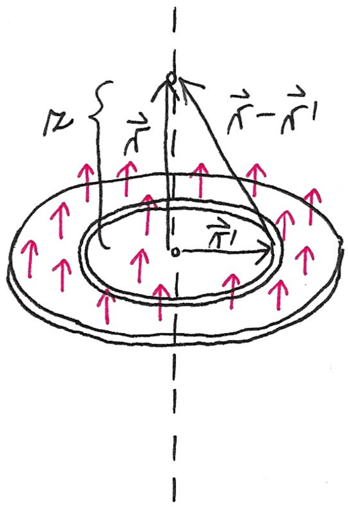
EMP, 1. A PISNI IZPIT

1 OKROGLA PLOŠČA Z ELEKTRIČNIMI DIPOLI



a) $V_{DIP} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{\vec{p}_{ei}(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$ za en dipol

$dV = \dfrac{1}{4\pi\varepsilon_0}\dfrac{d\vec{p}_e\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$ za porazdelitev dipolov

(1)+ $\left\{ d\vec{p}_e\cdot(\vec{r}-\vec{r}\,') = dp_e\cdot z \right.$, $|\vec{r}-\vec{r}\,'| = \sqrt{z^2+r'^2}$

(2)+ $\left\{ dp_e = \underbrace{2\pi r'\,dr'}_{dS'}\cdot\lambda_e \right.$

(3)+
$$dV = \frac{1}{4\pi\varepsilon_0}\frac{2\pi r'\,dr'\,z\,\lambda_e}{(z^2+r'^2)^{\frac{3}{2}}} = \frac{\lambda_e z}{4\varepsilon_0}\frac{d(z^2+r'^2)}{(z^2+r'^2)^{\frac{3}{2}}}$$
$$V(z) = \frac{\lambda_e z}{4\varepsilon_0}\int_0^a \frac{d(z^2+r'^2)}{(z^2+r'^2)^{\frac{3}{2}}} = \frac{\lambda_e z}{4\varepsilon_0}(-2)\frac{1}{\sqrt{z^2+r'^2}}\Bigg|_0^a$$

(4)+
$$V(z) = -\frac{\lambda_e z}{2\varepsilon_0}\left(\frac{1}{\sqrt{z^2+a^2}}-\frac{1}{|z|}\right) = \boxed{\frac{\lambda_e}{2\varepsilon_0}\left(\frac{z}{|z|}-\frac{z}{\sqrt{z^2+a^2}}\right)}$$

$\frac{z}{|z|} \rightarrow \mathrm{sgn}(z) = \pm 1$

b) $z \gg a$ & $z > 0$

(5)+
$$V(z) = \frac{\lambda_e}{2\varepsilon_0}\left(1-\left(1+\frac{a^2}{z^2}\right)^{-\frac{1}{2}}\right) \approx \frac{\lambda_e}{2\varepsilon_0}\left(1-1+\frac{1}{2}\frac{a^2}{z^2}\right) = \frac{\lambda_e a^2}{4\varepsilon_0 z^2}$$

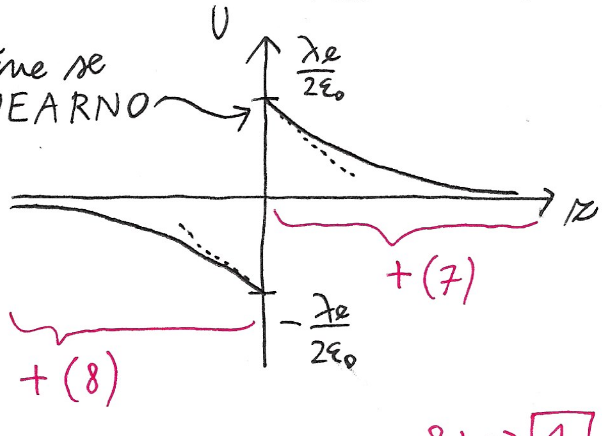
(6)+
$$V(z) = \frac{\lambda_e \pi a^2}{4\pi\varepsilon_0 z^2} = \boxed{\frac{p_e}{4\pi\varepsilon_0 z^2}}$$
dipolni člen za $\boxed{\cos\vartheta = 1,\ \vartheta = \frac{\pi}{2}}$

c) $z \to \infty$: $V \to 0$

$z \to -\infty$: $V \to 0$

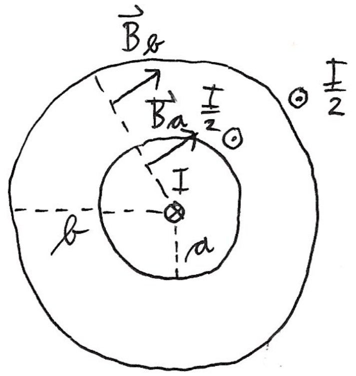
$z \to +0$: $\underbrace{V \to \frac{\lambda_e}{2\varepsilon_0}\left(1-\frac{z}{a}\right)}$ → začne se LINEARNO

