1. PISNI IZPIT

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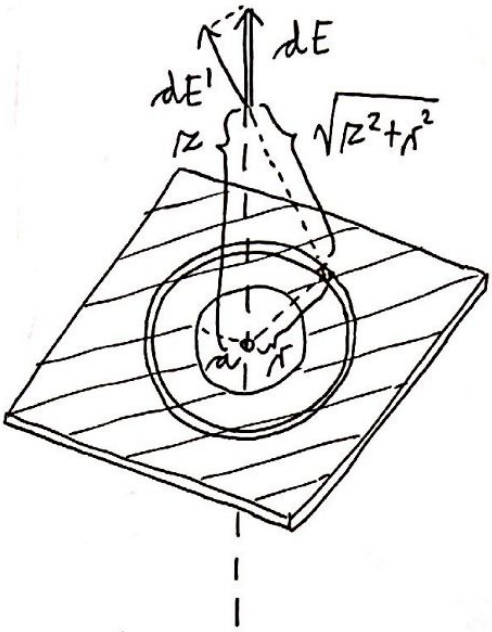


a)

$$dE' = \frac{de}{4\pi\varepsilon_0 (r^2+z^2)}$$

$$de = \sigma\, 2\pi r\, dr \quad \text{za kolobar}$$

$$dE = \frac{z}{\sqrt{r^2+z^2}}\, dE'$$

$$\Downarrow$$

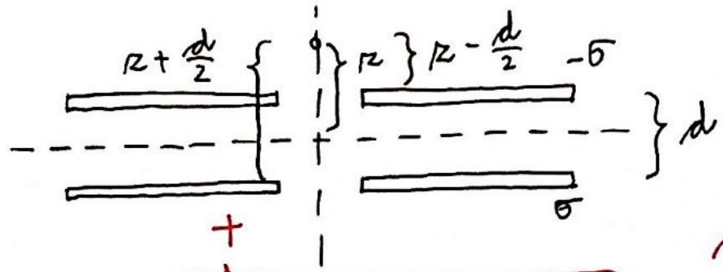
$$dE = \frac{z}{\sqrt{r^2+z^2}} \frac{\sigma r\, dr}{2\varepsilon_0 (r^2+z^2)} = \frac{z\sigma}{4\varepsilon_0} \frac{d(r^2+z^2)}{(r^2+z^2)^{\frac{3}{2}}}$$

1/4

$$E = \frac{z\sigma}{4\varepsilon_0} (-2) \frac{1}{\sqrt{z^2+r^2}}\Bigg|_a^\infty = \boxed{\frac{\sigma}{2\varepsilon_0} \frac{z}{\sqrt{z^2+a^2}}} \rightarrow$$

pri $z \gg a$ ima smiselno limito ravne plošče brez odprtine, $E \rightarrow \frac{\sigma}{2\varepsilon_0}$

b)



$$E_z(z) = E\left(z+\frac{d}{2}\right) - E\left(z-\frac{d}{2}\right) \approx d \cdot \frac{\partial E}{\partial z} =$$

1/4

$$= \frac{\sigma d}{2\varepsilon_0} \frac{1 \cdot \sqrt{z^2+a^2} - z \frac{2z}{2\sqrt{z^2+a^2}}}{z^2+a^2} = \frac{\sigma d}{2\varepsilon_0} \frac{z^2+a^2-z^2}{(z^2+a^2)^{\frac{3}{2}}} =$$

$$= \boxed{\frac{\sigma d}{2\varepsilon_0} \frac{a^2}{(z^2+a^2)^{\frac{3}{2}}}}$$

