2. KOLOKVIJ

1 a) $e = CU \,, \quad I = -\dot{e} = -C\dot{U} = \frac{U}{R}$ +

$$\dot{U} = -\frac{U}{RC} = -\frac{U}{\tau} \Rightarrow \underline{U = U_0 e^{-\frac{t}{\tau}}, \quad \tau = RC}$$

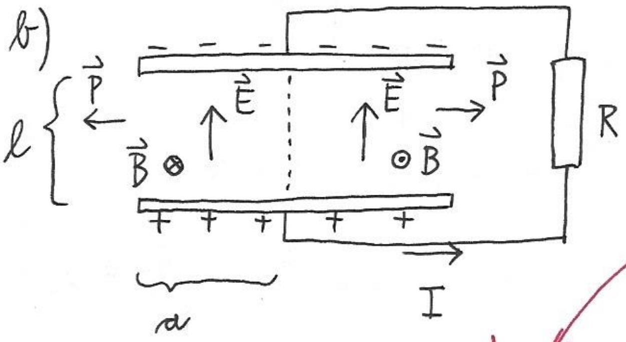
$\frac{1}{4}$

$$E = \frac{U}{l} = \boxed{\frac{U_0}{l} e^{-\frac{t}{\tau}}}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow B \cdot 2\pi r = \mu_0 \varepsilon_0 \pi r^2 \dot{E}$$

$$B = \frac{1}{2}\mu_0 \varepsilon_0 r \dot{E} = \boxed{-\frac{\mu_0 \varepsilon_0 U_0}{2\tau l} r e^{-\frac{t}{\tau}}}$$

b)



$\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \rightarrow$ kaže radialno navzven

$$P = \frac{1}{\mu_0} E|B| = \frac{\varepsilon_0 a}{2\tau}\left(\frac{U_0}{l}\right)^2 e^{-\frac{2t}{\tau}}$$

$$\int \vec{P} \cdot d\vec{S} = \overbrace{P \cdot 2\pi a l}^{+} = \overbrace{\frac{\varepsilon_0 \pi a^2}{l}}^{C} \frac{U_0^2}{\underset{\hookrightarrow RC}{\tau}} e^{-\frac{2t}{\tau}} = \overbrace{\boxed{\frac{U_0^2}{R} e^{-\frac{2t}{RC}}}}^{+}$$

c) $\frac{1}{4}$

$$W_e = \frac{1}{2}\varepsilon_0 E^2 \cdot V = \frac{\varepsilon_0}{2}\frac{U_0^2}{l^2} e^{-\frac{2t}{\tau}} \cdot Sl = \frac{1}{2} C U_0^2 e^{-\frac{2t}{\tau}}$$

$$\dot{W}_e = -\frac{1}{2} C U_0^2 \frac{2}{\underset{\hookrightarrow RC}{\tau}} e^{-\frac{2t}{\tau}} = \boxed{-\frac{U_0^2}{R} e^{-\frac{2t}{RC}}}$$

se kompenzirata

$\left(\frac{3}{4}+\right)$

$$W_m = \int \frac{B^2}{2\mu_0} \underset{\hookrightarrow l \cdot 2\pi r\, dr}{dV} = \frac{1}{2\mu_0} \int_0^a \frac{\mu_0^2 \varepsilon_0^2 U_0^2}{4\tau^2 l^2} e^{-\frac{2t}{\tau}} r^2 \cdot 2\pi l\, r\, dr =$$

$$= \frac{\pi}{4} \frac{\mu_0 \varepsilon_0^2 U_0^2}{\tau^2 l} e^{-\frac{2t}{\tau}} \underbrace{\int_0^a r^3 dr}_{\frac{a^4}{4}} = \frac{\mu_0 l}{16\pi} \frac{U_0^2}{\underset{\downarrow\, RC}{\tau^2}} \underbrace{\frac{\varepsilon_0^2 (\pi a^2)^2}{l^2}}_{C^2} =$$

$$= \frac{\mu_0 l}{16\pi} \frac{U_0^2}{R^2} e^{-\frac{2t}{\tau}}$$

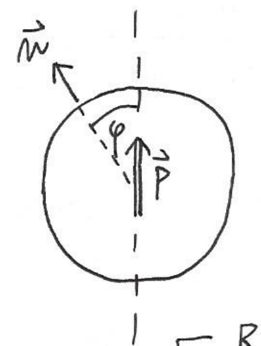
+ $$\dot{W}_m = -\frac{\mu_0 l}{16\pi}\frac{V_0^2}{R^2}\frac{2}{RC}e^{-\frac{2t}{\tau}} = \underbrace{-\frac{1}{8\pi}\frac{V_0^2}{R}\frac{\mu_0 l}{R^2 C}e^{-\frac{2t}{\tau}}}$$

