EMP, 1. KOLOKVIJ

1 NABITA KROGELNA LUPINA

a) – površina:

$$+\left\{ U(\vartheta) = U_0 \cos^2\vartheta = U_0\left[\frac{2}{3}\cdot\frac{1}{2}(3\cos^2\vartheta - 1) + \frac{1}{3}\right] = \underbrace{\frac{2U_0}{3}P_2(\cos\vartheta) + \frac{U_0}{3}}\right.$$

[rešitev za cel prostor lahko vsebuje le $P_2(\cos\vartheta)$ in konstanto

– zunaj krogle, $r > a$

$$+\left\{ U(r,\vartheta) = \sum_l B_l \frac{1}{r^{l+1}} P_l(\cos\vartheta) \rightarrow \frac{B_2}{r^3}P_2(\cos\vartheta) + \frac{B_0}{r}\right.$$

$$\Downarrow$$

$$+\left\{ \text{RP}: \quad \underbrace{B_2 = \frac{2U_0}{3}a^3, \quad B_0 = \frac{U_0}{3}a}\right.$$

– znotraj krogle, $r < a$

$$+\left\{ U(r,\vartheta) = \sum_l A_l r^l P_l(\cos\vartheta) \rightarrow A_2 r^l P_2(\cos\vartheta) + A_0\right.$$

$$\Downarrow$$

$$+\left\{ \text{RP}: \quad \underbrace{A_2 = \frac{2U_0}{3a^2}, \quad A_0 = \frac{U_0}{3}}\right.$$

– potencial

$$\boxed{U(r,\vartheta) = \begin{cases} \frac{2U_0}{3a^2}r^2 P_2(\cos\vartheta) + \frac{U_0}{3}, & r < a \\ \frac{2U_0 a^3}{3}\frac{1}{r^3}P_2(\cos\vartheta) + \frac{U_0 a}{3}\frac{1}{r}, & r > a \end{cases}}$$

b) – površinska gostota naboja

$$+\left\{ \frac{\sigma}{\varepsilon_0} = E^r_{ZUN} - E^r_{NOT} = 3\frac{2U_0a^3}{3}\frac{1}{a^4}P_2(\cos\vartheta) + \frac{U_0 a}{3}\frac{1}{a^2} + \right.$$

$$+ 2\frac{2U_0}{3a^2}a\,P_2(\cos\vartheta) =$$

$$+\left\{ = \frac{10U_0}{3a}\underbrace{P_2(\cos\vartheta)}_{\frac{3}{2}\cos^2\vartheta - \frac{1}{2}} + \frac{U_0}{3a} = \frac{5U_0}{a}\cos^2\vartheta - \frac{4U_0}{3a}\right.$$

$$\boxed{\sigma = \frac{\varepsilon_0 U_0}{a}\left(5\cos^2\vartheta - \frac{4}{3}\right)}$$

c) – skupni naboj

$$e = \int \sigma\, dS = \frac{\varepsilon_0 U_0}{a}\int_{\cos\vartheta=-1}^{1}\left(5\cos^2\vartheta - \frac{4}{3}\right)\overbrace{2\pi a^2\, d(\cos\vartheta)}^{dS} =$$

$$= 2\pi\varepsilon_0 U_0 a\left(5\cdot\frac{2}{3} - \frac{4}{3}\cdot 2\right) = \boxed{\frac{4}{3}\pi\varepsilon_0 U_0 a}$$

$\boxed{1}$

– skupni naboj na enostavnejši način: MONOPOLNI člen $\frac{B_0}{r}$ je ravno posledica skupnega naboja na krogelni lupini

$$\frac{e}{4\pi\varepsilon_0 r} = \frac{B_0}{r} \Rightarrow e = 4\pi\varepsilon_0 B_0 = 4\pi\varepsilon_0\frac{U_0}{3}a = \boxed{\frac{4}{3}\pi\varepsilon_0 U_0 a}$$

enak rezultat!

