

Elektromagnetnega polje: zbirka enačb za kolokvije in izpite

- Potencial električnega polja in Poissonova enačba zanj

$$\mathbf{E}(\mathbf{r}) = -\nabla U(\mathbf{r}) \quad \nabla^2 U(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\varepsilon_0} \quad (1)$$

- Potencial in električno polje porazdelitve nabojev

$$U(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_{(V)} \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \quad (2)$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_{(V)} \frac{\rho(\mathbf{r}')(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d^3\mathbf{r}' \quad (3)$$

- Električni dipol: potencial, električno polje in dipolni moment

$$U(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p}_e \cdot \mathbf{r}}{r^3} \quad (4)$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{3\mathbf{r}(\mathbf{p}_e \cdot \mathbf{r}) - \mathbf{p}_e r^2}{r^5}. \quad (5)$$

$$\mathbf{p}_e = \int_{(V)} \mathbf{r}' \rho(\mathbf{r}') d^3\mathbf{r}' \quad (6)$$

- Električna sila na porazdelitev nabojev v zunanjem električnem polju

$$\mathbf{F}_e = \int_V \rho(\mathbf{r}') \mathbf{E}(\mathbf{r}') d^3\mathbf{r}' \quad (7)$$

- Električna sila preko napetostnega tenzorja električnega polja

$$\mathbf{F}_e = \varepsilon_0 \oint_{(\partial V)} \left[\mathbf{E}(\mathbf{E} \cdot \mathbf{n}) - \frac{1}{2} E^2 \mathbf{n} \right] dS \quad (8)$$

- Vektorski potencial magnetnega polja in Poissonova enačba zanj

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \quad \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{j}(\mathbf{r}) \quad (9)$$

- Biot-Savartov zakon za vektorski potencial in magnetno polje porazdelitve tokov

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{(V)} \frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \quad (10)$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{(V)} \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d^3\mathbf{r}' \quad (11)$$

- Biot-Savartov zakon za vektorski potencial in magnetno polje tokovnega vodnika

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}'}{|\mathbf{r} - \mathbf{r}'|} \quad (12)$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}' \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \quad (13)$$

- Magnetni dipol: potencial, magnetno polje in dipolni moment

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 \mathbf{p}_m \times \mathbf{r}}{4\pi r^3} \quad (14)$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{3\mathbf{r}(\mathbf{p}_m \cdot \mathbf{r}) - \mathbf{p}_m r^2}{r^5} \quad (15)$$

$$\mathbf{p}_m = \frac{1}{2} \int_{(V)} \mathbf{r}' \times \mathbf{j}(\mathbf{r}') d^3\mathbf{r}' \quad (16)$$

- Magnetna sila na tokovno porazdelitev v zunanjem magnetnem polju

$$\mathbf{F}_m = \int_V \mathbf{j}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}') d^3\mathbf{r}' \quad (17)$$

- Magnetna sila preko napetostnega tenzorja magnetnega polja

$$\mathbf{F}_m = \frac{1}{\mu_0} \oint_{(\partial V)} \left[\mathbf{B}(\mathbf{B} \cdot \mathbf{n}) - \frac{1}{2} B^2 \mathbf{n} \right] dS \quad (18)$$

- Kontinuitetna enačba

$$\nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} \quad (19)$$

- Ohmov zakon

$$\mathbf{j} = \sigma \mathbf{E} \quad (20)$$

- Induktivnost

$$\Phi_i = \sum_k L_{ik} I_k \quad (21)$$

- Medsebojna induktivnost dveh tokovnih zank poljubne oblike

$$L_{ik} = \frac{\mu_0}{4\pi} \oint_{i,k} \frac{d\mathbf{l}_i d\mathbf{l}_k}{|\mathbf{r}(l_i) - \mathbf{r}(l_k)|} \quad (22)$$

- Inducirana napetost v zanki

$$U_i = -\frac{\partial \Phi}{\partial t} \quad (23)$$

- Električni tok v sklenjem tokovnem krogu

$$\dot{U} = L\ddot{I} + R\dot{I} + \frac{I}{C} \quad (24)$$

- Maxwellove enačbe v praznem prostoru

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\varepsilon_0} & \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \nabla \times \mathbf{B} &= \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \end{aligned} \quad (25)$$

- Poyntingov izrek

$$\frac{\partial w}{\partial t} + \nabla \cdot \mathbf{P} + \mathbf{j} \cdot \mathbf{E} = 0 \quad (26)$$

$$\mathbf{P} = \frac{1}{\mu_0} (\mathbf{E} \times \mathbf{B}) \quad w = \frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2} \frac{B^2}{\mu_0} \quad (27)$$

- Maxwellove enačbe v snovi

$$\nabla \cdot \mathbf{D} = \rho \quad \nabla \cdot \mathbf{B} = 0 \quad (28)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (29)$$

- Električna polja v snovi

$$\rho_v = -\nabla \cdot \mathbf{P} \quad \sigma_v = \mathbf{P} \cdot \mathbf{n} \quad (30)$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} \quad \mathbf{P} = \varepsilon_0 (\varepsilon - 1) \mathbf{E} \quad (31)$$

- Elektromagnetno valovanje v praznem prostoru

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(kz - \omega t)} \quad c_0 \mathbf{B}(\mathbf{r}, t) = \hat{e}_z \times \mathbf{E}(\mathbf{r}, t) \quad \omega = c_0 k \quad (32)$$

- Elektromagnetno valovanje v valovnem vodniku

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = 0 \quad \nabla^2 \mathbf{H}(\mathbf{r}, t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{H}(\mathbf{r}, t)}{\partial t^2} = 0 \quad (33)$$

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\boldsymbol{\rho}) e^{i(kz - \omega t)} \quad \mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\boldsymbol{\rho}) e^{i(kz - \omega t)} \quad (34)$$

$$\left(\nabla_{\perp}^2 + \frac{\omega^2}{c^2} - k^2 \right) \mathbf{E}(\boldsymbol{\rho}) = 0 \quad \left(\nabla_{\perp}^2 + \frac{\omega^2}{c^2} - k^2 \right) \mathbf{H}(\boldsymbol{\rho}) = 0 \quad (35)$$

$$c = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu_0}} \quad (36)$$

- TEM valovanje v valovnem vodniku

$$\nabla \times \mathbf{E} = i\mathbf{k} \times \mathbf{E} \quad \nabla \times \mathbf{H} = i\mathbf{k} \times \mathbf{H} \quad \omega = ck \quad c = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu_0}} \quad (37)$$

$$\mathbf{H} = \frac{1}{\mu_0 \omega} \mathbf{k} \times \mathbf{E} \quad \nabla_{\perp}^2 \mathbf{E}(\boldsymbol{\rho}) = 0 \quad \nabla_{\perp}^2 \mathbf{H}(\boldsymbol{\rho}) = 0 \quad (38)$$