EMP : 2. PISNI IZPIT

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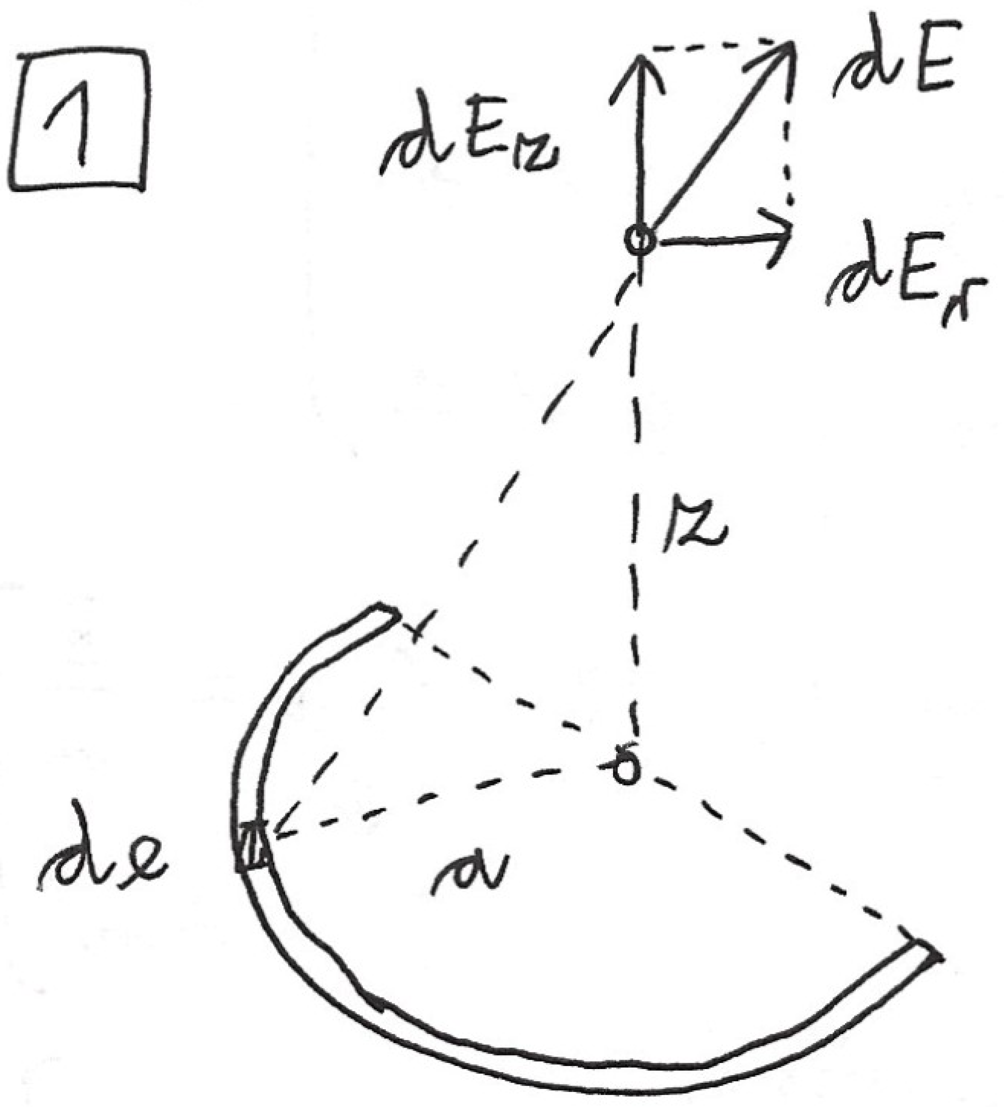
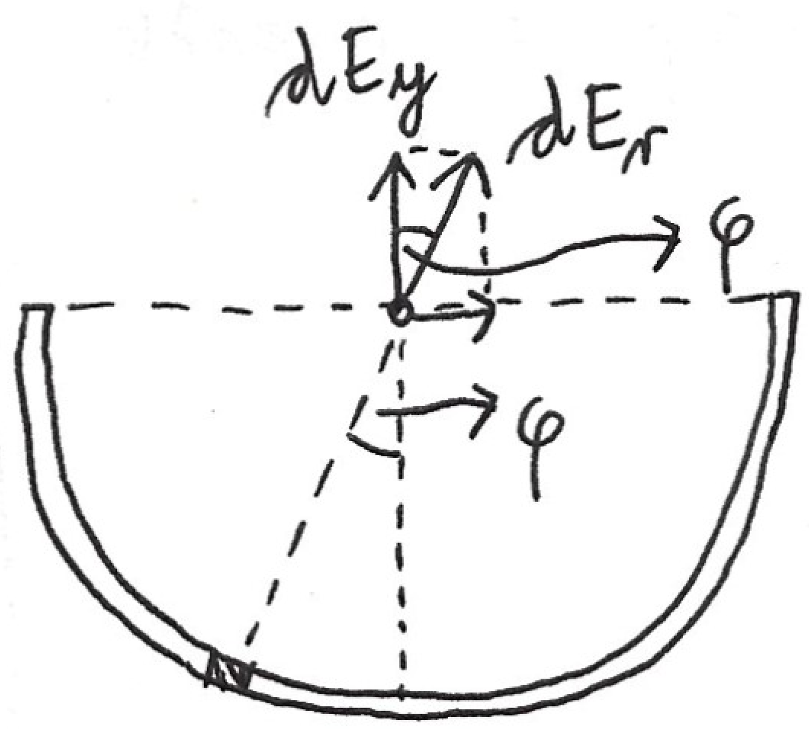


