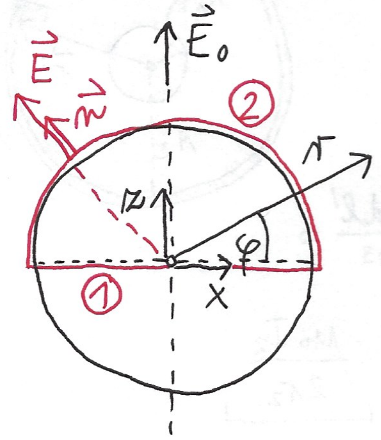
EMP, 2. PISNI IZPIT

1 KOVINSKI VALJ V ELEKTRIČNEM POLJU



+(1) $U_0(r,\varphi) = -E_0 r \sin\varphi$ homogeno polje

$$\Downarrow$$

$$\underline{U(r,\varphi) = -E_0 r \sin\varphi + \frac{B}{r}\sin\varphi}, \quad r > a$$

+(2) $\boxed{RP}$: $U(a,\varphi) = 0 \Rightarrow B = E_0 a^2$

$$\underline{U(r,\varphi) = -E_0 r \sin\varphi + \frac{E_0 a^2}{r}\sin\varphi}$$

+(3) – električno polje na površini krogle je PRAVOKOTNO na površino

$$E(a,\varphi) = E_r(a,\varphi) = -\frac{\partial U}{\partial r}\Big|_{r=a} = E_0 \sin\varphi + E_0\frac{a^2}{a^2}\sin\varphi = \underline{2E_0\sin\varphi}$$

– izračun SILE

+(4)
① $\vec{F}_{e1} = 0$, ker znotraj krogle NI POLJA

② $\vec{E} \parallel \vec{n} \Rightarrow \vec{E}(\vec{E}\cdot\vec{n}) = \vec{E}E = E^2\vec{n}$

$$\vec{F}_{e2} = \varepsilon_0 \int \left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS = \underline{\frac{\varepsilon_0}{2}\int E^2 \vec{n}\, dS}$$

+(5)

$$\vec{n} = \begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix}, \quad E = 2E_0\sin\varphi, \quad dS = l\,a\,d\varphi, \ \varphi \in [0,\pi]$$

+(6)

$$\vec{F}_{e2} = \frac{\varepsilon_0}{2}\int_0^{\pi} 4E_0^2 \sin^2\varphi \begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix} l\,a\,d\varphi =$$

+(7)

$$= 2\varepsilon_0 E_0^2\, l\, a \int_0^{\pi} \begin{bmatrix}\sin^2\varphi\, d(\sin\varphi)\\ -(1-\cos^2\varphi)\, d(\cos\varphi)\end{bmatrix} =$$

+(8)

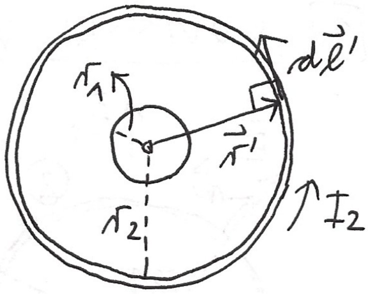
$$= 2\varepsilon_0 E_0^2\, l\, a \begin{bmatrix}\frac{1}{3}\sin^3\varphi\\ -\cos\varphi + \frac{1}{3}\cos^3\varphi\end{bmatrix}\Bigg|_{\varphi=0}^{\pi} = 2\varepsilon_0 E_0^2\, l\, a \begin{bmatrix}0\\ 2-\frac{2}{3}\end{bmatrix}$$

$$\boxed{\frac{\vec{F}_e}{l} = \frac{\vec{F}_{e2}}{l} = \frac{8}{3}\varepsilon_0 E_0^2\, a \cdot \hat{e}_y}$$

8+ = $\boxed{1}$

2 INDUKCIJA V DVEH ZANKAH

a) – magnetno polje v središču velike zanke



$$\vec{B}(\vec{r}) = \frac{\mu_0 I_2}{4\pi}\int \frac{d\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

+(1) $$\vec{B}(\vec{r}=0) = \frac{\mu_0 I_2}{4\pi}\int\frac{\vec{r}'\times d\vec{l}'}{r'^3} = \frac{\mu_0 I_2}{4\pi}\int\frac{r'\,dl'}{r'^3} =$$

