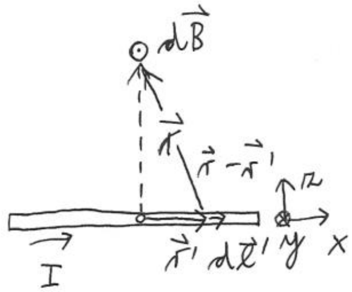
EMP – 3. PISNI IZPIT

1 a) – ena stranica



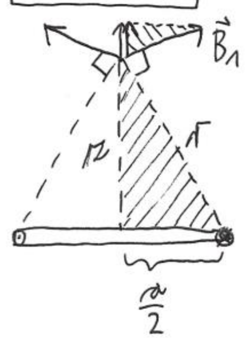
$$\vec B_1 = \frac{\mu_0 I}{4\pi}\int \frac{d\vec l' \times (\vec r - \vec r')}{|\vec r - \vec r'|^3}$$

$$\frac{\begin{bmatrix} dx' \\ 0 \\ 0\end{bmatrix}\times\begin{bmatrix} -x' \\ 0 \\ r\end{bmatrix}}{(x'^2+r^2)^{\frac{3}{2}}} = \frac{-r\,dx'\,\hat e_y}{(x'^2+r^2)^{\frac{3}{2}}}$$ } $\frac{1}{4}$ (12)

$$\vec B_1 = -\frac{\mu_0 I}{4\pi} r\,\hat e_y \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx'}{(x'^2+r^2)^{\frac{3}{2}}} = -\frac{\mu_0 I}{4\pi}\hat e_y \frac{1}{r}\, 2\,\frac{\frac{a}{2r}}{\sqrt{1+\left(\frac{a}{2r}\right)^2}} = -\frac{\mu_0 I \hat e_y}{4\pi}\frac{1a}{r\sqrt{r^2+\frac{a^2}{4}}}$$ + (4)

$$\frac{1}{r^2}\frac{\frac{x'}{r}}{\sqrt{1+\left(\frac{x'}{r}\right)^2}}\Bigg|_{-\frac{a}{2}}^{\frac{a}{2}}$$ + (3)

– cel okvir



$$\vec B = 4\,B_1\,\frac{\frac{a}{2}}{r}\,\hat e_z$$ + (5)

$$B = 4\,\frac{\mu_0 I}{4\pi}\frac{1a}{r\sqrt{r^2+\frac{a^2}{4}}}\,\frac{a}{2r}$$

$$B = \frac{\mu_0 I}{4\pi}\frac{2a^2}{r^2\sqrt{r^2+\frac{a^2}{4}}}$$

$$r = \sqrt{z^2+\frac{a^2}{4}}$$ + (6)

$$\boxed{B = \frac{\mu_0 I}{4\pi}\frac{2a^2}{\left(z^2+\frac{a^2}{4}\right)\sqrt{z^2+\frac{a^2}{2}}}}$$ + (7)

b) – velik z, $z \gg a$

$$B = \frac{\mu_0 I}{4\pi}\frac{2a^2}{z^2\cdot z} = \frac{\mu_0}{2\pi}\frac{I a^2}{z^3} = \boxed{\frac{\mu_0}{2\pi}\frac{p_m}{z^3}} \rightarrow$$ magnetni DIPOL + (8)

$$\boxed{B \propto \frac{1}{z^3}}$$

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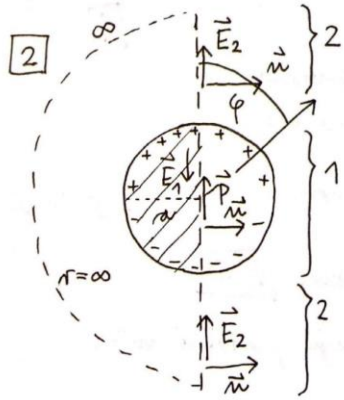

+ (1)

$$\sigma_v = \vec{P}\cdot\vec{n} = P\cos\varphi$$

$$U(r,\varphi) = \begin{cases} A r\cos\varphi, & r<a \\ \dfrac{B}{r}\cos\varphi, & r>a \end{cases}$$

RP1 : $U(a^-,\varphi) = U(a^+,\varphi) \Rightarrow \underline{Aa = \dfrac{B}{a}}$

RP2: $\sigma_v = \varepsilon_0\left[E_r(a^+) - E_r(a^-)\right] =$

$= \varepsilon_0\left(A\cos\varphi + \dfrac{B}{a^2}\cos\varphi\right) = 2\varepsilon_0 A\cos\varphi$

$$\sigma_v = 2\varepsilon_0 A\cos\varphi = P\cos\varphi \Rightarrow A = \frac{P}{2\varepsilon_0},\quad B = \frac{Pa^2}{2\varepsilon_0}$$

$$U(r,\varphi) = \begin{cases} \dfrac{P}{2\varepsilon_0} r\cos\varphi, & r<a \\ \dfrac{Pa^2}{2\varepsilon_0}\dfrac{1}{r}\cos\varphi, & r>a \end{cases}$$

+ (2)

$\rightarrow \underline{E_1 = -\dfrac{P}{2\varepsilon_0}}$ + (3)

$\rightarrow \underline{E_2(\varphi=0) = \dfrac{Pa^2}{2\varepsilon_0}\dfrac{1}{r^2}}$