$$dE = \frac{de}{4\pi\varepsilon_0}\frac{1}{a^2+z^2}$$ +

$$dE_z = \frac{z}{\sqrt{a^2+z^2}}\,dE = \frac{de}{4\pi\varepsilon_0}\frac{z}{(a^2+z^2)^{\frac{3}{2}}}$$ +

$$\underline{E_z = \frac{e}{4\pi\varepsilon_0}\frac{z}{(a^2+z^2)^{\frac{3}{2}}}}$$

$$dE_r = \frac{a}{\sqrt{a^2+z^2}}\,dE = \frac{de}{4\pi\varepsilon_0}\frac{a}{(a^2+z^2)^{\frac{3}{2}}}$$ +



$$dE_y = dE_r\cos\varphi$$

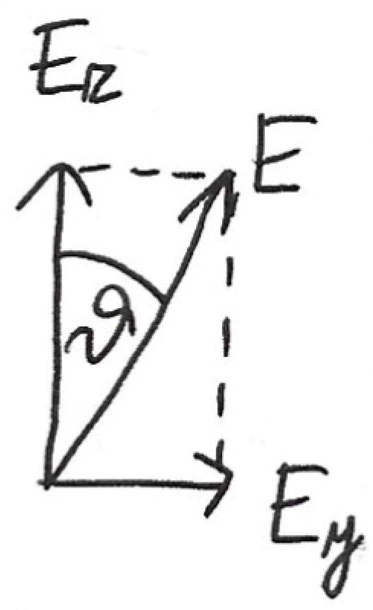
$$de = \mu\cdot a\,d\varphi\ ,\quad \mu = \frac{e}{\pi a}$$

$$dE_y = \frac{e}{4\pi\varepsilon_0}\frac{1}{\pi}\frac{a}{(a^2+z^2)^{\frac{3}{2}}}\cos\varphi\,d\varphi$$

$\frac{1}{4}$

$$E_y = \frac{e}{4\pi\varepsilon_0}\frac{1}{\pi}\frac{a}{(a^2+z^2)^{\frac{3}{2}}}\underbrace{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\varphi\,d\varphi}_{\sin\varphi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 2}$$ +

$$\underline{E_y = \frac{e}{4\pi\varepsilon_0}\frac{2}{\pi}\frac{a}{(a^2+z^2)^{\frac{3}{2}}}}$$



$$E = \sqrt{E_z^2+E_y^2} = \boxed{\frac{e}{4\pi\varepsilon_0}\frac{1}{(a^2+z^2)^{\frac{3}{2}}}\sqrt{z^2+\left(\frac{2}{\pi}a\right)^2}}$$ +

$$\operatorname{tg}\vartheta = \frac{E_y}{E_z} = \frac{2a}{\pi z} \Rightarrow \boxed{\vartheta = \operatorname{arctg}\frac{2a}{\pi z}}$$ +