$z \to -0$: $V \to \frac{\lambda_e}{2\varepsilon_0}\left(-1-\frac{z}{a}\right)$



8+ → 1

2 POSEBEN KOAKSIALNI VODNIK



– magnetno polje

(1)+ $\underline{r < a}$: $\mu_0 I = 2\pi r B_a \Rightarrow \underline{B_a = \frac{\mu_0 I}{2\pi r}}$

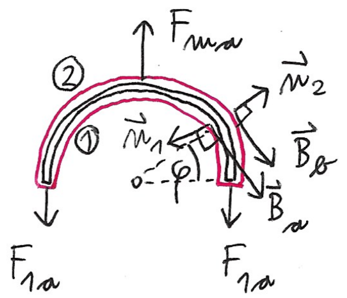
(2)+ $\underline{a < r < b}$: $\mu_0\left(I - \frac{I}{2}\right) = 2\pi r B_b$

$$\underline{B_b = \frac{\mu_0 I}{4\pi r}}$$

$\underline{r > b}$: $\mu_0\left(I - \frac{I}{2} - \frac{I}{2}\right) = 2\pi r B_{ZUN}$

$$\underline{B_{ZUN} = 0}$$

– notranja cev



(3)+ $\vec B \cdot \vec n = 0$ na obeh straneh

$\vec F_{ma} = \frac{1}{\mu_0}\oint\left(-\frac{1}{2}\right)B^2\vec n\,dS$, $dS = la\,d\varphi$

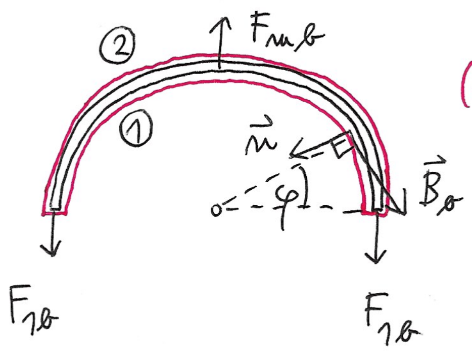
(4)+ $\vec F_{ma} = -\frac{1}{2\mu_0}\int_1 B_a^2(a)\vec n_1\,dS - \frac{1}{2\mu_0}\int_2 B_b^2(a)\vec n_2\,dS$

$$\vec F_{ma} = +\frac{la}{2\mu_0}\left[\left(\frac{\mu_0 I}{2\pi a}\right)^2 - \left(\frac{\mu_0 I}{4\pi a}\right)^2\right]\int_0^{\pi}\begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix}d\varphi$$

(5)+ $\vec F_{ma} = \frac{la}{2\mu_0}\frac{\mu_0^2 I^2}{16\pi^2 a^2}(4-1)\begin{bmatrix}0\\ 2\end{bmatrix} = \underline{\frac{3\mu_0 I^2 l}{16\pi^2 a}\hat e_y}$

(6)+ $F_{1a} = \frac{1}{2}F_{ma} \Longrightarrow \boxed{\frac{F_{1a}}{l} = \frac{3\mu_0 I^2}{32\pi^2 a}}$

– zunanja cev



(7)+ $\vec F_{mb} = -\frac{1}{2\mu_0}\int_1 B_b^2(b)\vec n\,dS$

$$\vec F_{mb} = \frac{la}{2\mu_0}\left(\frac{\mu_0 I}{4\pi b}\right)^2\int_0^{\pi}\begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix}d\varphi$$

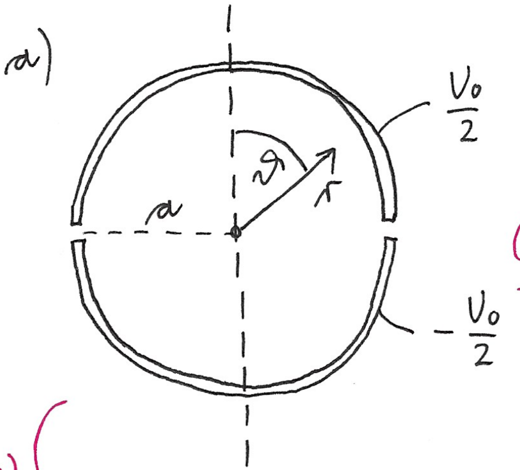
$$\vec F_{mb} = \frac{la}{2\mu_0}\frac{\mu_0^2 I^2}{16\pi^2 b^2}\begin{bmatrix}0\\ 2\end{bmatrix} = \frac{\mu_0 I^2 l}{16\pi^2 b}\hat e_y$$

8+ → $\boxed{1}$

(8)+ $F_{1b} = \frac{1}{2}F_{mb} \Rightarrow \boxed{\frac{F_{1b}}{l} = \frac{\mu_0 I^2}{32\pi^2 b}}$

3 PREPOLOVLJENA PREVODNA KROGELNA LUPINA

a)



