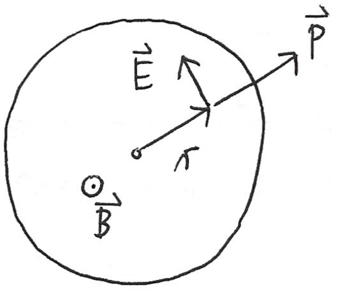
EMP, 1. B PISNI IZPIT

1 UGAŠANJE TULJAVE



a) $B = \frac{\mu_0 N I}{l}$, $I = I_0 - \alpha t$

+(1) $\nabla \times \vec E = -\frac{\partial \vec B}{\partial t}$ $\quad / \int d\vec S$

$\oint \vec E \cdot d\vec l = -\frac{\partial}{\partial t}\int \vec B \cdot d\vec S$

$2\pi r E = -\dot B \pi r^2 \Rightarrow \underline{E = -\dot B \frac{r}{2}}$

+(2) $\dot B = \frac{\mu_0 N}{l}\dot I = \frac{\mu_0 N}{l}(-\alpha) \Rightarrow \underline{E = \frac{\mu_0 N \alpha}{l}\cdot \frac{r}{2}}$

b) $\vec P = \frac{1}{\mu_0}\vec E \times \vec B$ → ima smer VEN IZ PLAŠČA tuljave

+(3) $P = \frac{1}{\mu_0} E B = \frac{1}{\mu_0}\frac{\mu_0 N \alpha}{l}\frac{r}{2}\frac{\mu_0 N I}{l} = \frac{\mu_0 N^2}{2 l^2}\alpha I \cdot r$

↳ $I_0 - \alpha t$

+(4) $\int P dS = P(a)\cdot 2\pi a l = \boxed{\frac{\mu_0 N^2 \pi a^2}{l}\alpha I} = L \alpha I$

c) +(5) $w_e = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0 \mu_0^2 N^2 \alpha^2}{8 l^2}\cdot r^2 \Rightarrow \underline{\dot w_e = 0}$,

ker E NI ČASOVNO odvisen!

+(6) $w_m = \frac{1}{2\mu_0}B^2 = \frac{\mu_0 N^2 I^2}{2 l^2}$ → NI ODVISEN od r

$W_m = w_m \cdot V = \frac{\mu_0 N^2 I^2}{2 l^2}\cdot \pi a^2 l = \frac{\mu_0 N^2 \pi a^2}{l}\cdot\frac{I^2}{2}$

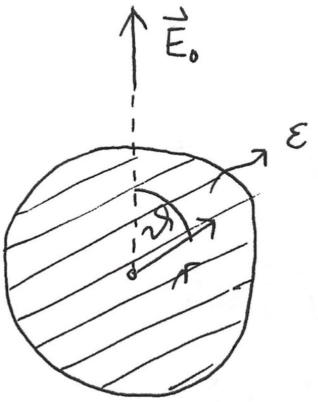
$\underbrace{\dot W_m = +\frac{\mu_0 N^2 \pi a^2}{l}\cdot 2\frac{I \dot I}{2}}_{+(7)} = \boxed{-\frac{\mu_0 N^2 \pi a^2}{l}\alpha I}$ +(8)

$\Downarrow$

$\boxed{\int P dS + \frac{\partial}{\partial t}(W_m + W_e) = 0}$

8+ = $\boxed{1}$

$\boxed{2}$ DIELEKTRIČNA KROGLA



$$U(r,\vartheta) = \begin{cases} \overbrace{-E_0 r\cos\vartheta + \frac{B}{r^2}\cos\vartheta}^{+(1)}, & r > a \\ A r\cos\vartheta \; \} +(2), & r < a \end{cases}$$

Zaradi RP v neskončnosti, $-E_0 r\cos\vartheta$, so namreč mogoči le členi s $\cos\vartheta$ ($l=1$).