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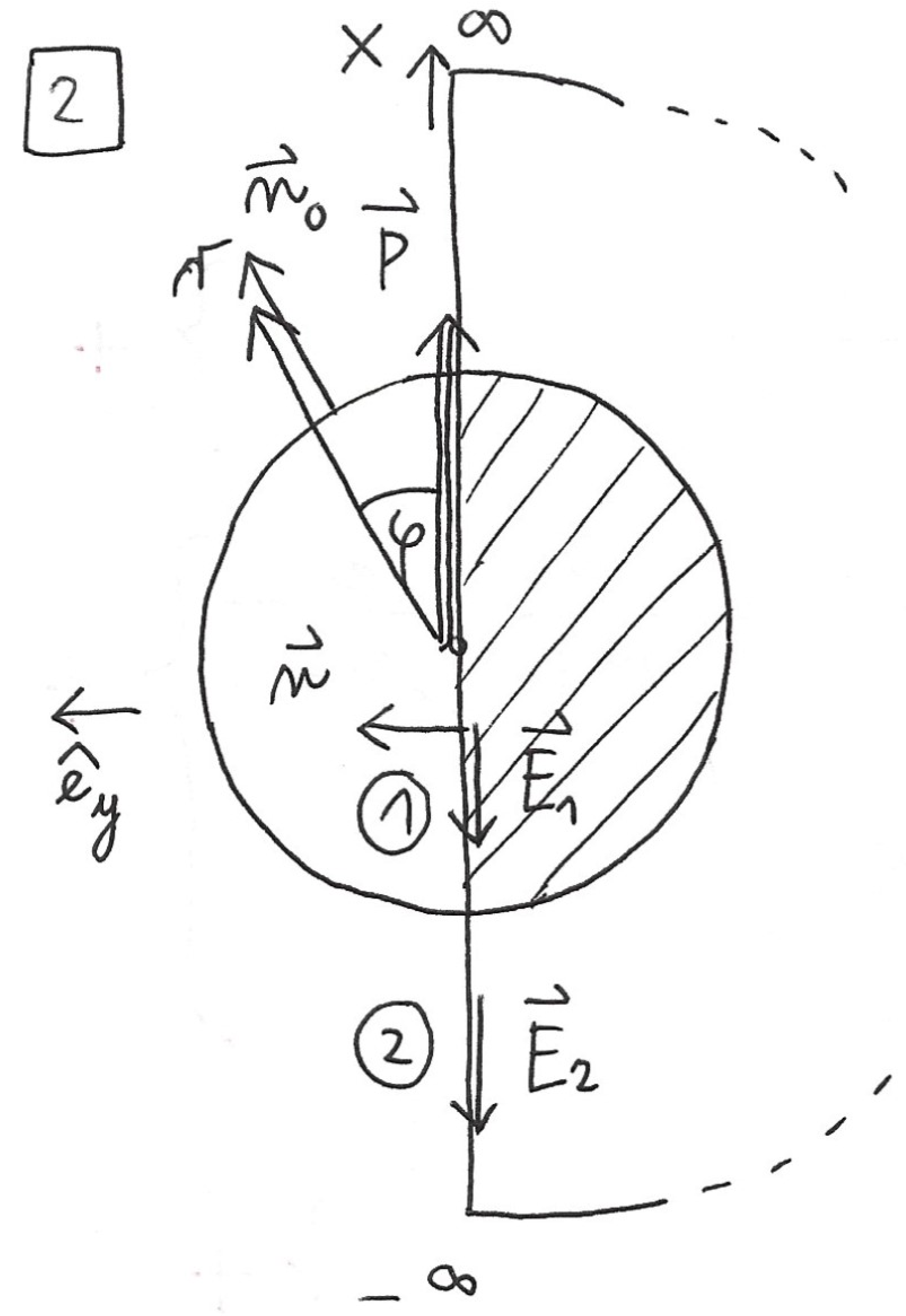

$$\sigma_v = \vec{P}\cdot\vec{n}_0 = P\cos\varphi$$

$$\rho_v = \vec{\nabla}\cdot\vec{P} = 0$$

$$U(r,\varphi) = \begin{cases} A r\cos\varphi\,, & \varphi < a \\ \frac{B}{r}\cos\varphi\,, & \varphi > a \end{cases}$$ +

– RP1 : zvezen potencial $\rightarrow A a = \frac{B}{a} \;\Downarrow\; B = A a^2$ +

