2. PISNI IZPIT

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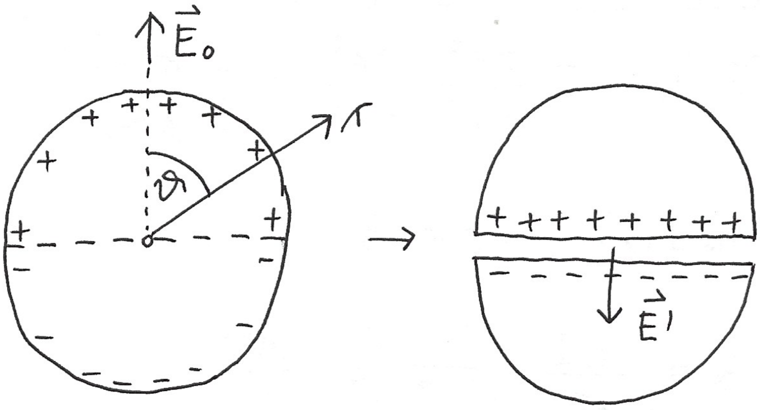

Naboj z oboda se enakomerno razporedi po notranjem preseku.

– neprerezana krogla (naloga iz vaj!)

$$+\left\{ U(r,\vartheta) = -E_0 r \cos\vartheta + \frac{B}{r^2}\cos\vartheta \right.$$

$$+\left\{ U(a,\vartheta) = \left(-E_0 a + \frac{B}{a^2}\right)\cos\vartheta = 0 \right.$$

$$\Downarrow$$

$$B = E_0 a^3$$

$$+\left\{ \underline{U(r,\vartheta) = E_0 \cos\vartheta \left(\frac{a^3}{r^2} - r\right)} \right.$$

– naboj, ki se inducira na obodu (naloga z vaj!)

$$+\left\{ \sigma_{IND} = \varepsilon_0 E\Big|_{r=a} = -\varepsilon_0 \frac{\partial U}{\partial r}\Big|_{r=a} = -\varepsilon_0 E_0 \cos\vartheta \left(-2\frac{a^3}{a^3} - 1\right) = 3\varepsilon_0 E_0 \cos\vartheta \right.$$

$$+\left\{ e_{IND} = \int \sigma_{IND}\, dS = \int_0^1 \underbrace{3\varepsilon_0 E_0 \cos\vartheta}_{\sigma_{IND}} \cdot \underbrace{2\pi a^2 d(\cos\vartheta)}_{dS} = 6\pi\varepsilon_0 E_0 a^2 \cdot \frac{1}{2} = \right.$$

$$= \underline{3\pi a^2 \varepsilon_0 E_0}$$

(zgornja polovica)

+ { – prerez deluje kot ploščati kondenzator

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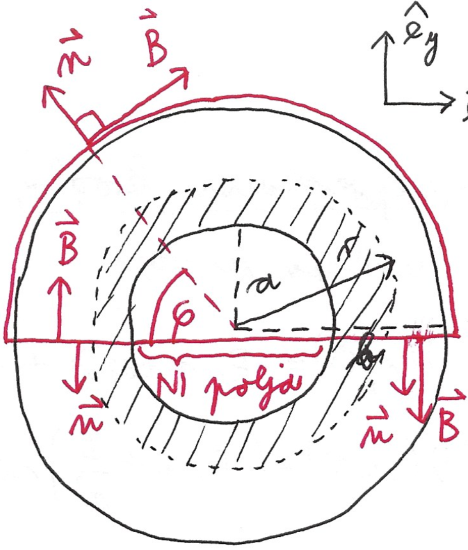
$$\frac{1}{4}\left\{ E' = \frac{\sigma'}{\varepsilon_0} = \frac{1}{\varepsilon_0}\frac{e_{IND}}{S'} = \frac{1}{\varepsilon_0}\frac{3\pi a^2 \varepsilon_0 E_0}{\pi a^2} = \boxed{3E_0} \right.$$

kaže NAVZDOL!