RP1 $\underline{U_{NOT}(a,\vartheta) = U_{ZUN}(a,\vartheta)} \rightarrow Aa = -E_0 a + \frac{B}{a^2}$ } +(3)

$$A + E_0 = \frac{B}{a^3}$$

RP2 $\underline{D_{\perp,NOT}(a,\vartheta) = D_{\perp,ZUN}(a,\vartheta)}$

$$\varepsilon_0\varepsilon\left(-\frac{\partial U_{NOT}}{\partial r}\right)_a = \varepsilon_0\left(-\frac{\partial U_{ZUN}}{\partial r}\right)_a \rightarrow -\varepsilon A = E_0 + 2B\frac{1}{a^3} \quad \} +(4)$$

$$-\varepsilon A = E_0 + 2(A+E_0)$$

$$-(\varepsilon+2)A = 3E_0$$

$\Leftarrow$

+(5) $\left\{ \begin{aligned} A &= -\frac{3}{\varepsilon+2}E_0 \\ B &= a^3(A+E_0) = \frac{\varepsilon-1}{\varepsilon+2}a^3E_0 \end{aligned} \right.$

a) +(6)

$$U_{NOT}(r,\vartheta) = -\frac{3}{\varepsilon+2}E_0 r\cos\vartheta = \underline{-\frac{3}{\varepsilon+2}E_0 z} \rightarrow \text{potencial HOMOGENEGA polja}$$

$$\vec{E}_{NOT} = -\frac{\partial U}{\partial z}\hat{e}_z = \boxed{\frac{3}{\varepsilon+2}\vec{E}_0}$$

b) DVA načina

b1) +(7)

$$\vec{D}_{NOT} = \varepsilon_0\varepsilon\vec{E}_{NOT}$$

$$\vec{P} = \vec{D}_{NOT} - \varepsilon_0\vec{E}_{NOT} = \varepsilon_0(\varepsilon-1)\vec{E}_{NOT}$$

+(8) $\boxed{\vec{P} = 3\varepsilon_0\frac{\varepsilon-1}{\varepsilon+2}\vec{E}_0}$

b2) iz DRUGEGA člena U_{ZUN}:

+(7) $\vec{p}_e = 4\pi\varepsilon_0\frac{\varepsilon-1}{\varepsilon+2}E_0 a^3$

+(8) $\vec{P} = \frac{1}{V}\vec{p}_e = \boxed{3\varepsilon_0\frac{\varepsilon-1}{\varepsilon+2}\vec{E}_0}$

$V \hookrightarrow \frac{4}{3}\pi a^3$

8+ = $\boxed{1}$

3 ENERGIJA V VALOVNEM VODNIKU

+(1)

TM način → $H_z = 0$, $E_z \neq 0$

$$\left[\frac{\partial^2}{\partial x^2} + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\right] E_z(x) = 0 \quad , \quad \text{RP}: \; E_z(0) = E_z(a) = 0$$

$$E_z(x) = E_0 \sin \varkappa x \quad , \quad \varkappa = \frac{n\pi}{a} \quad , \quad n = 1, 2, 3, \ldots \quad \text{(vse to dobro znano)}$$

a)
$$H_y = \frac{i}{\varkappa^2}\left(k \underbrace{\frac{\partial H_z}{\partial y}}_{0} + \omega \varepsilon_0 \frac{\partial E_z}{\partial x}\right) = i \underbrace{\frac{\omega \varepsilon_0 E_0}{\varkappa}}_{H_0} \cos \varkappa x = \boxed{i H_0 \cos \varkappa x}$$

$$H_0 \Rightarrow E_0 = \frac{\varkappa H_0}{\omega \varepsilon_0}$$

+(2)