– RP2 : Gauss na površini

$$\hookrightarrow \underbrace{\sigma_v S}_{e_v} = \varepsilon_0 S\left(E_\perp^{ZUN} - E_\perp^{NOT}\right) = \varepsilon_0 S\left(-\frac{\partial U^{ZUN}}{\partial r}\Big|_{r=a} + \frac{\partial U^{NOT}}{\partial r}\Big|_{r=a}\right)$$ +

$$P = \varepsilon_0 2A\,, \quad \left[\begin{aligned} A &= \frac{P}{2\varepsilon_0} \\ B &= \frac{P a^2}{2\varepsilon_0}\end{aligned}\right. \Leftarrow \quad \sigma_v = \varepsilon_0\left(\underbrace{\frac{B}{a^2}}_{A} + A\right)\cos\varphi = P\cos\varphi$$ +

$$U(r,\varphi) = \begin{cases} \frac{P}{2\varepsilon_0}\, r\cos\varphi = \frac{P}{2\varepsilon_0}\, x\,, & r < a \\ \frac{P a^2}{2\varepsilon_0}\frac{1}{r}\cos\varphi = \frac{P a^2}{2\varepsilon_0}\frac{x}{r^2}\,, & r > a \end{cases}$$

– integracijska ploskev : navpično čez polovico valja in sklenjena v ∞

$$E_1 = -\frac{\partial U^{NOT}}{\partial x}\Big|_{y=0} = -\frac{P}{2\varepsilon_0}$$

$$E_2 = -\frac{\partial U^{ZUN}}{\partial x}\Big|_{y=0} = \frac{P a^2}{2\varepsilon_0}\left(-\frac{\partial}{\partial x}\right)\left(\frac{1}{x}\right) = \frac{P a^2}{2\varepsilon_0}\frac{1}{x^2}$$

$$\vec{E}_{1,2} \perp \vec{n}$$

$$(\vec{E}\cdot\vec{n})\vec{E} - \frac{1}{2}E^2\vec{n} = -\frac{1}{2}E^2\vec{n}$$ $\frac{1}{4}$

$$\vec{F}_e = \varepsilon_0\left(-\frac{1}{2}\hat{e}_y\right)\left[2\int_0^a\left(\frac{P}{2\varepsilon_0}\right)^2 l\,dx + 2\int_a^\infty\left(\frac{P a^2}{2\varepsilon_0}\right)^2\frac{1}{x^4}\, l\,dx\right] =$$ +

$$= -\frac{\varepsilon_0}{2}\hat{e}_y\, 2\frac{P^2}{4\varepsilon_0^2}\, l\left[a + a^4\frac{1}{-3}\frac{1}{x^3}\Big|_a^\infty\right] =$$

$$= -\hat{e}_y\frac{P^2 l}{4\varepsilon_0}\left(a + \frac{1}{3}a\right) = -\hat{e}_y\frac{P^2 a}{3\varepsilon_0}\, l$$

+

$$\boxed{\frac{\vec{F}_e}{l} = -\frac{P^2 a}{3\varepsilon_0}\hat{e}_y}$$

sila na desno polovico torej kaže DESNO

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polovici valja se ODBIJATA

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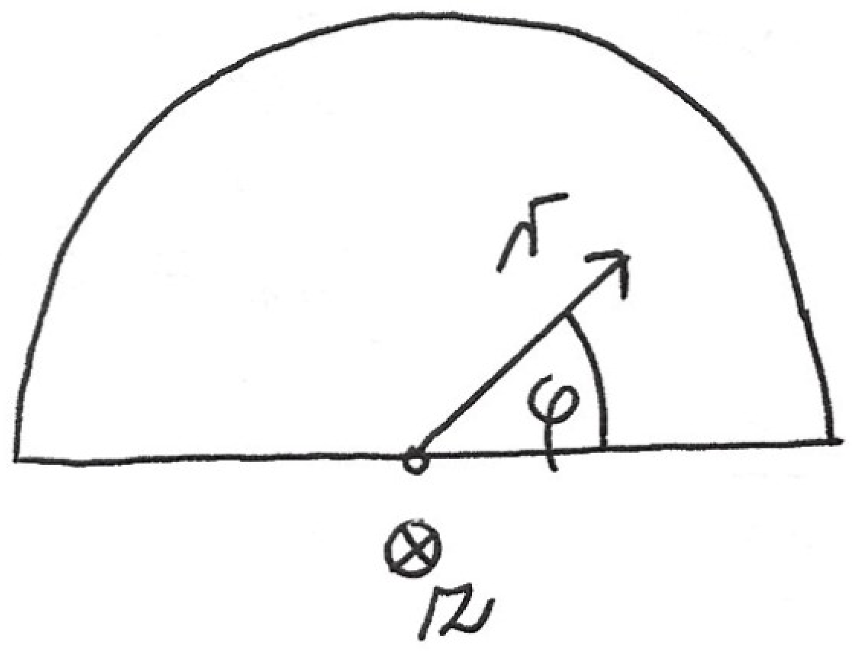

$$\left[\nabla_\perp^2 + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\right]\begin{Bmatrix}E_z\\ H_z\end{Bmatrix} = 0$$