$$\frac{\mu_0 l}{R^2 C} = \frac{\mu_0 l^2}{R^2\varepsilon_0 \pi a^2} = \frac{1}{\pi}\left(\frac{l}{a}\right)^2\frac{\mu_0}{\varepsilon_0}\frac{1}{R^2} = \frac{1}{\pi}\left(\frac{l}{a}\right)^2\left(\frac{Z_0}{R}\right)^2$$

$$\dot{W}_m = -\frac{1}{8\pi^2}\frac{V_0^2}{R}e^{-\frac{2t}{RC}}\left(\frac{Z_0}{R}\right)^2\left(\frac{l}{a}\right)^2 = \frac{1}{8\pi^2}\left(\frac{Z_0}{R}\right)^2\left(\frac{l}{a}\right)^2\dot{W}_e \underbrace{\ll \dot{W}_e}_{\text{zaradi } l \ll a}$$

$\boxed{1}$

$\boxed{2}$ – neprerezan valj



$$\sigma_v = \vec{P}\cdot\vec{n} = P\cos\varphi \quad \} +$$

$$V(r,\varphi) = \begin{cases} A r\cos\varphi, & r < a \\ \frac{B}{r}\cos\varphi, & r \geq a \end{cases} \quad \} +$$

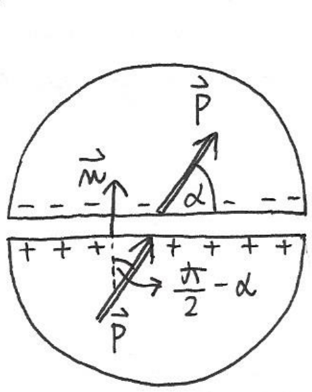
$$\text{RP1: } \sigma_v = \varepsilon_0\left[E_r^{ZUN} - E_r^{NOT}\right] = \varepsilon_0\left(\frac{B}{a^2} + A\right)\cos\varphi$$

$$\text{RP2: } V^{ZUN}(a,\varphi) = V^{NOT}(a,\varphi) \Rightarrow Aa = \frac{B}{a}, \quad \frac{B}{a^2} = A \quad \} +$$

$$P\cos\varphi = \varepsilon_0 2A\cos\varphi, \quad A = \frac{P}{2\varepsilon_0} \Rightarrow V^{NOT} = \frac{P}{2\varepsilon_0}\overbrace{r\cos\varphi}^{z}$$

$$\boxed{\vec{E}_{NOT} = -\frac{\vec{P}}{2\varepsilon_0}} \quad \} +$$

– prerezan valj



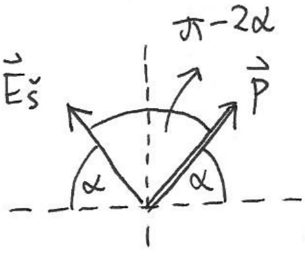
+ { spodnja ravna ploskev: $\sigma_v' = P\cos\left(\frac{\pi}{2}-\alpha\right) = P\sin\alpha$

$\Downarrow$

polje nabojev na prerezu: $\vec{E}' = \frac{\sigma_v'}{\varepsilon_0}\hat{e}_z = \frac{P\sin\alpha}{\varepsilon_0}\hat{e}_z$
(ploščati kondenzator)

+ $$\vec{E}_š = \vec{E}_{NOT} + \vec{E}' = -\frac{P}{2\varepsilon_0}\begin{bmatrix}\cos\alpha \\ \sin\alpha\end{bmatrix} + \frac{P}{2\varepsilon_0}\begin{bmatrix}0 \\ 2\sin\alpha\end{bmatrix}$$

$$\vec{E}_š = \frac{P}{2\varepsilon_0}\begin{bmatrix}-\cos\alpha \\ \sin\alpha\end{bmatrix} \Rightarrow \boxed{E_š = \frac{P}{2\varepsilon_0}} \quad \} +$$

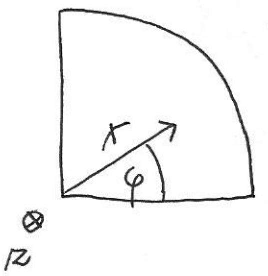


↳ simetrično od $\vec{P}$

$\boxed{1}$

+ { kot $\boxed{\pi-2\alpha}$ ← glede na navpično ravnino

$\boxed{3}$ – $\underline{\text{TM način}}$ $\rightarrow E_z \neq 0$



+ RP1: $E_z(\varphi = 0) = 0$
RP2: $E_z\left(\varphi = \frac{\pi}{2}\right) = 0$ (saj je $E_{\parallel} = 0$)

+ $\rightarrow \underline{E_z \propto \sin m\varphi}$

$\rightarrow \sin m\frac{\pi}{2} = 0 \Rightarrow \underline{m = 2, 4, 6, \ldots}$

($m = 0$ ne gre, saj je potem $E_z = 0$)

