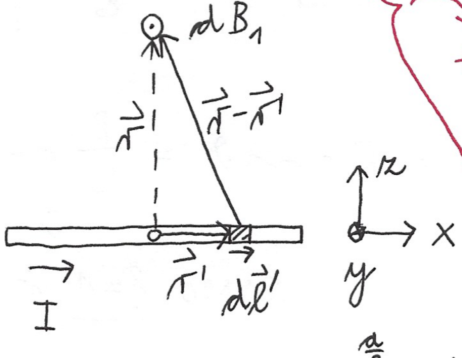
EMP : 2. PISNI IZPIT

1 TRIKOTNI OKVIR S TOKOM

a) – ena STRANICA

$$\vec{B}_1 = \frac{\mu_0 I}{4\pi}\int \underbrace{\frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}}$$



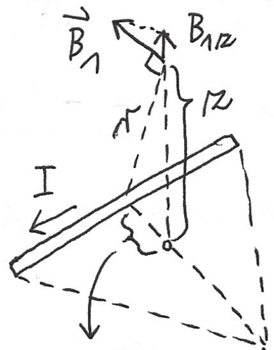
+(1)

$$\frac{\begin{bmatrix} dx' \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} -x' \\ 0 \\ r \end{bmatrix}}{(x'^2+r^2)^{\frac{3}{2}}} = \underbrace{\frac{-r\,dx'}{(x'^2+r^2)^{\frac{3}{2}}}\hat{e}_y}_{+(2)}$$

$$\vec{B}_1 = -\frac{\mu_0 I}{4\pi} r\hat{e}_y \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx'}{(x'^2+r^2)^{\frac{3}{2}}} = -\frac{\mu_0 I}{4\pi} r\hat{e}_y \left.\frac{x'}{r^2\sqrt{x'^2+r^2}}\right|_{-\frac{a}{2}}^{\frac{a}{2}} =$$

$$= \underline{-\frac{\mu_0 I}{4\pi}\frac{a}{r\sqrt{r^2+\frac{a^2}{4}}}\hat{e}_y} \quad +(3)$$

– cel OKVIR



$$\frac{1}{3}\cdot\frac{a\sqrt{3}}{2} = \frac{a\sqrt{3}}{6}$$

$$r^2 = z^2 + \left(\frac{a\sqrt{3}}{6}\right)^2 = z^2 + \frac{a^2}{12} \Rightarrow \underline{r = \sqrt{z^2+\frac{a^2}{12}}} \quad +(4)$$

$$\underbrace{B_{1z} = \frac{\frac{a\sqrt{3}}{6}}{r}B_1 \quad , \quad B = 3B_{1z}}_{} \quad +(5)$$

+(6)

$$B = \frac{a\sqrt{3}}{2}\,\frac{1}{r}\cdot\frac{\mu_0 I}{4\pi}\frac{a}{r\sqrt{r^2+\frac{a^2}{4}}} = \underline{2\frac{a^2\sqrt{3}}{4}\frac{\mu_0 I}{4\pi}\frac{1}{r^2\sqrt{r^2+\frac{a^2}{4}}}}$$

$$\boxed{B = 2\frac{a^2\sqrt{3}}{4}\frac{\mu_0 I}{4\pi}\frac{1}{\left(z^2+\frac{a^2}{12}\right)\sqrt{z^2+\frac{a^2}{3}}}}$$

b) – VELIK z , $z \gg a$

+(7) $$B = 2\frac{a^2\sqrt{3}}{4}\frac{\mu_0 I}{4\pi}\frac{1}{z^2\cdot z} = \underline{\frac{\mu_0 I}{2\pi}\frac{a^2\sqrt{3}}{4}\cdot\frac{1}{z^3}}$$

$$B_{DIP} = \frac{\mu_0}{4\pi}\frac{3z^2 p_m - z^2 p_m}{z^5} = \underbrace{\frac{\mu_0}{2\pi}\frac{p_m}{z^3}}_{+(8)}$$

$$\Rightarrow \boxed{p_m = I\frac{a^2\sqrt{3}}{4}}$$

$$\frac{a^2\sqrt{3}}{4} = S_\Delta$$

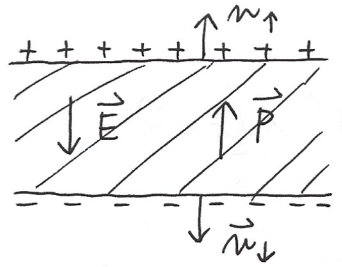
$$\Downarrow$$

$$\underline{p_m = I S_\Delta} \checkmark$$

8+ → $\boxed{1}$ (+ → $\frac{1}{8}$)

2 POLARIZIRANA PLOŠČA Z LUKNJO

a)

