^3\vec r'$$

$$\vec B(0) = -\frac{\mu_0}{4\pi}\int \frac{\vec j \times \vec r'}{r'^3}\, d^3\vec r'$$

$\downarrow$ $\vec j \perp \vec r'$ za 2

$$|\vec B| = -\frac{\mu_0}{4\pi}\int \frac{|\vec j|\, d^3\vec r'}{r'^2} \quad \Big\} \ \frac{1}{4}$$

– ravni del 1 $\rightarrow$ $B_1 = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+R^2}}$ } +

– krožni del 2 $\rightarrow$ $B_2 = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{I\, dl'}{r'^2}\underbrace{\frac{a}{\sqrt{a^2+R^2}}}_{\text{navpična komponenta}}$ } +

$$B_2 = \frac{\mu_0}{4\pi}\,\frac{I\cdot 2\pi a}{a^2+R^2}\,\frac{a}{\sqrt{a^2+R^2}} = \frac{\mu_0 I}{2\pi}\,\frac{\pi a^2}{(a^2+R^2)^{\frac{3}{2}}} \quad \Big\} +$$

– vsota 1 + 2 $\rightarrow$ vektorska vsota!

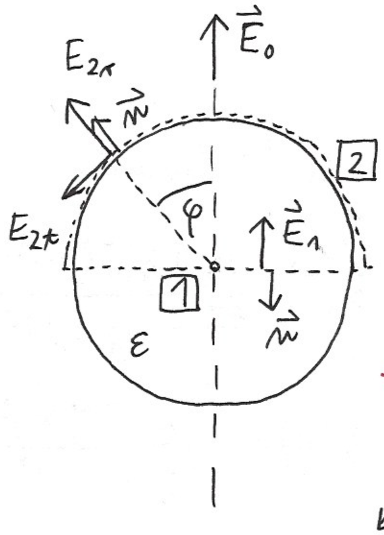
$$+\Big\{ B^2 = B_{1x}^2 + (B_2 + B_{1y})^2, \qquad \underbrace{B_{1x} = \frac{R}{\sqrt{a^2+R^2}}B_1, \quad B_{1y} = \frac{a}{\sqrt{a^2+R^2}}B_1}_{+}$$

$$B^2 = \left(\frac{\mu_0 I}{2\pi}\right)^2\left[\frac{R^2}{(a^2+R^2)^2} + \left(\frac{\pi a^2}{(a^2+R^2)^{\frac{3}{2}}} + \frac{a}{a^2+R^2}\right)^2\right]$$

$$\boxed{B = \frac{\mu_0 I}{2\pi}\sqrt{\frac{R^2}{(a^2+R^2)^2} + \left(\frac{\pi a^2}{(a^2+R^2)^{\frac{3}{2}}} + \frac{a}{a^2+R^2}\right)^2}} \quad \Big\} +$$

$\boxed{1}$

$\boxed{2}$



$$V(r,\varphi) = \begin{cases} A r\cos\varphi & , \; r<a \\ -E_0 r\cos\varphi + \dfrac{B}{r}\cos\varphi & , \; r\geq a \end{cases}$$

- RP1: zveznost V pri $r=a$

$$Aa = -E_0 a + \frac{B}{a} \rightarrow A = -E_0 + \frac{B}{a^2}$$

- RP2: zveznost $D_\perp$ pri $r=a$, $D_\perp = -\varepsilon \dfrac{\partial V}{\partial r}$

$$-\varepsilon A = E_0 + \frac{B}{a^2}$$

$$\begin{cases} A = -\dfrac{2}{\varepsilon+1} E_0 \\ B = \dfrac{\varepsilon-1}{\varepsilon+1} a^2 E_0 \end{cases}$$

$\Downarrow$

- znotraj: $E_1 = -\dfrac{\partial V}{\partial z} = -A = \underbrace{\dfrac{2}{\varepsilon+1} E_0}$ homogeno polje

