List of equations to be used in written examinations in EMP

- Electric field potential and Poisson equation

$$\mathbf{E}(\mathbf{r}) = -\nabla U(\mathbf{r}) \qquad \nabla^2 U(\mathbf{r}) = -\frac{\rho(\mathbf{r})}{\varepsilon_0} \tag{1}$$

- Electric field of a charge distribution

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_{(V)} \frac{\rho(\mathbf{r}')(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d^3\mathbf{r}' \tag{2}$$

$$U(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0} \int_{(V)} \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \tag{3}$$

- Electric dipole field

$$U(\mathbf{r}) = \frac{\mathbf{p}_e \cdot \mathbf{r}}{4\pi\varepsilon_0 r^3} \tag{4}$$

$$\mathbf{E}(\mathbf{r}) = \frac{3(\mathbf{p}_e \cdot \mathbf{r})\mathbf{r} - \mathbf{p}_e r^2}{4\pi\varepsilon_0 r^5}. \tag{5}$$

$$\mathbf{p}_e = \int \mathbf{r}'\rho(\mathbf{r}') \, \mathrm{d}^3\mathbf{r}' \tag{6}$$

- Electric force on a charge distribution

$$\mathbf{F}_e = \varepsilon_0 \oint_{(\partial V)} \left[\mathbf{E}(\mathbf{E} \cdot \mathbf{n}) - \frac{1}{2}E^2\mathbf{n} \right] dS \tag{7}$$

- Magnetic force on a current distribution in an external field

$$\mathbf{F}_m = \int_V \mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r}) \, d^3\mathbf{r} \tag{8}$$

- Magnetic field vector potential and Poisson equation

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) \qquad \nabla^2 \mathbf{A}(\mathbf{r}) = -\mu_0 \mathbf{j}(\mathbf{r}) \tag{9}$$

- Biot-Savart law

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{j}(\mathbf{r}')d^3\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \tag{10}$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_{(V)} \frac{\mathbf{j}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} d^3\mathbf{r}' \tag{11}$$

- Inductivity

$$\Phi_i = \sum_k L_{ik} I_k \tag{12}$$

- Mutual inductivity of two current loops

$$L_{ik} = \frac{\mu_0}{4\pi} \oint_{i,k} \frac{d\mathbf{l}_i d\mathbf{l}_k}{|\mathbf{r}(l_i) - \mathbf{r}(l_k)|} \tag{13}$$

- Magnetic dipole field

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{p}_m \times \mathbf{r}}{r^3} \tag{14}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{3\mathbf{r}\,(\mathbf{p}_m \cdot \mathbf{r}) - \mathbf{p}_m\, r^2}{r^5} \tag{15}$$

$$\mathbf{p}_m = \frac{1}{2} \int \mathbf{r}' \times \mathbf{j}(\mathbf{r}')\ d^3\mathbf{r}' \tag{16}$$

- Magnetic force on a current distribution

$$\mathbf{F}_m = \frac{1}{\mu_0} \oint_{(\partial V)} \left[\mathbf{B}(\mathbf{B} \cdot \mathbf{n}) - \frac{1}{2} B^2 \mathbf{n} \right] dS \tag{17}$$

- Electric conductivity

$$\mathbf{j} = \sigma \mathbf{E} \tag{18}$$

- Electric current in a closed loop

$$\dot{U} = L\ddot{I} + R\dot{I} + \frac{I}{C} \tag{19}$$

- Maxwell equations in vacuum

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0$$
$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \tag{20}$$

- Continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} \tag{21}$$

- Poynting theorem

$$\frac{\partial w}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{P} + \mathbf{j} \cdot \mathbf{E} = 0 \tag{22}$$

$$\mathbf{P} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) \qquad w = \tfrac{1}{2}\varepsilon_0 E^2 + \tfrac{1}{2}\frac{B^2}{\mu_0} \tag{23}$$

- Electromagnetic field momentum and angular momentum densities

$$\mathbf{g} = \varepsilon_0 (\mathbf{E} \times \mathbf{B}) \qquad \gamma = \mathbf{r} \times \mathbf{g} \tag{24}$$

