# Electromagnetic field: list of equations to be used in written examinations

- Electric field potential and Poisson equation

$$\mathbf{E}(\mathbf{r}) = -\nabla U(\mathbf{r}) \qquad \nabla^2 U(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\varepsilon_0} \tag{1}$$

- Electric field potential and electric field of a charge distribution

$$U(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_{(V)} \frac{\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, d^3\mathbf{r}' \tag{2}$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_{(V)} \frac{\rho(\mathbf{r}')(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3\mathbf{r}' \tag{3}$$

- Electric dipole: potential, electric field in dipole moment

$$U(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{\mathbf{p}_e \cdot \mathbf{r}}{r^3} \tag{4}$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \frac{3\mathbf{r}(\mathbf{p}_e \cdot \mathbf{r}) - \mathbf{p}_e r^2}{r^5}. \tag{5}$$

$$\mathbf{p}_e = \int_{(V)} \mathbf{r}' \rho(\mathbf{r}')\, \mathrm{d}^3\mathbf{r}' \tag{6}$$

- Electric force on a charge distribution in an external electric field

$$\mathbf{F}_e = \int_V \rho(\mathbf{r}')\mathbf{E}(\mathbf{r}')\, d^3\mathbf{r}' \tag{7}$$

- Electric force using the electric stress tensor

$$\mathbf{F}_e = \varepsilon_0 \oint_{(\partial V)} \left[\mathbf{E}(\mathbf{E}\cdot\mathbf{n}) - \frac{1}{2}E^2\mathbf{n}\right] dS \tag{8}$$

- Magnetic field vector potential and Kirchhoff equation

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \qquad \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{j}(\mathbf{r}) \tag{9}$$

- Biot-Savart law for the vector potential and for the magnetic field of the electric current distribution

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{(V)} \frac{\mathbf{j}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\, d^3\mathbf{r}' \tag{10}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{(V)} \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3}\, d^3\mathbf{r}' \tag{11}$$

- Biot-Savart law for the vector potential and for the magnetic field of the electric current conductor

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}'}{|\mathbf{r}-\mathbf{r}'|} \tag{12}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}' \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \tag{13}$$

- Magnetic dipole: vector potential, magnetic field and dipole moment

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{\mathbf{p}_m \times \mathbf{r}}{r^3} \tag{14}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{3\mathbf{r}\,(\mathbf{p}_m \cdot \mathbf{r}) - \mathbf{p}_m\, r^2}{r^5} \tag{15}$$

$$\mathbf{p}_m = \frac{1}{2}\int_{(V)} \mathbf{r}' \times \mathbf{j}(\mathbf{r}')\; d^3\mathbf{r}' \tag{16}$$

- Magnetic force on the electric current distribution in an external magnetic field

$$\mathbf{F}_m = \int_V \mathbf{j}(\mathbf{r}') \times \mathbf{B}(\mathbf{r}')\, d^3\mathbf{r}' \tag{17}$$

- Magnetic force using the magnetic stress tensor

$$\mathbf{F}_m = \frac{1}{\mu_0}\oint_{(\partial V)} \left[\mathbf{B}(\mathbf{B}\cdot\mathbf{n}) - \frac{1}{2}B^2\mathbf{n}\right] dS \tag{18}$$

- Continuity equation

$$\boldsymbol{\nabla}\cdot\mathbf{j} = -\frac{\partial \rho}{\partial t} \tag{19}$$

- Ohm's law

$$\mathbf{j} = \sigma\mathbf{E} \tag{20}$$

- Inductance

$$\Phi_i = \sum_k L_{ik} I_k \tag{21}$$

- Mutual inductance of two current loops of arbitrary shape

$$L_{ik} = \frac{\mu_0}{4\pi}\oint_{i,k} \frac{d\mathbf{l}_i d\mathbf{l}_k}{|\mathbf{r}(l_i) - \mathbf{r}(l_k)|} \tag{22}$$

- Induced voltage in a loop

$$U_i = -\frac{\partial \Phi}{\partial t} \tag{23}$$

- Electric current in a closed current loop

$$\dot{U} = L\ddot{I} + R\dot{I} + \frac{I}{C} \tag{24}$$

- Maxwell equations in empty space

$$\begin{aligned} \boldsymbol{\nabla}\cdot\mathbf{E} &= \frac{\rho}{\varepsilon_0} & \boldsymbol{\nabla}\cdot\mathbf{B} &= 0 \\ \boldsymbol{\nabla}\times\mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} & \boldsymbol{\nabla}\times\mathbf{B} &= \mu_0\mathbf{j} + \mu_0\varepsilon_0\frac{\partial \mathbf{E}}{\partial t} \end{aligned} \tag{25}$$

- Poynting theorem

$$\frac{\partial w}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{P} + \mathbf{j}\cdot\mathbf{E} = 0 \tag{26}$$

$$\mathbf{P} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) \qquad w = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\frac{B^2}{\mu_0} \tag{27}$$

- Maxwell equations in matter

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho \qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0 \tag{28}$$

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \boldsymbol{\nabla} \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \tag{29}$$

- Electric field in matter

$$\rho_v = -\boldsymbol{\nabla} \cdot \mathbf{P} \qquad \sigma_v = \mathbf{P} \cdot \mathbf{n} \tag{30}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} \qquad \mathbf{P} = \varepsilon_0 (\varepsilon - 1) \mathbf{E} \tag{31}$$

- Electromagnetic waves in empty space

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{i(kz-\omega t)} \qquad c_0 \mathbf{B}(\mathbf{r},t) = \hat{e}_z \times \mathbf{E}(\mathbf{r},t) \qquad \omega = c_0 k \tag{32}$$

- Electromagnetic waves in a waveguide

$$\nabla^2 \mathbf{E}(\mathbf{r},t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r},t)}{\partial t^2} = 0 \qquad \nabla^2 \mathbf{H}(\mathbf{r},t) - \frac{1}{c^2} \frac{\partial^2 \mathbf{H}(\mathbf{r},t)}{\partial t^2} = 0 \tag{33}$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\boldsymbol{\rho}) e^{i(kz-\omega t)} \qquad \mathbf{H}(\mathbf{r},t) = \mathbf{H}(\boldsymbol{\rho}) e^{i(kz-\omega t)} \tag{34}$$

$$\left( \nabla_\perp^2 + \frac{\omega^2}{c^2} - k^2 \right) \mathbf{E}(\boldsymbol{\rho}) = 0 \qquad \left( \nabla_\perp^2 + \frac{\omega^2}{c^2} - k^2 \right) \mathbf{H}(\boldsymbol{\rho}) = 0 \tag{35}$$

$$c = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu_0}} \tag{36}$$

- TEM waves in a waveguide

$$\nabla \times \mathbf{E} = i\mathbf{k} \times \mathbf{E} \qquad \nabla \times \mathbf{H} = i\mathbf{k} \times \mathbf{H} \qquad \omega = ck \qquad c = \frac{1}{\sqrt{\varepsilon \varepsilon_0 \mu_0}} \tag{37}$$

$$\mathbf{H} = \frac{1}{\mu_0 \omega} \mathbf{k} \times \mathbf{E} \qquad \nabla_\perp^2 \mathbf{E}(\boldsymbol{\rho}) = 0 \qquad \nabla_\perp^2 \mathbf{H}(\boldsymbol{\rho}) = 0 \tag{38}$$