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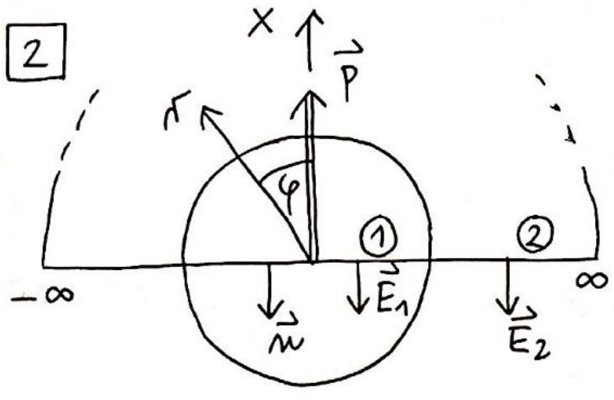
2



$$\left.\begin{array}{l} \sigma_v = \vec{P}\cdot\vec{n} = P\cos\varphi \\ \rho_v = \vec{\nabla}\cdot\vec{P} = 0 \\ \rightarrow U(r,\varphi) = \begin{cases} A r\cos\varphi, & r<a \\ \frac{B}{r}\cos\varphi, & r>a \end{cases} \end{array}\right\} +$$

– RP1: zvezen potencial $\rightarrow \; Aa = \frac{B}{a}, \quad B = Aa^2 \quad \} +$

– RP2: Gaussov izrek na površini $\rightarrow \; \underbrace{\sigma_v}_{e_v} S = \varepsilon_0 S\left(E_\perp^{ZUN} - E_\perp^{NOT}\right)$ +

$$\sigma_v S = \varepsilon_0 S\left(-\frac{\partial U^{ZUN}}{\partial r}\Big|_{r=a} + \frac{\partial U^{NOT}}{\partial r}\Big|_{r=a}\right)$$

$$\sigma_v = \varepsilon_0\left(\frac{B}{a^2} + A\right)\cos\varphi = P\cos\varphi \;\Rightarrow\; P = \varepsilon_0\cdot 2A, \quad A = \frac{P}{2\varepsilon_0}, \quad B = \frac{Pa^2}{2\varepsilon_0} \quad +$$

$$\boxed{U(r,\varphi) = \begin{cases} \frac{P}{2\varepsilon_0} r\cos\varphi, & r<a \\ \frac{Pa^2}{2\varepsilon_0}\frac{1}{r}\cos\varphi, & r>a \end{cases}} \begin{array}{l} = \frac{P}{2\varepsilon_0} x, \quad x = r\cos\varphi \\ = \frac{Pa^2}{2\varepsilon_0}\frac{x}{r^2} \end{array}$$

– integracijska ploskev: čez polovico valja in sklenjena v ∞

$$\frac{1}{4}\left\{\begin{array}{l} E_1 = -\frac{\partial U^{NOT}}{\partial x}\Big|_{x=0} = -\frac{P}{2\varepsilon_0} \\ E_2 = -\frac{\partial U^{ZUN}}{\partial x}\Big|_{x=0} = -\frac{Pa^2}{2\varepsilon_0}\frac{1}{r^2} \end{array}\right\} \begin{array}{l} \vec{E}_{1,2} \parallel \vec{n} \\ (\vec{E}\cdot\vec{n})\vec{E} - \frac{1}{2}E^2\vec{n} = \frac{1}{2}E^2\vec{n} \end{array}$$

$$+\left\{ \vec{F}_e = \varepsilon_0\left[2\int_0^a \frac{1}{2}\left(\frac{P}{2\varepsilon_0}\right)^2 l\,dr + 2\int_a^\infty \frac{1}{2}\left(\frac{Pa^2}{2\varepsilon_0}\right)^2\frac{1}{r^4} l\,dr\right](-\hat{e}_x) = \right.$$

$$= -2\varepsilon_0\,\hat{e}_x\frac{1}{2}\frac{P^2}{4\varepsilon_0^2} l\left(a + a^4\frac{1}{(-3)}\frac{1}{r^3}\Big|_a^\infty\right) =$$

$$+ \quad = -\hat{e}_x\frac{P^2 l}{4\varepsilon_0}\left(a + \frac{1}{3}a\right) = -\hat{e}_x\frac{2P^2 l a}{3\varepsilon_0}$$

$$\boxed{\frac{\vec{F}_e}{l} = -\frac{\cancel{2}P^2 a}{3\varepsilon_0}\hat{e}_x}$$