– disperzijska relacija : $\omega = c_0\sqrt{\varkappa^2+k^2}$ +

– s plazmo : $c_0 \to \frac{c_0}{\sqrt{\varepsilon}}$, $\varepsilon = 1-\frac{\omega_p^2}{\omega^2}$ +

$$\varkappa^2 = \frac{\omega^2}{c_0^2}\left(1-\frac{\omega_p^2}{\omega^2}\right) - k^2 = \frac{\omega^2}{c_0^2} - \frac{\omega_p^2}{c_0^2} - k^2$$

$$\tilde\omega = c_0\sqrt{\varkappa^2 + \underbrace{\frac{\omega_p^2}{c_0^2}}_{\left(\frac{\pi}{a}\right)^2} + k^2}$$

– $\boxed{\text{TM}}$: $E_z \neq 0$

+ RP1 : $E_z(\varphi = 0) = 0$, $E_z \propto \sin m\varphi$

RP2 : $E_z(\varphi = \pi) = 0$, $\sin m\pi = 0$ $\Rightarrow$ $m = 1, 2, 3, \ldots$

($m = 0$ bi vodil do $E_z = 0$

+ RP3 : $E_z(r = a) = 0$, $J_m(\varkappa a) = 0$ $\Rightarrow$ $\varkappa a = \xi_{mn}$

$\omega = c_0\sqrt{\left(\frac{\xi_{mn}}{a}\right)^2 + k^2}$, najmanjši ničli : $\xi_{11} = 3.83$, $\xi_{21} = 5.14$

$$\Delta\omega = \omega_{21} - \omega_{11} = (5.14 - 3.83)\frac{c_0}{a} = 1.31\frac{c_0}{a}$$

+ plazma: $\Delta\tilde\omega = \tilde\omega_{21} - \tilde\omega_{11} = \frac{c_0}{a}\left(\sqrt{\xi_{21}^2+\pi^2} - \sqrt{\xi_{11}^2+\pi^2}\right) = 1.07\frac{c_0}{a}$

$$\boxed{\frac{\Delta\tilde\omega}{\Delta\omega} = 0.82}$$

– $\boxed{\text{TE}}$: $H_z \neq 0$

+ RP1 : $\frac{\partial H_z}{\partial\varphi}(\varphi = 0) = 0$, $H_z \propto \cos m\varphi$

RP2 : $\frac{\partial H_z}{\partial\varphi}(\varphi = \pi) = 0$, $\sin m\pi = 0$ $\Rightarrow$ $m = 0, 1, 2, \ldots$

($m = 0$ gre, saj $H_z \propto \cos m\varphi$

RP3 : $\frac{\partial H_z}{\partial r}(r = a) = 0$, $J_m'(\varkappa a) = 0$

$\varkappa a = \xi'_{mn}$

$$+ \left\{ \omega' = c_0 \sqrt{\left(\frac{\xi'_{mn}}{a}\right)^2 + k^2} \right.$$

, najmanjši ničli : $\xi'_{11} = 1.84$

$\xi'_{21} = 3.05$

$$+ \left\{ \begin{array}{l} \Delta\omega' = \omega'_{21} - \omega'_{11} = \frac{c_0}{a}\left(\xi'_{21} - \xi'_{11}\right) = \underbrace{1.21\,\frac{c_0}{a}} \\ \text{plazma: } \Delta\tilde{\omega}' = \tilde{\omega}'_{21} - \tilde{\omega}'_{11} = \frac{c_0}{a}\left(\sqrt{\xi'^2_{21} + \pi^2} - \sqrt{\xi'^2_{11} + \pi^2}\right) = \underbrace{0.74\,\frac{c_0}{a}} \\ \boxed{\frac{\Delta\tilde{\omega}'}{\Delta\omega'} = 0.61} \end{array} \right.$$

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