- Maxwell equations in matter

$$\boldsymbol{\nabla} \cdot \mathbf{D} = \rho \qquad \boldsymbol{\nabla} \cdot \mathbf{B} = 0$$
$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \boldsymbol{\nabla} \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \tag{25}$$

- Electric fields in matter

$$\rho_v = -\boldsymbol{\nabla} \cdot \mathbf{P} \qquad \sigma_v = \mathbf{P} \cdot \mathbf{n} \tag{26}$$

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E} \qquad \mathbf{P} = \varepsilon_0 (\varepsilon - 1)\mathbf{E} \tag{27}$$

- Magnetic fields in matter

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \qquad \mathbf{H} = \frac{\mathbf{B}}{\mu \mu_0} \qquad \mathbf{M} = (1 - \frac{1}{\mu})\frac{\mathbf{B}}{\mu_0} \tag{28}$$

- Electromagnetic wave in vacuum

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 e^{i(kz-\omega t)} \qquad c_0\mathbf{B}(\mathbf{r},t) = \hat{e}_z \times \mathbf{E}(\mathbf{r},t) \qquad \omega = kc_0 \qquad (29)$$

- Electromagnetic wave in a waveguide

$$\nabla^2\mathbf{E}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2\mathbf{E}(\mathbf{r},t)}{\partial t^2} = 0 \qquad \nabla^2\mathbf{H}(\mathbf{r},t) - \frac{1}{c^2}\frac{\partial^2\mathbf{H}(\mathbf{r},t)}{\partial t^2} = 0 \qquad (30)$$

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}(\rho)e^{i(kz-\omega t)} \qquad \mathbf{H}(\mathbf{r},t) = \mathbf{H}(\rho)e^{i(kz-\omega t)} \qquad (31)$$

$$\left(\nabla_\perp^2 + \frac{\omega^2}{c^2} - k^2\right)\mathbf{E}(\rho) = 0 \qquad \left(\nabla_\perp^2 + \frac{\omega^2}{c^2} - k^2\right)\mathbf{H}(\rho) = 0 \qquad (32)$$

- TEM wave in a waveguide

$$\omega = kc \qquad \mathbf{H}(\rho) = Z^{-1}\hat{e}_z \times \mathbf{E}(\rho) \qquad Z = \sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}} \qquad \nabla_\perp^2\mathbf{E}(\rho) = 0 \qquad \nabla_\perp^2\mathbf{H}(\rho) = 0 \quad (33)$$

- Radiation field of a current distribution

$$\mathbf{B}(\mathbf{r},t) = -\frac{\mu_0}{4\pi c_0 r}\frac{\mathbf{r}}{r} \times \int_{V'} \frac{\partial \mathbf{j}(\mathbf{r}',t-\frac{r}{c_0}+\frac{\mathbf{r}}{r}\frac{\mathbf{r}'}{c_0})}{\partial t} d^3\mathbf{r}'$$

$$\mathbf{E}(\mathbf{r},t) = -\frac{\mathbf{r}}{r} \times c_0\mathbf{B}(\mathbf{r},t) \qquad (34)$$

- Dipole radiation of an electric and magnetic dipole

$$\mathbf{B}(\mathbf{r},t) = -\frac{\mu_0}{4\pi c_0 r}\frac{\mathbf{r}}{r} \times \ddot{\mathbf{p}}_e(t-\frac{r}{c_0}) \qquad\qquad \mathbf{E}(\mathbf{r},t) = -\frac{\mathbf{r}}{r} \times c_0\mathbf{B}(\mathbf{r},t)$$

$$\mathbf{B}(\mathbf{r},t) = -\frac{\mu_0}{4\pi c_0^2 r}\frac{\mathbf{r}}{r} \times \left(\frac{\mathbf{r}}{r} \times \ddot{\mathbf{p}}_m(t-\frac{r}{c_0})\right) \qquad \mathbf{E}(\mathbf{r},t) = -\frac{\mathbf{r}}{r} \times c_0\mathbf{B}(\mathbf{r},t) \qquad (35)$$