+(2) $$= \frac{\mu_0 I_2}{4\pi}\int\frac{dl'}{r_2^2} = \frac{\mu_0 I_2}{4\pi r_2^2}\underbrace{\int dl'}_{2\pi r_2} = \underline{\frac{\mu_0 I_2}{2 r_2}}$$

– zaradi $r_1 \ll r_2$, lahko vzamemo, da je takšno polje v celotni manjši zanki

+(3) $$B_1 = \frac{\mu_0 I_2}{2r_2} \Rightarrow \Phi_1 = B_1 \pi r_1^2 = \underbrace{\frac{\pi\mu_0}{2}\frac{r_1^2}{r_2}}_{L_{12}} I_2 \Rightarrow \boxed{L_{12} = \frac{\pi\mu_0}{2}\frac{r_1^2}{r_2}}$$

b) +(4) $$\underline{I_1 = I_0\left(1-\frac{t}{t_0}\right)}$$, saj je potem res $I_1(t=t_0)=0$

+(5) $$U_{g2} = R_2 I_2 + L_2\dot{I}_2 + L_{21}\dot{I}_1$$ ← enačba za tokokrog druge zanke

$U_{g2} \downarrow 0$, $R_2 I_2 \downarrow 0$

+(6) $$\dot{I}_2 = -\frac{L_{21}}{L_2}\dot{I}_1 = \frac{L_{21}}{L_2}\frac{I_0}{t_0}$$

$$\frac{L_{21}}{L_2} = \frac{\frac{\pi\mu_0}{2}\frac{r_1^2}{r_2}}{\mu_0 r_2 \ln(8\alpha)} = \frac{\pi}{2}\left(\frac{r_1}{r_2}\right)^2\frac{1}{\ln(8\alpha)}$$

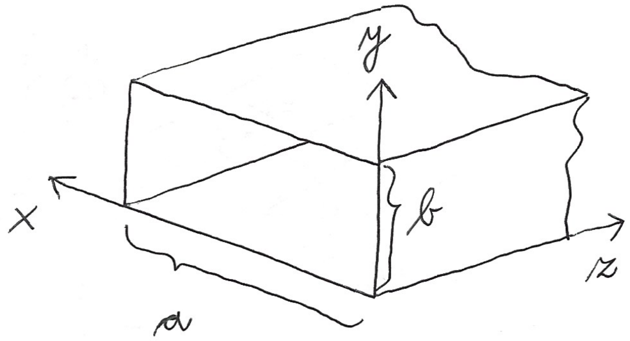
+(7) $$I_2(t) = \frac{L_{21}}{L_2}\frac{I_0}{t_0}t = \boxed{\frac{\pi}{2}\left(\frac{r_1}{r_2}\right)^2\frac{1}{\ln(8\alpha)} I_0 \frac{t}{t_0}}$$

+(8) $$I_2^{FIN} = \underline{I_2(t=t_0)} = \boxed{\frac{\pi}{2}\left(\frac{r_1}{r_2}\right)^2\frac{1}{\ln(8\alpha)} I_0}$$ končna vrednost toka

8+ = $\boxed{1}$

$\boxed{3}$ ELEKTRIČNI TOKOVI NA POVRŠINI VALOVNEGA VODNIKA

TE način $\rightarrow$ $E_z = 0$, $H_z \neq 0$



+ (1)

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\right] \underbrace{H_z(x,y)}_{X(x)Y(y)} = 0$$

$$X''Y + XY'' + \varkappa^2 XY = 0 \Rightarrow \underbrace{\frac{X''}{X}}_{-\varkappa_x^2} = -\underbrace{\frac{Y''}{Y}}_{-\varkappa_y^2} - \varkappa^2 \Rightarrow \varkappa_x^2 + \varkappa_y^2 = \varkappa^2$$

cos

$$X'' + \varkappa_x^2 X = 0 \Rightarrow X \propto \cos \varkappa_x x$$

$$Y'' + \varkappa_y^2 Y = 0 \Rightarrow Y \propto \cos \varkappa_y y$$

če že upoštevamo $\boxed{RP1}$ $\frac{\partial H_z}{\partial n}(x=0 \text{ ali } y=0) = 0$

+ (2)

$\boxed{RP2}$ $\frac{\partial H_z}{\partial n}(x=a \text{ ali } y=b) = 0 \Rightarrow \varkappa_x = \frac{m\pi}{a}, \quad \varkappa_y = \frac{n\pi}{b}$

$$\varkappa_x^2 + \varkappa_y^2 = \varkappa^2 \Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \frac{\omega^2}{c_0^2} - k^2$$

veja

$$\omega_{mn} = c_0 \sqrt{k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$m = 0, 1, 2, \ldots$ $n = 0, 1, 2, \ldots$ NE oba 0!