- zunaj $(r=a)$:

$$E_{2r} = -\frac{\partial V}{\partial r}\bigg|_{r=a} = E_0\cos\varphi + \frac{\varepsilon-1}{\varepsilon+1}E_0\cos\varphi = \underbrace{\frac{2\varepsilon}{\varepsilon+1}E_0\cos\varphi}$$

radialni del

$$E_{2t} = -\frac{1}{r}\frac{\partial V}{\partial \varphi}\bigg|_{r=a} = -E_0\sin\varphi + \frac{\varepsilon-1}{\varepsilon+1}E_0\sin\varphi = \underbrace{-\frac{2}{\varepsilon+1}E_0\sin\varphi}$$

tangentni del

$$\vec{F}_e = \begin{cases} \varepsilon_0\varepsilon \oint\left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS & \text{znotraj} \\ \varepsilon_0 \oint\left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS & \text{zunaj} \end{cases}$$

- presek (1): $\vec{E}(\vec{E}\cdot\vec{n}) = E_1^2\vec{n} \Rightarrow \vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n} = \frac{1}{2}E_1^2\vec{n}$

$$\vec{F}_{e1} = \varepsilon_0\varepsilon\,\frac{1}{2}\left(\frac{2}{\varepsilon+1}E_0\right)^2\vec{n}\cdot\overbrace{2al}^{S} = -\frac{4\varepsilon_0\varepsilon a E_0^2}{(\varepsilon+1)^2}\, l\,\hat{e}_z$$

$$\underbrace{\frac{\vec{F}_{e1}}{l} = -\frac{4\varepsilon_0\varepsilon}{(\varepsilon+1)^2}\, a E_0^2\,\hat{e}_z}$$

- obod (2): $\vec{E}\cdot\vec{n} = E_{2r}$, $\vec{E}(\vec{E}\cdot\vec{n}) = E_{2r}^2\vec{n} + E_{2r}E_{2t}\vec{t}$

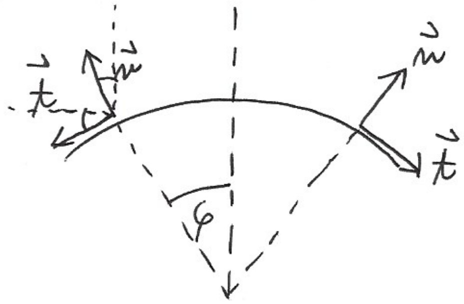
$$E^2\vec{n} = (E_{2r}^2 + E_{2t}^2)\vec{n}$$

$$\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n} = \frac{1}{2}E_{2r}^2\vec{n} - \frac{1}{2}E_{2t}^2\vec{n} + E_{2r}E_{2t}\vec{t} =$$

$$= \frac{1}{2}\left(E_{2r}^2 - E_{2t}^2\right)\vec{n} + E_{2r}E_{2t}\vec{t}$$

$$E_{2r}^2 - E_{2t}^2 = \frac{4E_0^2}{(\varepsilon+1)^2}\left(\varepsilon^2\cos^2\varphi - \sin^2\varphi\right)$$

$$E_{2r}E_{2t} = -\frac{4E_0^2}{(\varepsilon+1)^2}\,\varepsilon\sin\varphi\cos\varphi$$



$\vec n \to$ preživi navpična, $\underbrace{\cos\varphi}$

$\vec t \to$ preživi navpična, $\underbrace{-\sin\varphi}$

$dS = a l\, d\varphi$

$$\vec F_{e2} = \varepsilon_0 \frac{4E_0^2}{(\varepsilon+1)^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac12\left(\varepsilon^2\cos^2\varphi-\sin^2\varphi\right)\cos\varphi - \varepsilon\sin\varphi\cos\varphi\,(-\sin\varphi)\right] a l\, d\varphi \cdot \hat e_r$$

$$\vec F_{e2} = \frac{4\varepsilon_0}{(\varepsilon+1)^2}\, a l E_0^2 \int_{-1}^{1}\left[\frac{\varepsilon^2}{2}\left(1-\sin^2\varphi\right) - \frac12\sin^2\varphi + \varepsilon\sin^2\varphi\right] d(\sin\varphi) \overset{\hat e_r}{=}$$

$$= \frac{4\varepsilon_0}{(\varepsilon+1)^2}\, a l E_0^2 \underbrace{\left(\frac{\varepsilon^2}{2}\cdot\frac43 - \frac12\cdot\frac23 + \varepsilon\cdot\frac23\right)}_{\frac13\left(2\varepsilon^2+2\varepsilon-1\right)}\hat e_r$$

$$\underbrace{\frac{\vec F_{e2}}{l} = \frac{\frac43\varepsilon_0\left(2\varepsilon^2+2\varepsilon-1\right)}{(\varepsilon+1)^2}\, a E_0^2\, \hat e_r}$$

$$\frac{2\varepsilon^2+2\varepsilon-1}{3} - \frac{3\varepsilon}{3} = \frac{2\varepsilon^2-\varepsilon-1}{3}$$

