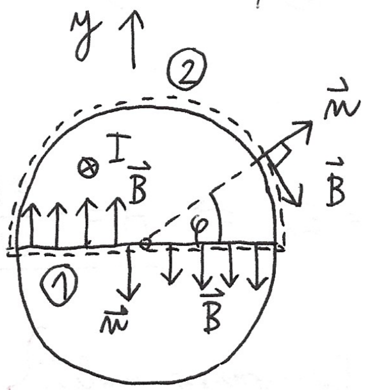
EMP, 1. PISNI IZPIT

1 MAGNETNA SILA V VODNIKU S TOKOM

$$j = \frac{k}{r}, \quad I(r) = \int j\, dS = \int_0^r \frac{k}{r'} 2\pi r' dr' = 2\pi k r \quad \Big\} 1+$$

Ampere: $\mu_0 I(r) = 2\pi r \cdot B$

$$\mu_0 2\pi k r = 2\pi r B \Rightarrow \underline{B = \mu_0 k} \quad \text{konstanta} \quad \Big\} 2+$$



$$\vec{F}_m = \frac{1}{\mu_0} \oint \left[\vec{B}(\vec{B}\cdot\vec{n}) - \frac{1}{2}B^2\vec{n}\right] dS$$

1 $\vec{B} \parallel \vec{n} \Rightarrow \vec{B}(\vec{B}\cdot\vec{n}) = B^2\vec{n}$

$$\vec{F}_{m1} = \frac{1}{2\mu_0} \int B^2 \vec{n}\, dS \quad \Big\} 3+$$

$$4+ \Big\{ \vec{F}_{m1} = \frac{1}{2\mu_0} (\mu_0 k)^2 (-\hat{e}_y)\, 2al = \underline{-\mu_0 k^2 a l\, \hat{e}_y}$$

2 $\vec{B} \perp \vec{n} \Rightarrow \vec{B}\cdot\vec{n} = 0$

$$\vec{F}_{m2} = -\frac{1}{2\mu_0} \int B^2 \vec{n}\, dS \quad \Big\} 5+$$

$$\underbrace{\vec{F}_{m2} = -\frac{1}{2\mu_0}(\mu_0 k)^2 \int_0^\pi \begin{bmatrix} \cos\varphi \\ \sin\varphi \end{bmatrix} a l\, d\varphi}_{6+} = \underbrace{-\frac{1}{2}\mu_0 k^2 a l \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \underline{-\mu_0 k^2 a l\, \hat{e}_y}}_{7+}$$

$$\frac{\vec{F}_m}{l} = \frac{\vec{F}_{m1} + \vec{F}_{m2}}{l} = -2\mu_0 k^2 a\, \hat{e}_y$$

$$I_0 = 2\pi k a \Rightarrow k^2 = \frac{I_0^2}{4\pi^2 a^2}$$

$$\boxed{\frac{\vec{F}_m}{l} = -\frac{\mu_0 I_0^2}{2\pi^2 a}\hat{e}_y} \quad \Big\} 8+$$

$\boxed{2}$ PREREZANA DIELEKTRIČNA KROGLA V POLJU

$$U(r,\vartheta) = \begin{cases} A r\cos\vartheta & , r<a \\ -E_0 r\cos\vartheta + \dfrac{B}{r^2}\cos\vartheta & , r\geq a \end{cases} \quad \Big\} \; 1+$$

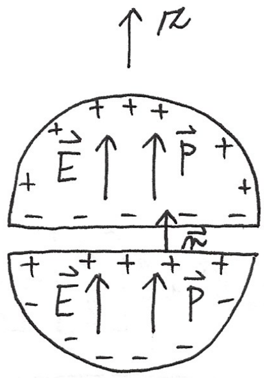
$\boxed{RP1}$ zveznost U :

$\boxed{RP2}$ zveznost $D_\perp = \varepsilon_0 \varepsilon \dfrac{\partial U}{\partial r}$:

$$\left[\begin{array}{ll} A = -E_0 + \dfrac{B}{a^3} & (1) \quad \} \; 2+ \\ -\varepsilon A = E_0 + \dfrac{2B}{a^3} & (2) \quad \} \; 3+ \end{array}\right.$$

$$4+ \begin{cases} (1)+(2): & (\varepsilon-1)A = -\dfrac{3B}{a^3} \\ (1): & A = -E_0 - \dfrac{1}{3}(\varepsilon-1)A \Rightarrow -\dfrac{\varepsilon+2}{3}A = E_0 \Rightarrow \underline{A = -\dfrac{3E_0}{\varepsilon+2}} \end{cases}$$

znotraj krogle: $U = -\dfrac{3E_0}{\varepsilon+2}\overbrace{r\cos\vartheta}^{z} \Rightarrow$ homogeno polje
$$\underline{\vec{E} = \frac{3\vec{E}_0}{\varepsilon+2}} \quad \Big\} \; 5+$$