$\boxed{RP}$: $$U(a,\vartheta) = \left\{ \begin{matrix} \frac{V_0}{2} \ , & 0<\vartheta<\frac{\pi}{2} \\ -\frac{V_0}{2} \ , & \frac{\pi}{2}<\vartheta<\pi \end{matrix} \right\} = f(\vartheta)$$

(1) + $\boxed{r<a}$: $$\underbrace{U(r,\vartheta) = \sum_{\ell=0}^{\infty} A_\ell \left(\frac{r}{a}\right)^{\ell} P_\ell(\cos\vartheta)}$$

RP : $$U(a,\vartheta) = \sum_{\ell=0}^{\infty} A_\ell \, P_\ell(\cos\vartheta) \Big/ \int_{-1}^{1} P_{\ell'}(\cos\vartheta)\, d(\cos\vartheta)$$

(2) +

$$\underbrace{-\frac{V_0}{2}\int_{-1}^{0} P_{\ell'}(x)\,dx}_{+\int_0^1 P_{\ell'}(-x)\,dx \,=\, (-1)^{\ell'}\int_0^1 P_{\ell'}(x)\,dx} + \frac{V_0}{2}\int_{0}^{1} P_{\ell'}(x)\,dx = \sum_{\ell=0}^{\infty} A_\ell \underbrace{\int_{-1}^{1} P_\ell(x) P_{\ell'}(x)\,dx}_{\frac{2}{2\ell+1}\delta_{\ell\ell'}} = A_{\ell'}\frac{2}{2\ell'+1}$$

(3) +

$\underline{\text{sodi } \ell'}$ $\rightarrow$ $\underline{A_{\ell'} = 0}$

$\underline{\text{lihi } \ell'}$ $\rightarrow$ $$\underline{A_{\ell'} = \frac{2\ell'+1}{2} V_0 \int_0^1 P_{\ell'}(x)\,dx}$$

(4) +

$$A_1 = \frac{3}{2}V_0 \int_0^1 x\,dx = \underline{\frac{3}{4}V_0}$$

$$A_3 = \frac{7}{2}V_0 \int_0^1 \frac{1}{2}(5x^3-3x)\,dx = \frac{7}{2}V_0\left(\frac{5}{8}-\frac{3}{4}\right) = \underline{-\frac{7}{16}V_0}$$

$$\boxed{U(r,\vartheta) = \frac{3}{4}V_0 \frac{r}{a} P_1(\cos\vartheta) - \frac{7}{16}V_0\left(\frac{r}{a}\right)^3 P_3(\cos\vartheta) + \dots}$$

$\boxed{r>a}$: $$\underbrace{U(r,\vartheta) = \sum_{\ell=0}^{\infty} B_\ell \left(\frac{a}{r}\right)^{\ell+1} P_\ell(\cos\vartheta)}$$ + (5)

RP : $$U(a,\vartheta) = \sum_{\ell=0}^{\infty} B_\ell \, P_\ell(\cos\vartheta)$$ povsem ENAKA enačba kot za $r<a$

$\Downarrow$

(6) +

$\underline{B_\ell = 0}$, sodi ℓ

$\underline{B_1 = \frac{3}{4}V_0}$

$\underline{B_3 = -\frac{7}{16}V_0}$

$$\boxed{V(r,\vartheta) = \frac{3}{4} V_0 \left(\frac{a}{r}\right)^2 P_1(\cos\vartheta) - \frac{7}{16} V_0 \left(\frac{a}{r}\right)^4 P_3(\cos\vartheta) + \cdots}$$

b) - središče krogelne lupine, $r = 0$

↳ ko odvajamo po r ali $\cos\vartheta$ in postavimo $r=0$, preživi LE PRVI ČLEN

(7)+

$$V(r,\vartheta) = \frac{3}{4} V_0 \frac{r}{a} \cos\vartheta + \cdots = \frac{3}{4} V_0 \frac{z}{a} + \cdots$$

$$E_z = -\frac{\partial V}{\partial r}\Big|_{r=0} = \boxed{-\frac{3}{4}\frac{V_0}{a}}$$

- skupni p_e → v izrazu za $r > a$ moramo prepoznati DIPOLNI ČLEN

(8)+

$$V(r,\vartheta) = \frac{3}{4} V_0 \left(\frac{a}{r}\right)^2 \cos\vartheta + \cdots = \underbrace{\frac{3}{4} V_0 a^2 \frac{\cos\vartheta}{r^2}}_{\text{ravno dipolni člen}} + \cdots$$

$$\frac{3}{4} V_0 a^2 \frac{\cos\vartheta}{r^2} = \frac{p_e}{4\pi\varepsilon_0} \frac{\cos\vartheta}{r^2}$$

$$\Downarrow$$

$$\boxed{p_e = 3\pi\varepsilon_0 V_0 a^2}$$

Vsi ostali načini izračuna obeh količin so veliko bolj ZAMUDNI.

8+ → $\boxed{1}$