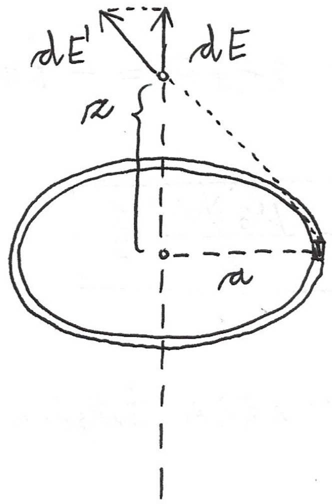
EMP : 3. PISNI IZPIT

1 NABITI KROŽNI ZANKI



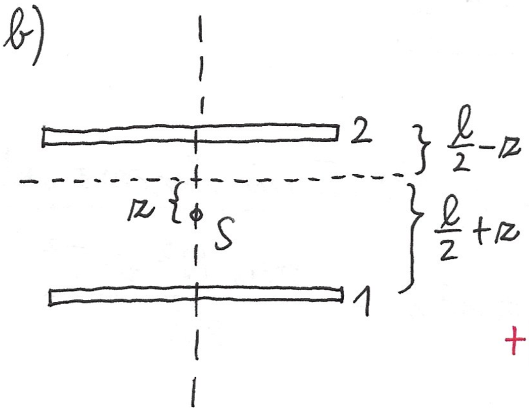
a)
$$dE' = \frac{de}{4\pi\varepsilon_0 (a^2+z^2)}$$
$$de = \frac{e}{2\pi a}\, d\varphi$$
+(1)

$$dE = \frac{z}{\sqrt{a^2+z^2}}\, dE' = \frac{e\, d\varphi}{8\pi^2 \varepsilon_0} \frac{z}{(a^2+z^2)^{\frac{3}{2}}}$$ +(2)

$$E = \int_{\varphi=0}^{2\pi} dE = \boxed{\frac{e}{4\pi\varepsilon_0} \frac{z}{(a^2+z^2)^{\frac{3}{2}}}}$$ +(3)

preverimo: $z = 0 \rightarrow E = 0$ ✓

b)



$$E_{1,2} = \pm \frac{e}{4\pi\varepsilon_0} \frac{\frac{\ell}{2} \pm z}{\left[a^2 + \left(\frac{\ell}{2} \pm z\right)^2\right]^{\frac{3}{2}}}$$ +(4)

+(5)
$$E = E_1 + E_2 = \frac{e}{4\pi\varepsilon_0}\left[\frac{\frac{\ell}{2}+z}{\left[a^2+\left(\frac{\ell}{2}+z\right)^2\right]^{\frac{3}{2}}} - \frac{\frac{\ell}{2}-z}{\left[a^2+\left(\frac{\ell}{2}-z\right)^2\right]^{\frac{3}{2}}}\right]$$

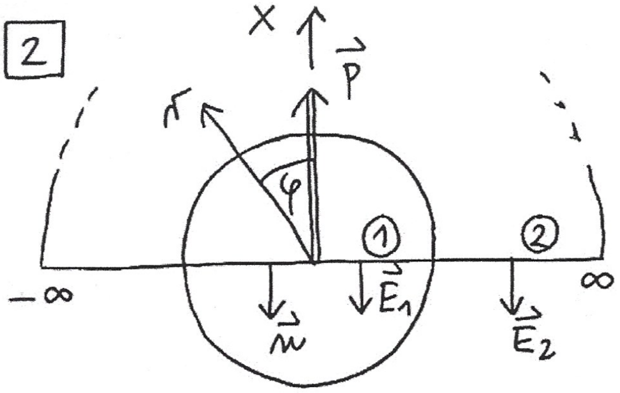
+(6)
$$\left.\frac{\partial E}{\partial z}\right|_{z=0} = \frac{e}{4\pi\varepsilon_0}\left[\frac{\left(a^2+\frac{\ell^2}{4}\right)^{\frac{3}{2}} - \frac{\ell}{2}\frac{3}{2}\left(a^2+\frac{\ell^2}{4}\right)^{\frac{1}{2}} 2\frac{\ell}{2}}{\left(a^2+\frac{\ell^2}{4}\right)^3} - \frac{-\left(a^2+\frac{\ell^2}{4}\right)^{\frac{3}{2}} + \frac{\ell}{2}\frac{3}{2}\left(a^2+\frac{\ell^2}{4}\right)^{\frac{1}{2}} 2\frac{\ell}{2}}{\left(a^2+\frac{\ell^2}{4}\right)^3}\right]$$

+(7)
$$\left.\frac{\partial E}{\partial z}\right|_{z=0} = 0 \Rightarrow \left(a^2+\frac{\ell^2}{4}\right)^{\frac{3}{2}} - \frac{3}{4}\ell^2\left(a^2+\frac{\ell^2}{4}\right)^{\frac{1}{2}} = 0$$

+(8)
$$a^2 + \frac{\ell^2}{4} - \frac{3\ell^2}{4} = a^2 - \frac{\ell^2}{2} = 0 \Rightarrow \boxed{\ell = \sqrt{2}\, a}$$

$8+ = \boxed{1}$

2



$$\sigma_v = \vec{P}\cdot\vec{n} = P\cos\varphi$$

$$\rho_v = \vec{\nabla}\cdot\vec{P} = 0$$

$$U(r,\varphi) = \begin{cases} A r\cos\varphi\,, & r<a \\ \frac{B}{r}\cos\varphi\,, & r>a \end{cases}$$

+(1)

– RP1 : zvezen potencial $\rightarrow$ $Aa = \frac{B}{a}$, $B = Aa^2$ +(2)