$b < a$ $\Rightarrow$ NAJNIŽJA veja ima $\boxed{m = 1, \; n = 0}$

$\Downarrow$

$\varkappa = \frac{\pi}{a} \leftarrow H_z(x,y) = H_0 \cos \frac{\pi x}{a}$ NI odvisnosti od y!

+ (3)

polja

podane 4 ENAČBE : $E_x = 0$ $\qquad E_0 \rightarrow$ PODANA amplituda

$$\boxed{E_y} = i \overbrace{H_0 \frac{\omega \mu_0 a}{\pi}}^{E_0} \sin \frac{\pi x}{a} = \boxed{i E_0 \sin \frac{\pi x}{a}}$$

$$H_0 = E_0 \frac{\pi}{\omega \mu_0 a} \Leftarrow$$

$$\boxed{H_x} = -i H_0 \frac{k a}{\pi} \sin \frac{\pi x}{a} = \boxed{-i E_0 \frac{k}{\omega \mu_0} \sin \frac{\pi x}{a}}$$

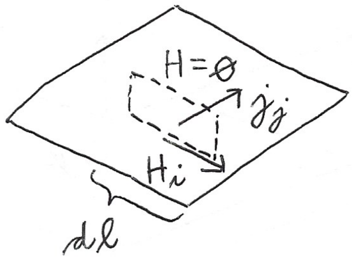
$\downarrow$

$$\boxed{H_z = E_0 \frac{\pi}{\omega \mu_0 a} \cos \frac{\pi x}{a}}$$

$H_y = 0$

- izpeljava POVRŠINSKEGA TOKA iz ustreznega RP

ustrezna Maxwellova enačba v integralski obliki



+ (4)

$$\vec{\nabla}\times\vec{H} = \vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \Big/ \int d\vec{S}$$

$$\underbrace{\oint \vec{H}\cdot d\vec{l}}_{H_i\,dl} = \underbrace{\int \vec{j}\cdot d\vec{S}}_{dI_j} + \varepsilon_0 \frac{\partial}{\partial t}\underbrace{\int \vec{E}\cdot d\vec{S}}$$

gre proti NIČ, ko narisano zanko ZOŽIMO

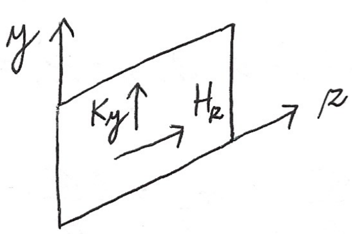
+ (5)

$$\Downarrow$$

$$\boxed{H_i = \frac{dI_j}{dl} = K_j}\ , \text{ kjer je } \underline{i \perp j}$$

izpeljana enačba za K_j

a) - površinski tokovi v NAVPIČNIH STENAH



+ (6) $K_z = H_y = \underline{0} \Longrightarrow$ NAVPIČNA smer!

+ (7)

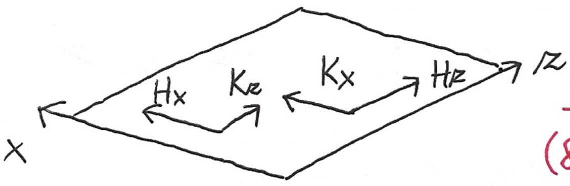
$$K_y = H_z = \frac{E_0 \pi}{\mu_0 \omega a}\cos\frac{\pi x}{a}\Big|_{x=0,a}$$

$$\boxed{K_y = \frac{E_0\pi}{\mu_0\omega a}\begin{cases} 1, & x = 0 \\ -1, & x = a \end{cases}}$$

NASPROTNA na nasprotnih stenah

NEODVISNA od y, $\underline{K_{y0} = \frac{E_0\pi}{\mu_0\omega a}}$

b) - površinski tokovi v VODORAVNIH STENAH



+ (8)

$$K_x = H_z = \frac{E_0\pi}{\mu_0\omega a}\cos\frac{\pi x}{a}$$

$$\boxed{K_x\left(x = \frac{a}{2}\right) = \underline{0}}$$

na SREDINSKIH črtah NI PREČNEGA toka

$$\underline{|K_x|(x = 0, a) = \frac{E_0\pi}{\mu_0\omega a}}$$

8+ = $\boxed{1}$