+(3)
$$E_x = \frac{i}{\varkappa^2}\left(k \frac{\partial E_z}{\partial x} + \omega \mu_0 \underbrace{\frac{\partial H_z}{\partial y}}_{0}\right) = i \frac{k}{\varkappa} E_0 \cos \varkappa x = \boxed{i \frac{k H_0}{\omega \varepsilon_0} \cos \varkappa x}$$

$$E_z = E_0 \sin \varkappa x = \boxed{\frac{\varkappa H_0}{\omega \varepsilon_0} \sin \varkappa x}$$

+(4)
$$\langle w_m \rangle = \left\langle \frac{1}{2} \mu_0 H^2 \right\rangle = \frac{1}{4} \mu_0 |H_y|^2 = \frac{1}{4} \mu_0 H_0^2 \cos^2 \varkappa x$$

$$\int \langle w_m \rangle \, dS = \int_0^a \int_0^b \frac{1}{4} \mu_0 H_0^2 \cdot \cos^2 \varkappa x \, dx \, dy = \boxed{\frac{1}{4} \mu_0 H_0^2 \cdot \frac{a}{2} b}$$

+(5)
$$\langle w_e \rangle = \left\langle \frac{1}{2} \varepsilon_0 E^2 \right\rangle = \frac{\varepsilon_0}{4}\left(|E_x|^2 + |E_z|^2\right) = \frac{H_0^2}{4 \varepsilon_0 \omega^2}\left(k^2 \cos^2 \varkappa x + \varkappa^2 \sin^2 \varkappa x\right)$$

+(6)
$$\int \langle w_e \rangle \, dS = \int_0^a \int_0^b \frac{H_0^2}{4 \varepsilon_0 \omega^2}\left(k^2 \cos^2 \varkappa x + \varkappa^2 \sin^2 \varkappa x\right) dx \, dy =$$

$$= \frac{H_0^2}{4 \varepsilon_0 \omega^2} \underbrace{\left(k^2 + \varkappa^2\right)}_{\frac{\omega^2}{c_0^2}} \frac{a}{2} b = \frac{H_0^2}{4 \varepsilon_0 \underbrace{c_0^2}_{\frac{1}{\varepsilon_0 \mu_0}}} \cdot \frac{a}{2} b = \boxed{\frac{1}{4} \mu_0 H_0^2 \cdot \frac{a}{2} b}$$

⇓

$$\boxed{\int \langle w_m \rangle \, dS = \int \langle w_e \rangle \, dS = \frac{1}{8} \mu_0 H_0^2 a b}$$

b) $$\frac{\omega^2}{c_0^2} - k^2 = \varkappa^2 \Rightarrow \omega = c_0\sqrt{k^2+\varkappa^2} = \sqrt{c_0^2k^2 + \underbrace{c_0^2\varkappa^2}_{\omega_{MIN}^2}}$$

+(7)

$$\underline{\omega = \sqrt{c_0^2k^2 + \omega_{MIN}^2}}$$ disperzijska relacija

$$v_g = \frac{\partial\omega}{\partial k} = \frac{2c_0^2k}{2\underbrace{\sqrt{c_0^2k^2+\omega_{MIN}^2}}_{\omega}} = \frac{c_0^2k}{\omega}$$

$$k = \frac{1}{c_0}\sqrt{\omega^2-\omega_{MIN}^2} \Rightarrow v_g = c_0\frac{\sqrt{\omega^2-\omega_{MIN}^2}}{\omega} = \boxed{c_0\sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}}}$$

+(8)

$$v_t = \frac{\int\langle p\rangle dS}{\int\langle w_e\rangle dS + \int\langle w_m\rangle dS} = \frac{\frac{1}{4}Z_0H_0^2ab\sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}}}{2\cdot\frac{1}{8}\mu_0H_0^2ab} =$$

$$= \underbrace{\frac{Z_0}{\mu_0}}\sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}} = \boxed{c_0\sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}}} \Rightarrow \boxed{v_t = v_g}$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{1}{\mu_0} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = c_0$$

To pomeni, da se energija EM valovanja res širi z grupno hitrostjo.

8+ = $\boxed{1}$