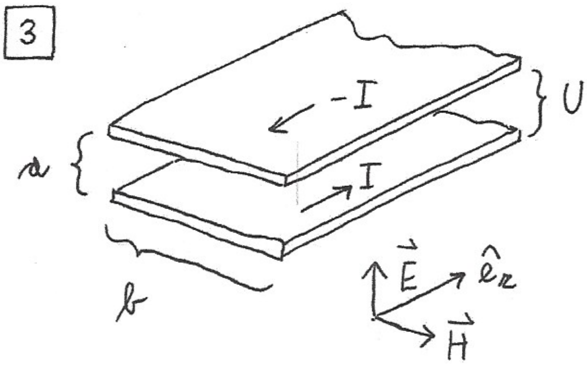
$\Downarrow$

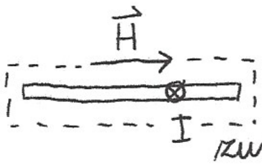
$$\boxed{\frac{\vec F_e}{l} = \frac{\varepsilon_0\left(2\varepsilon^2-\varepsilon-1\right)}{3(\varepsilon+1)^2}\, a E_0^2\, \hat e_r}$$

test: $\varepsilon = 1 \Rightarrow \frac{\vec F_e}{l} = 0$ ✓

$\boxed{1}$

3





zunaj ni polja!

a) $\left.\omega = kc = k\dfrac{c_0}{\sqrt{\varepsilon}}\right\}$ +

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\Downarrow$$

+ $\left\{\begin{array}{l} \omega = \dfrac{k c_0}{\sqrt{1-\dfrac{\omega_p^2}{\omega^2}}} \\ k^2 c_0^2 = \omega^2 - \omega_p^2 \rightarrow \boxed{\omega = \sqrt{\omega_p^2 + k^2 c_0^2}} \end{array}\right.$

$\boxed{\frac{1}{4}}$

b) $\left.\vec{H} = \dfrac{1}{Z}\hat{e}_z \times \vec{E}, \quad Z = \sqrt{\dfrac{\mu_0}{\varepsilon\varepsilon_0}} = \dfrac{Z_0}{\sqrt{\varepsilon}}\right\}$ +

$$H = \frac{\sqrt{\varepsilon}}{Z_0} E$$

– Ampere za posamezno ploščo:

$\left.\mu_0 I = Bb \Rightarrow B = \dfrac{\mu_0 I}{b} \rightarrow \underline{H = \dfrac{I}{b}}\right\}$ +

– definicija napetosti:

$\left.U = Ea \Rightarrow \underline{E = \dfrac{U}{a}}\right\}$ +

$\left.\begin{array}{l} \dfrac{I}{b} = \dfrac{1}{Z_0}\sqrt{1-\dfrac{\omega_p^2}{\omega^2}}\,\dfrac{U}{a} \\ Z' = \dfrac{U}{I} = \boxed{Z_0 \dfrac{a}{b} \dfrac{1}{\sqrt{1-\dfrac{\omega_p^2}{\omega^2}}}} \end{array}\right\}$ $\frac{1}{4}$

– primer $Z' = Z_0 \Rightarrow \sqrt{1-\dfrac{\omega_p^2}{\omega^2}} = \dfrac{a}{b}$

$$\frac{\omega_p^2}{\omega^2} = 1 - \left(\frac{a}{b}\right)^2$$

+ $\left\{ \boxed{\omega = \dfrac{\omega_p}{\sqrt{1-\left(\frac{a}{b}\right)^2}}} > \omega_p \right.$ → pri taki frekvenci je $\varepsilon > 0$, kakor mora biti!

$\boxed{\frac{3}{4}}$