$\underline{E_2(\varphi=\pi) = \dfrac{Pa^2}{2\varepsilon_0}\dfrac{1}{r^2}}$ + (4)

$$\vec{F}_e = \varepsilon_0 \oint \Big[\vec{E}\underbrace{(\vec{E}\cdot\vec{n})}_{0} - \frac{1}{2}E^2\vec{n}\Big]dS = -\frac{\varepsilon_0}{2}\vec{n}\oint E^2 dS$$

$F_\infty = 0$, ker gre $E^2 \propto \dfrac{1}{r^4}$, $S \propto r$

+ $\vec{F}_1 = -\dfrac{\varepsilon_0}{2}\vec{n}\, E_1^2\, 2al = -\vec{n}\,\varepsilon_0 a l E_1^2 = \underline{-\vec{n}\,\dfrac{P^2 al}{4\varepsilon_0}}$ (5)

$\dfrac{1}{4}$ $\vec{F}_2 = -\dfrac{\varepsilon_0}{2}\vec{n}\cdot 2\cdot\displaystyle\int_a^\infty E_2^2\, l\, dr = -\vec{n}\,\varepsilon_0\,\dfrac{P^2a^4}{4\varepsilon_0^2}\, l \underbrace{\int_a^\infty \frac{dr}{r^4}}_{-\frac{1}{3}\frac{1}{r^3}\Big|_a^\infty = \frac{1}{3a^3}}$ (67)

$\vec{F}_2 = \underline{-\vec{n}\,\dfrac{P^2al}{12\varepsilon_0}}$

$$\vec{F} = -\vec{n}\,\frac{P^2al}{\varepsilon_0}\left(\frac{1}{4}+\frac{1}{12}\right) = -\vec{n}\,\frac{P^2al}{3\varepsilon_0}$$

+ $\boxed{\dfrac{\vec{F}}{l} = -\vec{n}\,\dfrac{P^2a}{3\varepsilon_0}}$ → sila kaže v smeri $-\vec{n}$, kar je proti LEVI (istovrstni naboji na površini se ODBIJAJO) (8)

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KOMENTAR : Električno polje tik ob površini valja NI PRAVOKOTNO na valj.

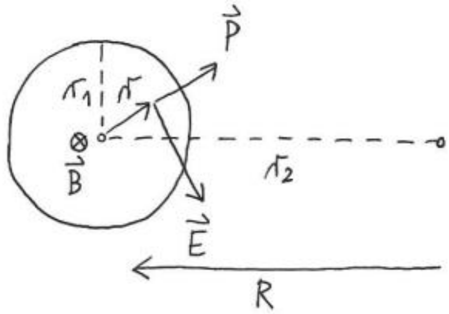
$E_r = -\dfrac{\partial U}{\partial r}\Big|_{r=a} = \dfrac{P}{2\varepsilon_0}\cos\varphi$

$E_\varphi = -\dfrac{1}{r}\dfrac{\partial U}{\partial\varphi}\Big|_{r=a} = \dfrac{P}{2\varepsilon_0}\sin\varphi \;\boxed{\neq 0}$

$\Rightarrow \vec{E} \nparallel \vec{n}$

Izračun sile preko integrala po zunanji površini valja je zato nekoliko zapletenejši.

3



a) – magnetno polje

(1) + $\vec{\nabla}\times\vec{B} = \mu_0 \vec{j} \rightarrow B\cdot 2\pi R = \mu_0 N I$

$$B = \frac{\mu_0 N I}{2\pi R} \approx \frac{\mu_0 N I}{2\pi r_2}\text{, ker } r_2 \gg r_1$$

– električno polje

(2) + $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow 2\pi r E = -\dot{B}\pi r^2$ (zanka s polmerom r)

(3) + $E = -\dot{B}\frac{r}{2} = \boxed{-\frac{\mu_0 N \dot{I} r}{4\pi r_2}}$

$\dot{I} < 0 \Rightarrow \vec{E}$ ima v levem delu toroidne tuljave tangentno smer, in sicer v smislu urinega kazalca

b) Poyntingov vektor $\rightarrow \vec{P} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$ ima smer VEN iz plašča tuljave

(4) + $P = \frac{1}{\mu_0}EB = -\underbrace{\frac{NI}{2\pi r_2}}_{B/\mu_0}\cdot\underbrace{\frac{\mu_0 N \dot{I} r_1}{4\pi r_2}}_{E} = -\frac{\mu_0 N^2 r_1}{8\pi^2 r_2^2} I\dot{I}$ ← na površini

(5) + $\int P\,dS = P\cdot\underbrace{2\pi r_1\cdot 2\pi r_2}_{\text{površina tuljave}} = \boxed{-\frac{\mu_0 N^2 r_1^2}{2 r_2} I\dot{I}}$

c) energija EM valovanja

(6) + $W_m = \frac{B^2}{2\mu_0}\cdot\underbrace{\pi r_1^2\cdot 2\pi r_2}_{\text{prostornina tuljave}} = \frac{\mu_0 N^2 I^2}{8\pi^2 r_2^2}\cdot 2\pi^2 r_1^2 r_2 = \frac{\mu_0 N^2 I^2 r_1^2}{4 r_2}$

(7) + $\dot{W}_m = \frac{\mu_0 N^2 r_1^2}{4 r_2}\underbrace{(I^2)^{\cdot}}_{2I\dot{I}} = \boxed{\frac{\mu_0 N^2 r_1^2}{2 r_2} I\dot{I}} = -\int P\,dS$ ✓

(8) + $\dot{W}_e = 0$, saj je $\dot{I} = \text{konst} \Rightarrow E^2 = \text{konst}$

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