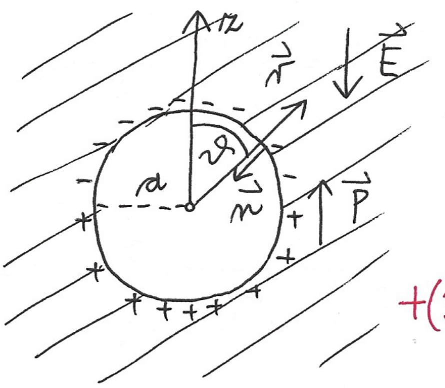


$$\left.\begin{aligned} \sigma_{v\uparrow} &= \vec{P}\cdot\vec{n}_\uparrow = P = \sigma \\ \sigma_{v\downarrow} &= \vec{P}\cdot\vec{n}_\downarrow = -P = -\sigma \end{aligned}\right\} +(1)$$

kaže DOL

ploščati kondenzator: $\left. E = \dfrac{\sigma}{\varepsilon_0} = \boxed{\dfrac{P}{\varepsilon_0}} \right\} +(2)$

b)



$$U_E = E z = E r\cos\vartheta = \frac{P}{\varepsilon_0} r\cos\vartheta$$

$\Downarrow$

SAMO $l=1$ v rešitvi

$$+(3)\left\{ U(r,\vartheta) = \begin{cases} A r\cos\vartheta & , r \le a \\ \dfrac{P}{\varepsilon_0} r\cos\vartheta + \dfrac{B\cos\vartheta}{r^2} & , r > a \end{cases} \right.$$

$\boxed{\text{RP1}}$ zveznost $U(r,\vartheta)$, pri $r = a$

$$\left. A a = \frac{P}{\varepsilon_0} a + \frac{B}{a^2} \Rightarrow A = \frac{P}{\varepsilon_0} + \frac{B}{a^3} \right\} +(4)$$

kaže NAVZNOTER

$\boxed{\text{RP2}}$

$$+(5)\left\{ \begin{aligned} &\sigma_v = \varepsilon_0 \left[E_r^{ZUN}(a) - E_r^{NOT}(a) \right] \quad , \quad \sigma_v = \vec{P}\cdot\vec{n} = -P\cos\vartheta \\ &-\frac{P}{\varepsilon_0}\cos\vartheta = -\frac{P}{\varepsilon_0}\cos\vartheta + 2\frac{B}{a^3}\cos\vartheta + A\cos\vartheta \Rightarrow A = -\frac{2B}{a^3} \end{aligned} \right.$$

$$-\frac{2B}{a^3} = \frac{P}{\varepsilon_0} + \frac{B}{a^3} \Rightarrow \boxed{B = -\frac{P a^3}{3\varepsilon_0}} \quad \& \quad \boxed{A = \frac{2P}{3\varepsilon_0}}$$

$$\left. U(r,\vartheta) = \begin{cases} \dfrac{2P}{3\varepsilon_0} r\cos\vartheta & , r \le a \\ \dfrac{P}{\varepsilon_0} r\cos\vartheta - \dfrac{P a^3}{3\varepsilon_0}\dfrac{\cos\vartheta}{r^2} & , r > a \end{cases} \right\} +(6)$$

$$\left. U_{NOT} = \frac{2P}{3\varepsilon_0} z = -E_{NOT} z \Rightarrow \text{HOMOGENO}, \ \boxed{E_{NOT} = -\frac{2P}{3\varepsilon_0}} \right\} +(7)$$

NAVZDOL

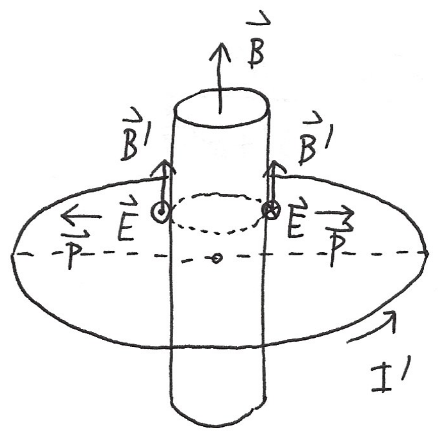
$$+(8)\left\{ U_{ZUN,\,DIP} = -\frac{P a^3}{3\varepsilon_0}\frac{\cos\vartheta}{r^2} = \frac{p_e}{4\pi\varepsilon_0}\frac{\cos\vartheta}{r^2} \Rightarrow p_e = -4\pi\varepsilon_0 \frac{P a^3}{3\varepsilon_0} \right.$$

$$\boxed{p_e = -\frac{4\pi}{3} P a^3} = -PV$$

NAVZDOL

$8+ \rightarrow \boxed{1} \quad (+ \rightarrow \frac{1}{8})$

3 POYNTINGOV VEKTOR PRI TULJAVI



a) +(1)

$$B = \frac{\mu_0 N I}{l} = \mu_0 n I$$

$$\dot{B} = \mu_0 n \dot{I} = -\mu_0 n \alpha$$

$$\dot{\Phi}_m = \dot{B}\,\pi a^2 = -\mu_0 n \pi a^2 \alpha \rightarrow R I'$$

$$\boxed{I' = \frac{1}{R}\mu_0 n \pi a^2 \alpha} = \text{konst}$$

tok v ZANKI

b) +(2)