– RP2 : Gaussov izrek na površini $\rightarrow$ $\underbrace{\sigma_v}_{\rho_v} S = \varepsilon_0 S\left(E_\perp^{ZUN} - E_\perp^{NOT}\right)$ (3) +

$$\sigma_v S = \varepsilon_0 S\left(-\frac{\partial U^{ZUN}}{\partial r}\Big|_{r=a} + \frac{\partial U^{NOT}}{\partial r}\Big|_{r=a}\right)$$

$$\sigma_v = \varepsilon_0\left(\frac{B}{a^2}+A\right)\cos\varphi = P\cos\varphi \Rightarrow P = \varepsilon_0\cdot 2A,\quad A = \frac{P}{2\varepsilon_0}$$

$$B = \frac{Pa^2}{2\varepsilon_0}$$

$$\boxed{U(r,\varphi) = \begin{cases} \frac{P}{2\varepsilon_0} r\cos\varphi\,, & r<a \\ \frac{Pa^2}{2\varepsilon_0}\frac{1}{r}\cos\varphi\,, & r>a \end{cases}} \begin{matrix} = \frac{P}{2\varepsilon_0}x\,, \quad x = r\cos\varphi \\ = \frac{Pa^2}{2\varepsilon_0}\frac{x}{r^2} \end{matrix}$$

+(4)

– integracijska ploskev : čez polovico valja in sklenjena v ∞

$\frac{1}{4}$

$$E_1 = -\frac{\partial U^{NOT}}{\partial x}\Big|_{x=0} = -\frac{P}{2\varepsilon_0} \quad (5)$$

$$E_2 = -\frac{\partial U^{ZUN}}{\partial x}\Big|_{x=0} = -\frac{Pa^2}{2\varepsilon_0}\frac{1}{r^2} \quad (6)$$

$\vec{E}_{1,2} \parallel \vec{n}$

$$(\vec{E}\cdot\vec{n})\vec{E} - \frac{1}{2}E^2\vec{n} = \frac{1}{2}E^2\vec{n}$$

+(7)

$$\vec{F}_e = \varepsilon_0\left[2\int_0^a \frac{1}{2}\left(\frac{P}{2\varepsilon_0}\right)^2 l\,dr + 2\int_a^\infty \frac{1}{2}\left(\frac{Pa^2}{2\varepsilon_0}\right)^2\frac{1}{r^4} l\,dr\right](-\hat{e}_x) =$$

+(8)

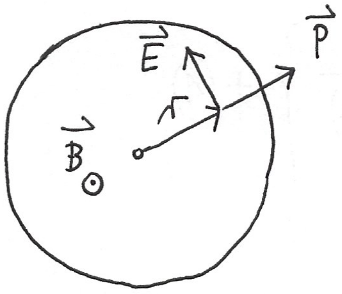
$$= -2\varepsilon_0\,\hat{e}_x\frac{1}{2}\frac{P^2}{4\varepsilon_0^2}l\left(a + a^4\frac{1}{(-3)}\frac{1}{r^3}\Big|_a^\infty\right) =$$

$$= -\hat{e}_x\frac{P^2 l}{4\varepsilon_0}\left(a+\frac{1}{3}a\right) = -\hat{e}_x\frac{2P^2 l a}{3\varepsilon_0}$$

$$\boxed{\frac{\vec{F}_e}{l} = -\frac{\cancel{2}P^2 a}{3\varepsilon_0}\hat{e}_x}$$

1 = 8+

3 UGAŠANJE TULJAVE



a) $B = \dfrac{\mu_0 N I}{l}$

+(1) $\left\{ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \Rightarrow 2\pi r E = -\dot{B} \pi r^2 \right.$

+(2) $\left\{ E = -\dot{B}\dfrac{r}{2} = \boxed{-\dfrac{\mu_0 N \alpha}{l} \dfrac{r}{2}} \right.$

b) $\vec{P} = \dfrac{1}{\mu_0} \vec{E} \times \vec{B} \Rightarrow$ ima smer VEN IZ PLAŠČA tuljave

+(3) $\left\{ P = \dfrac{1}{\mu_0} |EB| = \dfrac{1}{\mu_0} \dfrac{\mu_0 N \alpha}{l} \dfrac{r}{2} \dfrac{\mu_0 N I}{l} = \dfrac{\mu_0}{2} \dfrac{N^2}{l^2} \alpha I \cdot r \right.$

+(4) $\left\{ \oint P \, dS = P(a) \cdot 2\pi a l = \boxed{\pi \mu_0 \dfrac{N^2}{l} \alpha a^2 I} \right.$

+(5) $\Big\{$ c) $W_e = \dfrac{1}{2} \varepsilon_0 E^2 \cdot V \Rightarrow \underline{\dot{W}_e = 0}$, saj $\underline{E \text{ ni časovno odvisen!}}$

+(6) $\left\{ W_m = \dfrac{1}{2\mu_0} B^2 \cdot V = \dfrac{1}{2\mu_0} \dfrac{\mu_0^2 N^2 I^2}{l^2} \pi a^2 l = \dfrac{\pi \mu_0 N^2 a^2 I^2}{2l} \right.$

$$\underbrace{\dot{W}_m = \frac{\pi \mu_0 N^2 a^2}{2l} 2 I \underbrace{\dot{I}}_{\alpha}}_{+(7)} = \boxed{\pi \mu_0 \frac{N^2}{l} \alpha a^2 I}$$

$$\Downarrow$$

$$\oint P \, dS = (W_e + W_m)^{\cdot} \quad \checkmark$$

+(8)

$8+ = \boxed{1}$