$\boxed{2}$ SILA MED POLOVICAMA KROGLE

$$\rho = \frac{\alpha}{r} \Rightarrow e(r) = \int_0^r \frac{\alpha}{r'}4\pi r'^2\, dr' = 4\pi\alpha\int_0^r r'\, dr' = 2\pi\alpha r^2$$

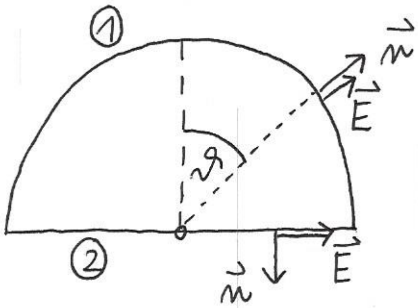
naboj do polmera r

– Gaussov izrek za kroglo polmera r $(r \leq a)$:

$$e(r) = 2\pi\alpha r^2 = \varepsilon_0 E(r)\cdot 4\pi r^2 \Rightarrow \underline{E(r) = \frac{\alpha}{2\varepsilon_0}}$$

konstanta! (radialna smer)

– sila na zgornjo polovico krogle



$$\vec{F}_1 = \varepsilon_0\int\left[\vec{E}(\overbrace{\vec{E}\cdot\vec{n}}^{E}) - \frac{1}{2}E^2\vec{n}\right]dS =$$

$$= \frac{\varepsilon_0}{2}E^2\int\vec{n}\, dS =$$

$$= \frac{\varepsilon_0}{2}\left(\frac{\alpha}{2\varepsilon_0}\right)^2\int^1\begin{bmatrix}\sin\vartheta\cos\varphi\\ \sin\vartheta\sin\varphi\\ \cos\vartheta\end{bmatrix}\overbrace{a^2\, d\varphi\, d(\cos\vartheta)}^{dS} =$$

$$= \frac{\pi\alpha^2 a^2}{4\varepsilon_0}\begin{bmatrix}0\\ 0\\ \frac{1}{2}\end{bmatrix} = \underline{\frac{\pi\alpha^2 a^2}{8\varepsilon_0}\hat{e}_z}$$

$\boxed{1}$

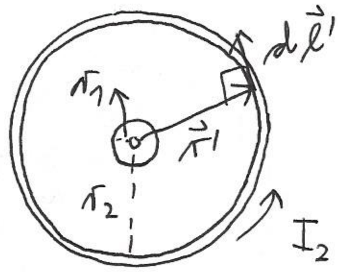
$$\vec{F}_2 = \varepsilon_0\int\left[\vec{E}(\overbrace{\vec{E}\cdot\vec{n}}^{0}) - \frac{1}{2}E^2\vec{n}\right]dS = -\frac{\varepsilon_0}{2}E^2\vec{n}\int dS =$$

$$= -\frac{\varepsilon_0}{2}\left(\frac{\alpha}{2\varepsilon_0}\right)^2(-\hat{e}_z)\pi a^2 = \underline{\frac{\pi\alpha^2 a^2}{8\varepsilon_0}\hat{e}_z} \Rightarrow \vec{F} = \vec{F}_1 + \vec{F}_2 = \boxed{\frac{\pi\alpha^2 a^2}{4\varepsilon_0}\hat{e}_z}$$

sila je ODBOJNA!

3 INDUKCIJA V DVEH ZANKAH

a) – magnetno polje v središču velike zanke



$$\vec{B}(\vec{r}) = \frac{\mu_0 I_2}{4\pi}\int \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

1/4

$$\vec{B}(\vec{r}=0) = \frac{\mu_0 I_2}{4\pi}\int \frac{\vec{r}'\times d\vec{l}'}{r'^3} \Rightarrow B(\vec{r}=0) = \frac{\mu_0 I_2}{4\pi}\int\frac{r'\,dl'}{r'^3} = \frac{\mu_0 I_2}{4\pi}\int\frac{dl'}{r_2^2} =$$

$$= \frac{\mu_0 I_2}{4\pi r_2^2}\underbrace{\int dl'}_{2\pi r_2} = \boxed{\frac{\mu_0 I_2}{2 r_2}}$$

– ker je $r_1 \ll r_2$, je magnetno polje v manjši zanki HOMOGENO

+

$$B_1 = \frac{\mu_0 I_2}{2 r_2} \Rightarrow \Phi_1 = B_1 \pi r_1^2 = \underbrace{\frac{\pi\mu_0}{2}\frac{r_1^2}{r_2}}_{L_{12}} I_2 \Rightarrow \boxed{L_{12} = \frac{\pi\mu_0}{2}\frac{r_1^2}{r_2}}$$

b) $I_1 = I_0\, e^{-\frac{t}{\tau}}$

1/4

$\Phi_2 = L_{21} I_1 = L_{12} I_1$ → pretok v večji zanki

$U_2 = -\dot{\Phi}_2 = -L_{12}\dot{I}_1$ → inducirana napetost

$U_2 = L_2 \dot{I}_2$ → U_2 požene tok I_2

$$\dot{I}_2 = -\frac{L_{12}}{L_2}\dot{I}_1 = \frac{L_{12}}{L_2} I_0 \frac{1}{\tau} e^{-\frac{t}{\tau}}$$

