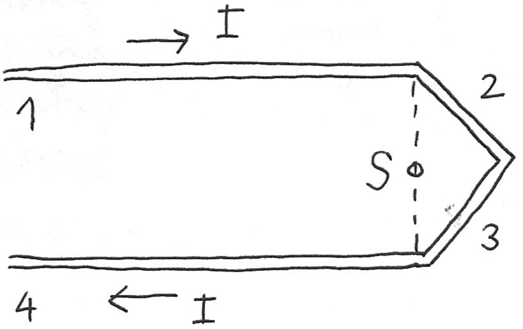
3. PISNI IZPIT

1 PREPOGNJEN VODNIK

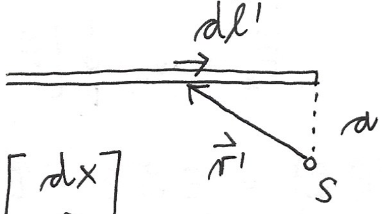
$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$\vec{r} = 0$ (izhodišče = S)

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{\vec{r}' \times d\vec{l}'}{r'^3}$$



- segment 1

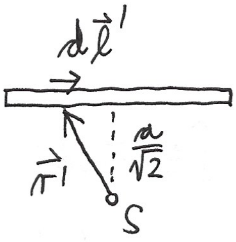


+(1) $\left\{ \vec{r}' = \begin{bmatrix} x \\ d \\ 0 \end{bmatrix}, \quad d\vec{l}' = \begin{bmatrix} dx \\ 0 \\ 0 \end{bmatrix} \right.$

+(2) $\left\{ \underline{\vec{r}' \times d\vec{l}' = \begin{bmatrix} 0 \\ 0 \\ -d\,dx \end{bmatrix}} \right.$, $r' = \sqrt{x^2+d^2}$ $\rightarrow$ $\overbrace{B_1 = -\frac{\mu_0 I d}{4\pi} \int_{-\infty}^{0} \frac{dx}{(x^2+d^2)^{\frac{3}{2}}}}^{+(3)}$

+(4) $$\left\{ \underline{B_1 = -\frac{\mu_0 I}{4\pi d} \int_{-\infty}^{0} \frac{d\left(\frac{x}{d}\right)}{\left[1+\left(\frac{x}{d}\right)^2\right]^{\frac{3}{2}}}} \right. = -\frac{\mu_0 I}{4\pi d} \underbrace{\frac{1}{\sqrt{1+\left(\frac{d}{x}\right)^2}}}_{\frac{x}{\sqrt{x^2+d^2}}} \Bigg|_{-\infty}^{0} = \underbrace{\underline{-\frac{\mu_0 I}{4\pi d}}}_{+(5)}$$

- segment 2



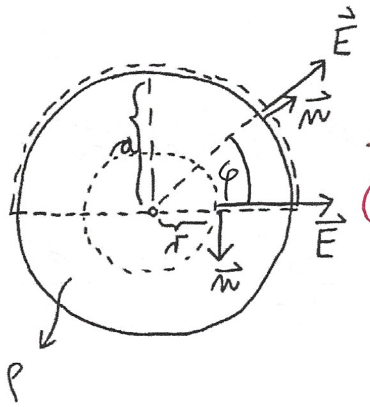
+(6) { vse podobno kot zgoraj, le $\underline{d \rightarrow \frac{d}{\sqrt{2}}}$ in meji $\underline{\left[-\frac{d}{\sqrt{2}}, \frac{d}{\sqrt{2}}\right]}$

$$\underline{B_2 = -\frac{\mu_0 I}{4\pi \frac{d}{\sqrt{2}}} \int_{-\frac{d}{\sqrt{2}}}^{\frac{d}{\sqrt{2}}} \frac{dx}{\left(x^2+\frac{d^2}{2}\right)^{\frac{3}{2}}}} = -\frac{\mu_0 I}{4\pi \frac{d}{\sqrt{2}}} \underbrace{\frac{x}{\sqrt{x^2+\frac{d^2}{2}}} \Bigg|_{-\frac{d}{\sqrt{2}}}^{\frac{d}{\sqrt{2}}}}_{\sqrt{2}} = \underbrace{\underline{-\frac{\mu_0 I}{2\pi d}}}_{+(7)}$$

- segmenta 3 & 4 → enako kot 1 & 2
- skupaj: $B = 2(B_1 + B_2) = \boxed{-\frac{3\mu_0 I}{2\pi d}}$ } +(8)

8+ → $\boxed{1}$

$\boxed{2}$ SILA NA POLOVICO NABITEGA VALJA



+(1)

$$\varepsilon_0 E \cdot \underbrace{2\pi r l}_{S} = \rho \cdot \underbrace{\pi r^2 l}_{V}$$

→ Gauss za valj s polmerom r, ZNOTRAJ, $r \leq a$!

$$E = \frac{\rho r}{2\varepsilon_0}$$

- obod valja

$$\vec F_e = \varepsilon_0 \oint \left[\vec E(\vec E \cdot \vec n) - \frac{1}{2}E^2 \vec n\right] dS$$

+(2)

$$\vec E \parallel \vec n \Rightarrow \vec E(\vec E \cdot \vec n) = E \vec n E = E^2 \vec n$$

$$\vec F_{e1} = \frac{\varepsilon_0}{2}\int E^2 \vec n \, dS$$

+(3)

$$E = \frac{\rho a}{2\varepsilon_0}, \quad \vec n = \begin{bmatrix}\cos\varphi \\ \sin\varphi\end{bmatrix}, \quad dS = a\,d\varphi \cdot l$$