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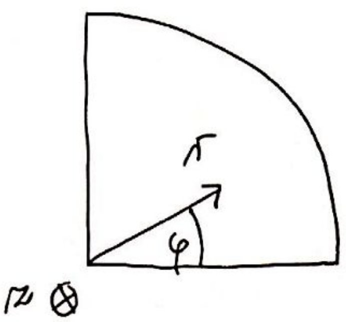
$\boxed{3}$



$$\left[\nabla_\perp^2 + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\right]\begin{Bmatrix}E_z\\ H_z\end{Bmatrix} = 0$$

- disperzijska relacija: $\omega = c_0\sqrt{\varkappa^2 + k^2}$ +

- s plazmo: $c_0 \to \frac{c_0}{\sqrt{\varepsilon}}$, $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$

$$\varkappa^2 = \frac{\omega^2}{c_0}\left(1 - \frac{\omega_p^2}{\omega^2}\right) - k^2 = \frac{\omega^2}{c_0^2} - \frac{\omega_p^2}{c_0^2} - k^2$$

$$\tilde{\omega} = c_0\sqrt{\varkappa^2 + \underbrace{\frac{\omega_p^2}{c_0^2}}_{\to\left(\frac{\pi}{a}\right)^2} + k^2}$$ +

- $\boxed{\text{TM}}$: $E_z \neq 0$

+ RP1: $E_z(\varphi = 0) = 0$, $E_z \propto \sin m\varphi$

RP2: $E_z\left(\varphi = \frac{\pi}{2}\right) = 0$, $\sin m\frac{\pi}{2} = 0 \Rightarrow m = 2, 4, 6, \dots$

($m = 0$ bi vodil do $E_z = 0$)

+ RP3: $E_z(r = a) = 0$, $J_m(\varkappa a) = 0 \Rightarrow \varkappa a = \xi_{mn}$

$$\omega = c_0\sqrt{\left(\frac{\xi_{mn}}{a}\right)^2 + k^2}$$, najmanjši ničli: $\xi_{21} = 5.14$, $\xi_{41} = 7.59$

+ $$\Delta\omega = \omega_{41} - \omega_{21} = (7.59 - 5.14)\frac{c_0}{a} = 2.45\frac{c_0}{a}$$

plazma: $$\Delta\tilde{\omega} = \tilde{\omega}_{41} - \tilde{\omega}_{21} = \frac{c_0}{a}\left(\sqrt{\xi_{41}^2 + \pi^2} - \sqrt{\xi_{21}^2 + \pi^2}\right) = 2.19\frac{c_0}{a}$$

$$\boxed{\frac{\Delta\tilde{\omega}}{\Delta\omega} = 0.89}$$

- $\boxed{\text{TE}}$: $H_z \neq 0$

+ RP1: $\frac{\partial H_z}{\partial\varphi}(\varphi = 0) = 0$, $H_z \propto \cos m\varphi$

RP2: $\frac{\partial H_z}{\partial\varphi}\left(\varphi = \frac{\pi}{2}\right) = 0$, $\sin m\frac{\pi}{2} = 0 \Rightarrow m = 0, 2, 4, 6, \dots$

($m = 0$ je v redu, saj je $H_z \propto \cos m\varphi$)

RP3: $\frac{\partial H_z}{\partial r}(r = a) = 0$, $J_m'(\varkappa a) = 0$

$\varkappa a = \xi_{mn}'$

$+\left\{ \omega' = c_0 \sqrt{\left(\frac{\xi'_{mn}}{a}\right)^2 + k^2} \right.$, najmanjši ničli: $\left[\begin{array}{l} \xi'_{21} = 3.05 \\ \xi'_{01} = 3.83 \end{array} \right.$

$$\Delta\omega' = \omega'_{01} - \omega'_{21} = \frac{c_0}{a}\left(\xi'_{01} - \xi'_{21}\right) = \underline{0.78\,\frac{c_0}{a}}$$

plazma: $\Delta\tilde{\omega}' = \tilde{\omega}'_{01} - \tilde{\omega}'_{21} = \frac{c_0}{a}\left(\sqrt{\xi'^2_{01} + \pi^2} - \sqrt{\xi'^2_{21} + \pi^2}\right) = \underline{0.58\,\frac{c_0}{a}}$

$$\boxed{\frac{\Delta\tilde{\omega}'}{\Delta\omega'} = 0.74}$$

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