1/4

$$I_2(t) - \underbrace{I_2(0)}_{0} = \frac{L_{12}}{L_2}\frac{I_0}{\tau}\,\tau\left(-e^{-\frac{t}{\tau}}\right)\Big|_0^t = \frac{L_{12}}{L_2} I_0 \left(1-e^{-\frac{t}{\tau}}\right)$$

$$\boxed{I_2(t) = \frac{\pi}{2}\left(\frac{r_1}{r_2}\right)^2 \frac{1}{\ln 8\alpha} I_0\left(1-e^{-\frac{t}{\tau}}\right)}$$

$$\frac{\frac{\pi\mu_0}{2}\frac{r_1^2}{r_2}}{\mu_0 r_2 \ln(8\alpha)} = \frac{\pi}{2}\left(\frac{r_1}{r_2}\right)^2\frac{1}{\ln 8\alpha}$$

+

– končna vrednost toka (za $t \to \infty$) → $t \gg \tau$

$$\boxed{I_2^{FIN} = \frac{\pi}{2}\left(\frac{r_1}{r_2}\right)^2 \frac{1}{\ln 8\alpha} I_0}$$

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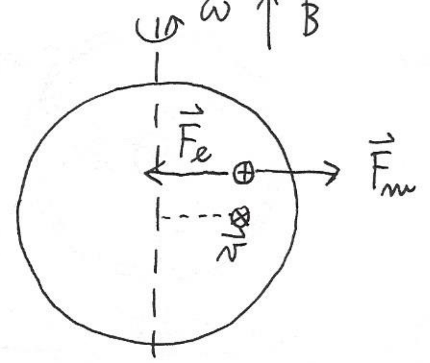
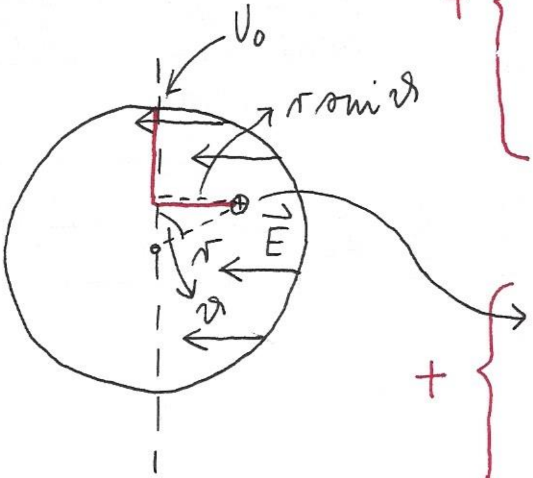
4 VRTEČA SE KROGLA V HOMOGENEM MAGNETNEM POLJU



– zaradi prerazporeditve naboja nastane električno polje $\vec{E}$, in sicer takšno, da električna sila na vsak naboj $\vec{F}_e = e\vec{E}$ ravno uravnoteži magnetno silo $\vec{F}_m = e\vec{v}\times\vec{B}$

$$\Downarrow$$



$$\underline{\vec{E} = -\vec{v}\times\vec{B}}$$

$$E = -vB = -\omega\,\overbrace{r\sin\vartheta}^{\rho}\cdot B$$

kaže proti osi vrtenja

v navpični smeri ni prispevka

$$U = U_0 - \int \vec{E}\cdot d\vec{r} = U_0 - \int E\,d\rho =$$

$$= U_0 + \int_0^{r\sin\vartheta} \omega\rho B\,d\rho = \underline{U_0 + \frac{1}{2}\omega B r^2 \sin^2\vartheta}$$

$$\sin^2\vartheta = 1-\cos^2\vartheta = 1 - \frac{2}{3}\underbrace{\left(\frac{3}{2}\cos^2\vartheta - \frac{1}{2}\right)}_{P_2(\cos\vartheta)} - \frac{1}{3} = \frac{2}{3} - \frac{2}{3}P_2(\cos\vartheta)$$

$\frac{1}{4}$

$$\underline{U(r,\vartheta) = \left(U_0 + \frac{1}{3}\omega B r^2\right) - \frac{1}{3}\omega B r^2 P_2(\cos\vartheta)} \rightarrow \text{potencial znotraj krogle}$$