$\boxed{2}$



$\hat{e}_y$, $\hat{e}_x$ – Amperov zakon pri polmeru r:

$$I(r) = I\,\frac{\pi r^2 - \pi a^2}{\pi b^2 - \pi a^2} = I\,\frac{r^2 - a^2}{b^2 - a^2}$$

$$\mu_0 I(r) = 2\pi r \cdot B$$

$$\underline{B = \frac{\mu_0 I}{2\pi}\,\frac{1}{b^2 - a^2}\left(r - \frac{a^2}{r}\right)}$$

$$\vec{F}_m = \frac{1}{\mu_0}\oint\left[\vec{B}(\vec{B}\cdot\vec{n}) - \frac{1}{2}B^2\vec{n}\right]dS$$

– obod: $\vec{B}\cdot\vec{n} = 0 \Rightarrow \vec{F}_{m1} = -\frac{1}{2\mu_0}B^2(b)\int \vec{n}\,dS$

$$\vec{F}_{m1} = -\frac{1}{2\mu_0}\,\frac{\mu_0^2 I^2}{4\pi^2}\,\frac{1}{b^2}\int_0^\pi \vec{n}\,\underbrace{l b\,d\varphi}_{dS} = -\frac{\mu_0 I^2 l}{8\pi^2 b}\underbrace{\int_0^\pi \vec{n}\,d\varphi}$$

$$\underline{\vec{F}_{m1} = -\frac{\mu_0 I^2 l}{4\pi^2 b}\,\hat{e}_y} \qquad \int_0^\pi \sin\varphi\,d\varphi\,\hat{e}_y = 2\hat{e}_y$$

– prerez: – levo $\vec{B}\cdot\vec{n} = -B$, $\vec{B}(\vec{B}\cdot\vec{n}) = -B\vec{B} = B^2\vec{n}$; – desno $\vec{B}\cdot\vec{n} = B$, $\vec{B}(\vec{B}\cdot\vec{n}) = B\vec{B} = B^2\vec{n}$ } enako

$$\vec{F}_{m2} = \frac{1}{\mu_0}\int\left(B^2\vec{n} - \frac{1}{2}B^2\vec{n}\right)dS = \frac{1}{2\mu_0}\int B^2\vec{n}\,dS$$

$$\vec{F}_{m2} = \underbrace{2}_{\text{levo \& desno}}\cdot\frac{1}{2\mu_0}\,\frac{\mu_0^2 I^2}{4\pi^2}\,\frac{1}{(b^2-a^2)^2}(-\hat{e}_y)\int_a^b\left(r - \frac{a^2}{r}\right)^2\underbrace{l\,dr}_{dS} =$$

$$= -\frac{\mu_0 I^2 l}{4\pi^2(b^2-a^2)^2}\underbrace{\int_a^b\left(r^2 - 2a^2 + \frac{a^4}{r^2}\right)dr}_{\frac{1}{3}b^3 - \frac{1}{3}a^3 - 2a^2 b + 2a^3 + a^3 - \frac{a^4}{b} = \frac{8}{3}a^3 - 2a^2 b - \frac{a^4}{b} + \frac{1}{3}b^3} =$$

$$\underline{= -\frac{\mu_0 I^2 l}{4\pi^2 b}\cdot\frac{1}{3}\,\frac{b^4 + 8a^3 b - 6a^2 b^2 - 3a^4}{(b^2-a^2)^2}\,\hat{e}_y}$$

– vsota (tehnični del naloge)

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$$\vec{F}_m = \vec{F}_{m1} + \vec{F}_{m2} = -\frac{\mu_0 I^2 l}{12\pi^2 b} \frac{b^4 + 8a^3 b - 6a^2b^2 - 3a^4 + 3(b^2-a^2)^2}{(b^2-a^2)^2}\,\hat{e}_y$$

$$b^4 + 8a^3b - 6a^2b^2 \underline{-3a^4} + 3b^4 - 6a^2b^2 \underline{+3a^4} =$$

$$= 4b^4 + 8a^3b - 12a^2b^2 = 4b(b^3 - 3a^2b + 2a^3) =$$

$$= 4b(b^3 - a^2b + 2a^3 - 2a^2b) =$$

$$= 4b[b\underbrace{(b^2-a^2)}_{(b-a)(b+a)} - 2a^2(b-a)] = 4b(b-a)\underbrace{(b^2+ab-2a^2)}_{(b-a)(b+2a)} =$$

$$= 4b(b-a)^2(b+2a)$$

$$\vec{F}_m = -\frac{\mu_0 I^2 l}{12\pi^2 b}\,\frac{4b(b-a)^2(b+2a)}{(b-a)^2(b+a)^2}\,\hat{e}_y = \boxed{-\frac{\mu_0 I^2 l}{3\pi^2}\,\frac{2a+b}{(a+b)^2}\,\hat{e}_y}$$

PRIVLAČNA, saj kaže dol!

$\boxed{1}$

Mogoč je tudi LAŽJI dokaz v drugo smer, kjer imenovalec in števec zadnjega izraza razširimo z $(b-a)^2$ in nato primerjamo števca.