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow E\cdot 2\pi a = -\dot{B}\cdot \pi a^2$$ ob notranji PLOSKVI TULJAVE

$$\underline{E = \mu_0 n \alpha \frac{a}{2}}$$ (nasprotna smer urinega kazalca)

+(3)

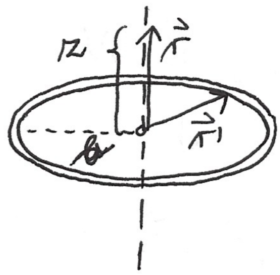
$\vec{B}'$ tik ob ZUNANJI ploskvi tuljave kaže NAVZGOR (zaradi Lenzovega pravila)

↳ SAMO od ZANKE! ← zunaj je polje tuljave ničelno!

- $\boxed{\text{podproblem}}$: izračun magnetnega polja ZANKE na njeni OSI

zanka je veliko širša od tuljave

$$\vec{B}'(\vec{r}) = \frac{\mu_0 I'}{4\pi}\int \frac{d\vec{l}'\times(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} =$$



+(4)

$$= \frac{\mu_0 I'}{4\pi}\int_0^{2\pi} \underbrace{\frac{1}{(z^2+b^2)^{\frac{3}{2}}}}_{\frac{1}{|\vec{r}-\vec{r}'|^3}} \underbrace{b\,d\varphi \begin{bmatrix} -\sin\varphi \\ \cos\varphi \\ 0 \end{bmatrix}}_{d\vec{l}'} \times \underbrace{\begin{bmatrix} -b\cos\varphi \\ -b\sin\varphi \\ z \end{bmatrix}}_{\vec{r}-\vec{r}'} =$$

$$= \frac{\mu_0 I'}{4\pi}\int_0^{2\pi} \frac{b\,d\varphi}{(z^2+b^2)^{\frac{3}{2}}} \begin{bmatrix} z\cos\varphi \\ z\sin\varphi \\ b \end{bmatrix} = \underline{\frac{\mu_0 I' b^2}{2}\frac{1}{(z^2+b^2)^{\frac{3}{2}}}\hat{e}_z}$$

$$\underline{B' = \frac{\mu_0 I'}{2}\frac{b^2}{(b^2+z^2)^{\frac{3}{2}}}}$$ (znani rezultat)

+(5)

$$\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B}' \quad \to \quad \text{kaže VEN iz tuljave}$$

$$P = \frac{1}{\mu_0} E B' = \frac{1}{\mu_0} \cdot \mu_0 n \alpha \frac{a}{2} \cdot \frac{\mu_0}{2} \frac{\mu_0 n \pi a^2 \alpha}{R} \frac{b^2}{(b^2+z^2)^{\frac{3}{2}}} =$$

$$= \underline{\frac{1}{4R} \mu_0^2 n^2 \alpha^2 \pi a^3 \frac{b^2}{(b^2+z^2)^{\frac{3}{2}}}}$$

c) $$\underbrace{\int \vec{P} \cdot d\vec{S}}_{\text{nasprotna}} = -\int_{-\infty}^{\infty} P \cdot \underbrace{2\pi a \, dz}_{dS} = \quad +(6)$$

+(7)

$$= -\frac{1}{4R} \mu_0^2 n^2 \alpha^2 \pi a^3 b^2 \, 2\pi a \underbrace{\int_{-\infty}^{\infty} \frac{dz}{(b^2+z^2)^{\frac{3}{2}}}}$$

$$\frac{z}{b^2 \sqrt{z^2+b^2}} \Bigg|_{-\infty}^{\infty} = \frac{1}{b^2 \sqrt{1+\frac{b^2}{z^2}}} \Bigg|_{0}^{\infty} = \frac{2}{b^2}$$

$$\boxed{\int \vec{P} \cdot d\vec{S} = -\frac{1}{R} \mu_0^2 n^2 \alpha^2 (\pi a^2)^2}$$

E, B' NISTA časovno odvisna $\Rightarrow \frac{\partial W_e}{\partial t}, \frac{\partial W_m}{\partial t} = 0$

+(8)

$$\int \vec{j} \cdot \vec{E} \, dV = R I'^2 = R \left(\frac{1}{R} \mu_0 n \alpha \pi a^2 \right)^2 = \boxed{\frac{1}{R} \mu_0^2 n^2 \alpha^2 (\pi a^2)^2}$$

$$\Downarrow$$

$$\underline{\int \vec{P} \cdot d\vec{S} + \int \vec{j} \cdot \vec{E} \, dV = 0}$$

$8+ \to \boxed{1} \quad (+ \to \frac{1}{8})$