RP3: $E_z(r = a) = 0 \Rightarrow J_m(\varkappa a) = 0$

$$\varkappa a = \xi_{mn}$$

$$\varkappa^2 = \left(\frac{\xi_{mn}}{a}\right)^2 = \frac{\omega^2}{c_0^2} - k^2 \Rightarrow \underline{\omega = c_0\sqrt{\left(\frac{\xi_{mn}}{a}\right)^2 + k^2}}$$

najnižja dva pasova → najmanjši ničli pri dovoljenih m:

+
$$\Delta\omega_{TM} = \frac{c_0}{a}\left(\xi_{41} - \xi_{21}\right) \Leftarrow \begin{bmatrix} m = 2 \rightarrow \xi_{21} = 5.14 \\ m = 4 \rightarrow \xi_{41} = 7.59 \end{bmatrix} +$$

$$\boxed{\Delta\omega_{TM} = 2.45\frac{c_0}{a}}$$

– $\underline{\text{TE način}}$ $\rightarrow H_z \neq 0$

+ RP1: $\frac{\partial H_z}{\partial\varphi}(\varphi = 0) = 0$
RP2: $\frac{\partial H_z}{\partial\varphi}\left(\varphi = \frac{\pi}{2}\right) = 0$ (saj je $H_{\perp} = 0$, to pa je povezano z odvodom po pravokotni koordinati)

+ $\rightarrow \underline{H_z \propto \cos m\varphi}$ $\rightarrow \sin m\frac{\pi}{2} = 0 \Rightarrow \underline{m = 0, 2, 4, \ldots}$

(zaradi $\cos m\varphi$ je $m = 0$ dovoljen)

RP3: $\frac{\partial H_z}{\partial r}(r = a) = 0 \Rightarrow J_m'(\varkappa a) = 0$

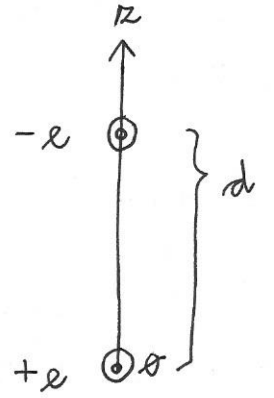
$$\varkappa a = \xi_{mn}' \rightarrow \underline{\omega = c_0\sqrt{\left(\frac{\xi_{mn}'}{a}\right)^2 + k^2}}$$

+
$$\begin{bmatrix} m = 0 \rightarrow \xi_{01}' = 3.83 \\ m = 2 \rightarrow \xi_{21}' = 3.05 \end{bmatrix} \Rightarrow \boxed{\Delta\omega_{TE} = 0.78\frac{c_0}{a}} \rightarrow \boxed{\frac{\Delta\omega_{TM}}{\Delta\omega_{TE}} = 3.1} + \boxed{1}$$

4 – realni del impedance → sevalni upor, ki smo ga za dipolno anteno z $d \ll \lambda$ izpeljali na vajah

$$\frac{1}{4}+ \left\{ \underline{Z_R = \frac{2\pi}{3} Z_0 \left(\frac{d}{\lambda}\right)^2} \right. , \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377\,\Omega$$

– imaginarni del impedance → kapaciteta izvodov



$$\frac{1}{4} \left\{ \begin{aligned}
&U_{1,2} = \frac{\pm e}{4\pi\varepsilon_0 r_{1,2}} \\
&U = U_1 + U_2 = \frac{e}{4\pi\varepsilon_0 r_1} - \frac{e}{4\pi\varepsilon_0 r_2}
\end{aligned} \right.$$

– na osi z:

$$U(a) = \frac{e}{4\pi\varepsilon_0 a} - \frac{e}{4\pi\varepsilon_0 (d-a)}$$

$$U(d-a) = \frac{e}{4\pi\varepsilon_0 (d-a)} - \frac{e}{4\pi\varepsilon_0 a}$$

$$\Delta U = -U(d-a) + U(a) = \frac{2e}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{d-a}\right)$$

$$\Delta U = \frac{e}{2\pi\varepsilon_0 a}\left(1 - \frac{a}{d-a}\right) \approx \frac{e}{2\pi\varepsilon_0 a} \qquad (d \gg a)$$

$$C = \frac{e}{\Delta U} = 2\pi\varepsilon_0 a$$

$$\frac{1}{4} \left\{ Z_C = \frac{1}{\omega C} = \frac{\lambda}{2\pi c_0} \cdot \frac{1}{2\pi\varepsilon_0 a} = \frac{1}{4\pi^2}\frac{\lambda}{a}\sqrt{\frac{\mu_0}{\varepsilon_0}} = \underline{\frac{1}{4\pi^2} Z_0 \frac{\lambda}{a}} \right.$$

$$+ \left\{ Z = \frac{2\pi}{3} Z_0 \underbrace{\left(\frac{d}{\lambda}\right)^2}_{\ll 1} - i \frac{1}{4\pi^2} Z_0 \underbrace{\frac{\lambda}{a}}_{\gg 1} \right. \Rightarrow \boxed{|Z| \gg Z_0}$$

zato je tok majhen