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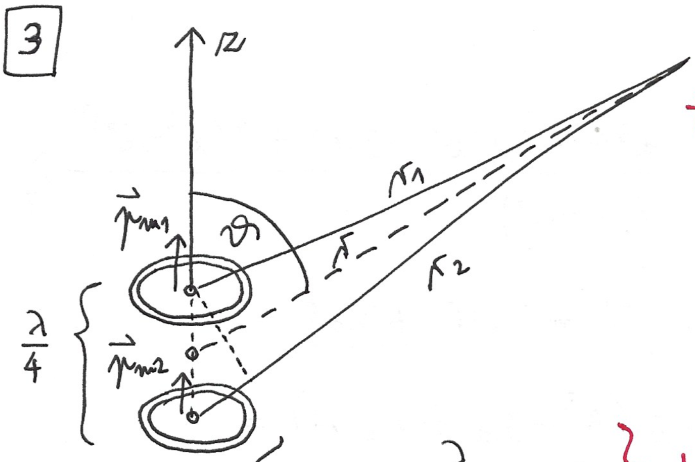

$$\underbrace{r_2 - r_1 = \frac{\lambda}{4}\cos\vartheta}$$ +

+ $$\vec{B} = -\frac{\mu_0}{4\pi c^2 r}\,\hat{e}_r \times \underbrace{\left[\hat{e}_r \times \ddot{\vec{p}}_m\left(t-\frac{r}{c}\right)\right]}$$

+ $$\ddot{p}_m\left(t-\frac{r}{c}\right)\sin\vartheta\,(-\hat{e}_\vartheta)$$

– prispevka obeh zank imata ENAKO SMER $-\hat{e}_\vartheta$, zato si ogledimo le velikosti

+
$$B = B_1 + B_2 = -\frac{\mu_0}{4\pi c^2 r}\sin\vartheta\left[\ddot{p}_{m1}\left(t-\frac{r_1}{c}\right) + \ddot{p}_{m2}\left(t-\frac{r_2}{c}\right)\right]$$

$$p_{m1}\left(t-\frac{r_1}{c}\right) = p_m\cos\omega\left(t-\frac{r_1}{c}\right)$$
$$p_{m2}\left(t-\frac{r_2}{c}\right) = -p_m\cos\omega\left(t-\frac{r_2}{c}\right)$$

[upoštevamo
$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$]

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$$B \propto \sin\vartheta\left[\cos\omega\left(t-\frac{r_1}{c}\right) - \cos\omega\left(t-\frac{r_2}{c}\right)\right] =$$

$$= -2\sin\vartheta\,\sin\left[\omega t - \frac{\omega}{c}\underbrace{\frac{r_1+r_2}{2}}_{r}\right]\sin\left[+\frac{\omega}{c}\frac{r_2-r_1}{2}\right] =$$

$$= -2\sin\vartheta\,\sin\omega\left(t-\frac{r}{c}\right)\sin\left(\frac{\omega}{2c}\cdot\frac{\lambda}{4}\cos\vartheta\right) =$$

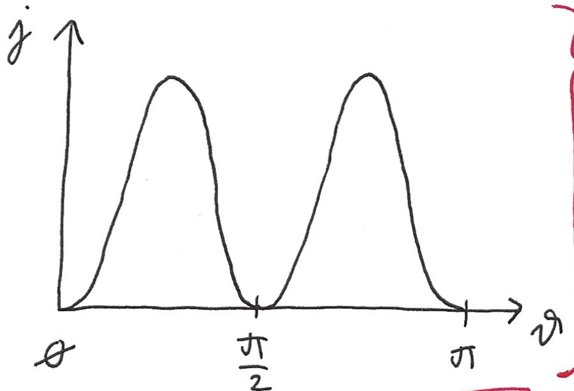
$$\left[\frac{\omega}{2c}\frac{\lambda}{4} = \frac{2\pi\nu\lambda}{8c} = \frac{\pi}{4}\right]$$

$$= -2\sin\vartheta\,\sin\omega\left(t-\frac{r}{c}\right)\sin\left(\frac{\pi}{4}\cos\vartheta\right)$$

+
$$j \propto \langle B^2\rangle = 4\sin^2\vartheta\,\underbrace{\left\langle\sin^2\omega\left(t-\frac{r}{c}\right)\right\rangle}_{\frac{1}{2}}\sin^2\left(\frac{\pi}{4}\cos\vartheta\right)$$

$$\boxed{j \propto \sin^2\vartheta\,\sin^2\left(\frac{\pi}{4}\cos\vartheta\right)}$$

$\vartheta = 0 \rightarrow 0$
$\vartheta = \pi \rightarrow 0$
$\vartheta = \frac{\pi}{2} \rightarrow 0$



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