+(4)

$$\vec F_{e1} = \frac{\varepsilon_0}{2}\frac{\rho^2 a^2}{4\varepsilon_0^2} a l \int_0^{\pi} \begin{bmatrix}\cos\varphi \\ \sin\varphi\end{bmatrix} \begin{matrix}\to 0 \\ \to 2\end{matrix} d\varphi = \frac{\rho^2 a^3 l}{4\varepsilon_0}\hat e_y$$

- presek valja (ploskev skozi središče)

+(5)

$$\vec E \perp \vec n \Rightarrow \vec E \cdot \vec n = 0$$

$$\vec F_{e2} = -\frac{\varepsilon_0}{2}\int E^2 \vec n \, dS$$

+(6)

$$E = \frac{\rho r}{2\varepsilon_0}, \quad \vec n = -\hat e_y, \quad dS = l\,dr$$

+(7)

$$\vec F_{e2} = \frac{\varepsilon_0}{2}\frac{\rho^2 l}{4\varepsilon_0^2}\hat e_y \int_{-a}^{a} r^2 dr = \frac{\rho^2 l}{8\varepsilon_0} 2\frac{a^3}{3}\hat e_y = \frac{\rho^2 a^3 l}{12\varepsilon_0}\hat e_y$$

+(8)

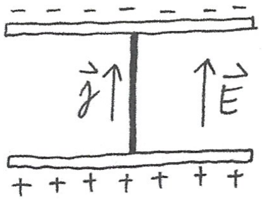
- skupaj

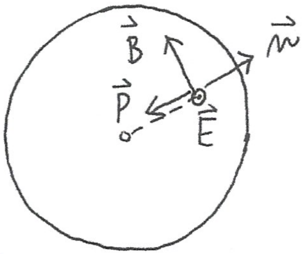
$$\vec F_e = \vec F_{e1} + \vec F_{e2} = \frac{\rho^2 a^3 l}{3\varepsilon_0}\hat e_y, \quad \boxed{\frac{\vec F_e}{l} = \frac{\rho^2 a^3}{3\varepsilon_0}\hat e_y}$$

kaže NAVZGOR za zgornjo polovico!

8+ → $\boxed{1}$

$\boxed{3}$ PRAZNJENJE KONDENZATORJA





a) tokovni krog:

+(1) $$\dot{U}_t = R\dot{I} + \frac{I}{C} = 0 \Rightarrow \frac{\dot{I}}{I} = -\frac{1}{RC} = -\frac{1}{\tau}$$

$$\underline{I = I_0 e^{-\frac{t}{\tau}}}$$

+(2) $$U(t) = RI(t) = RI_0 e^{-\frac{t}{\tau}} = U_0 e^{-\frac{t}{\tau}}$$

$$E(r,t) = \frac{U(t)}{l} = \boxed{\frac{U_0}{l} e^{-\frac{t}{\tau}}}, \quad \underline{\tau = RC = \frac{\varepsilon_0 \pi a^2 R}{l}}$$

$$\nabla \times \vec B = \varepsilon_0 \mu_0 \frac{\partial \vec E}{\partial t} + \mu_0 \vec j \quad / \int_0 d\vec S$$

+(3) $$B \cdot 2\pi r = \varepsilon_0 \mu_0 \dot{E}(t) \pi r^2 + \mu_0 \underbrace{\int \vec j \, d\vec S}_{I(t)}$$

+(4) $$B = \varepsilon_0 \mu_0 \frac{r}{2}\left(-\frac{1}{\tau}\right)\frac{U_0}{l} e^{-\frac{t}{\tau}} + \mu_0 \frac{I_0 e^{-\frac{t}{\tau}}}{2\pi r} =$$

$$= \frac{\mu_0 I_0 e^{-\frac{t}{\tau}}}{2\pi r}\left[1 - \varepsilon_0 \frac{r}{2} 2\pi r \underbrace{\frac{l}{\varepsilon_0 \pi a^2 R}}_{\frac{1}{\tau}} \frac{R}{l}\right] = \frac{\mu_0 I_0 e^{-\frac{t}{\tau}}}{2\pi r}\left(1 - \frac{r^2}{a^2}\right)$$

$$\boxed{B(r,t) = \frac{\mu_0 U_0}{2\pi R r} e^{-\frac{t}{\tau}}\left(1 - \frac{r^2}{a^2}\right)}$$

+(5) b) $\vec P = \frac{1}{\mu_0} \vec E \times \vec B$, $\vec P$ kaže proti središču (vodniku)

$$P = \frac{1}{\mu_0} E B = \underline{\frac{U_0^2}{2\pi l R} e^{-\frac{2t}{\tau}} \frac{1}{r}\left(1 - \frac{r^2}{a^2}\right)}$$

8+ → $\boxed{1}$

\+ → $\frac{1}{8}$

+(6) $$\underline{P(a) = 0} \Rightarrow \boxed{\int_{r=a} \vec P \cdot d\vec S = 0}$$

c) $r_0 \ll a$ → polmer vodnika ⇒ $P(r_0) = \frac{U^2(t)}{2\pi l R r_0}$ → $U_0^2 e^{-\frac{2t}{\tau}}$

+(7) $$\oint \vec P \cdot \underset{\parallel \vec n}{d\vec S} = -P(r_0) 2\pi r_0 l = -\underline{\frac{U^2(t)}{R_0}}$$

$r_0 \ll a \Rightarrow W = 0$

$$, \int \vec j \cdot \vec E \, dV = +j \overbrace{E l}^{U} \pi r_0^2 = UI = \underline{\frac{U^2(t)}{R}}$$

$$\boxed{\oint \vec P \cdot d\vec S + \int \vec j \cdot \vec E \, dV = 0}$$ +(8)

8+ → $\boxed{1}$