– potencial ZUNAJ krogle lahko vsebuje le $P_2(\cos\vartheta)$ in konstanto

$$U(r,\vartheta) = \frac{B_0}{r} + \frac{B_2}{r^3}P_2(\cos\vartheta)$$

– zveznost pri $r = a$

$$\frac{B_0}{a} = U_0 + \frac{1}{3}\omega B a^2 \Rightarrow B_0 = U_0 a + \frac{1}{3}\omega B a^3$$

$$\frac{B_2}{a^3} = -\frac{1}{3}\omega B a^2 \Rightarrow B_2 = -\frac{1}{3}\omega B a^5$$

$$\underline{U(r,\vartheta) = \left(U_0 a + \frac{1}{3}\omega B a^3\right)\frac{1}{r} - \frac{1}{3}\omega B a^5\frac{1}{r^3}P_2(\cos\vartheta)} \rightarrow \text{zunaj}$$

– določiter U_0 : ker je krogla nevtralna, potencial ne sme vsebovati MONOPOLNEGA člena → NIČ

$$U_0\, a + \frac{1}{3}\omega B a^3 = 0 \quad \Rightarrow \quad \underline{U_0 = -\frac{1}{3}\omega B a^2}$$

$$\Downarrow$$

$$\boxed{U(r,\vartheta) = \begin{cases} \frac{1}{3}\omega B (r^2 - a^2) - \frac{1}{3}\omega B r^2 P_2(\cos\vartheta) & , \; r < a \\ -\frac{1}{3}\omega B \frac{a^5}{r^3} P_2(\cos\vartheta) & , \; r > a \end{cases}}$$

– <u>površinska gostota naboja</u>

$$\boxed{\frac{\sigma}{\varepsilon_0}} = E_r^{ZUN} - E_r^{NOT} = -\omega B a P_2(\cos\vartheta) + \frac{2}{3}\omega B a - \frac{2}{3}\omega B a P_2(\cos\vartheta) =$$

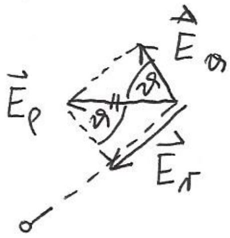
$$= \frac{2}{3}\omega B a - \frac{5}{3}\omega B a P_2(\cos\vartheta) =$$

$$= \frac{2}{3}\omega B a - \frac{5}{2}\omega B a \cos^2\vartheta + \frac{5}{6}\omega B a =$$

$$= \frac{9}{6}\omega B a - \frac{5}{2}\omega B a \cos^2\vartheta =$$

$$= \boxed{\frac{3}{2}\omega B a - \frac{5}{2}\omega B a \cos^2\vartheta}$$

– <u>prostorninska gostota naboja</u>



$$\vec{E} = -\omega r \sin\vartheta\, B \;\hat{e}_\rho$$

$$E_r = -\omega r \sin^2\vartheta\, B$$

$$E_\vartheta = -\omega r \sin\vartheta \cos\vartheta\, B$$

$$\boxed{\frac{\rho}{\varepsilon_0}} = \vec{\nabla}\cdot\vec{E} = -\omega B\, \vec{\nabla}\cdot \begin{bmatrix} r\sin^2\vartheta \\ r\sin\vartheta\cos\vartheta \\ 0 \end{bmatrix}_{\substack{r \\ \vartheta \\ \varphi}} =$$

$$= -\omega B \left[\frac{1}{r^2} 3r^2 \sin^2\vartheta + \frac{1}{r\sin\vartheta} r \left(2\sin\vartheta\cos^2\vartheta - \sin^3\vartheta\right)\right] =$$

$$= -\omega B \left(3\sin^2\vartheta + 2\cos^2\vartheta - \sin^2\vartheta\right) = \boxed{-2\omega B}$$

- skupni naboj:

$$e = \int \rho \, dV + \int \sigma \, dS = -2\varepsilon_0 \omega B \cdot \frac{4}{3}\pi a^3 + \frac{3}{2}\varepsilon_0 \omega B a \cdot 4\pi a^2 -$$

$$-\frac{5}{2}\varepsilon_0 \omega B a \cdot 2\pi a^2 \, \frac{\cos^3 \vartheta}{3}\Big|_{-1}^{1} =$$

$$= \pi \varepsilon_0 \omega B a^3 \underbrace{\left(-\frac{8}{3} + 6 - \frac{10}{3}\right)}_{0} = \boxed{0}$$

↳ kakor mora biti

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