$$6+ \Big\{ \vec{P} = \varepsilon_0(\varepsilon-1)\vec{E} = \frac{3(\varepsilon-1)}{\varepsilon+2}\varepsilon_0\vec{E}_0$$

$$7+ \begin{cases} \sigma_v = \vec{P}\cdot\vec{n} = \dfrac{3(\varepsilon-1)}{\varepsilon+2}\varepsilon_0 E_0 & \text{spodaj} \\ \vec{E}_v = \dfrac{\sigma_v}{\varepsilon_0}\hat{e}_z = \underline{\dfrac{3(\varepsilon-1)}{\varepsilon+2}\vec{E}_0} & \text{ploščati kondenzator} \end{cases}$$

$$8+ \Big\{ \vec{E}' = \vec{E} + \vec{E}_v = \frac{3\varepsilon-3+3}{\varepsilon+2}\vec{E}_0 = \boxed{\frac{3\varepsilon}{\varepsilon+2}\vec{E}_0}$$

$\boxed{1}$

$\vec{E}$: od nabojev na OBODU krogle in ZUNANJEGA POLJA

$\vec{E}_v$: od nabojev na ravnih ploskvah REŽE

$\boxed{3}$ ENERGIJSKI TOK V VALOVNEM VODNIKU, TEM / TM

$\boxed{\text{TEM}}$

$$\vec{H} = \frac{\vec{k}}{\omega\mu_0} \times \vec{E}$$

$$H_0 = \frac{k}{\omega\mu_0} E_0 = \frac{1}{c_0\mu_0} E_0 = \frac{1}{Z_0} E_0 \Rightarrow E_0 = Z_0 H_0$$ 1+

$$\vec{P} = \vec{E} \times \vec{H} \Rightarrow \langle P \rangle = \frac{1}{2} E_0 H_0 = \frac{1}{2} Z_0 H_0^2$$

$$\left\langle \int \vec{P} \cdot d\vec{S} \right\rangle = \langle P \rangle ab = \underline{\frac{1}{2} ab Z_0 H_0^2}$$ 2+

$\boxed{\text{TM}}$

$$H_z = 0, \quad E_z \neq 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \varkappa^2\right) E_z(x) = 0, \quad E_z(0) = E_z(a) = 0$$

$$\underline{E_z(x) = E_0 \sin \varkappa x}, \quad \varkappa = \frac{n\pi}{a}, \quad n = 1, 2, 3, \ldots$$ 3+

$$H_y = \frac{i}{\varkappa^2} \omega \varepsilon_0 \frac{\partial E_z}{\partial x} = i \underbrace{\frac{\omega \varepsilon_0 E_0}{\varkappa}}_{H_0} \cos \varkappa x$$

$$H_0 \Rightarrow E_0 = \frac{\varkappa H_0}{\omega \varepsilon_0}$$ 4+

$$E_x = \frac{i}{\varkappa^2} k \frac{\partial E_z}{\partial x} = i \frac{k}{\varkappa} E_0 \cos \varkappa x = i \frac{k H_0}{\omega \varepsilon_0} \cos \varkappa x$$ 5+

$$\langle P \rangle = \frac{1}{2} |E_x| |H_y| = \frac{1}{2} \frac{k}{\omega \varepsilon_0} H_0^2 \cos^2 \varkappa x$$

$$\omega^2 = c_0^2 (k^2 + \varkappa^2) = c_0^2 k^2 + \omega_{MIN}^2$$

$$k = \frac{1}{c_0} \sqrt{\omega^2 - \omega_{MIN}^2} \Rightarrow \underline{\frac{k}{\omega} = \frac{1}{c_0} \sqrt{1 - \frac{\omega_{MIN}^2}{\omega^2}}}$$ 6+

7+ $$\langle P \rangle = \frac{1}{2} \underbrace{\frac{1}{\varepsilon_0 c_0}}_{Z_0} H_0^2 \sqrt{1 - \frac{\omega_{MIN}^2}{\omega^2}} \cos^2 \varkappa x$$

$$\int_0^a \int_0^b \cos^2 \varkappa x \, dx \, dy$$

$$\underline{\langle \vec{P} \cdot d\vec{S} \rangle = \frac{1}{2} Z_0 H_0^2 \sqrt{1 - \frac{\omega_{MIN}^2}{\omega^2}} \cdot b \frac{a}{2}}$$

8+ $\boxed{\text{TEM / TM}}$

$$\frac{\langle \vec{P} \cdot d\vec{S} \rangle_{TEM}}{\langle \vec{P} \cdot d\vec{S} \rangle_{TM}} = \boxed{\frac{2}{\sqrt{1 - \frac{\omega_{MIN}^2}{\omega^2}}}} \geqslant 2 \Rightarrow \underline{\text{TEM je ugodnejši}}$$

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