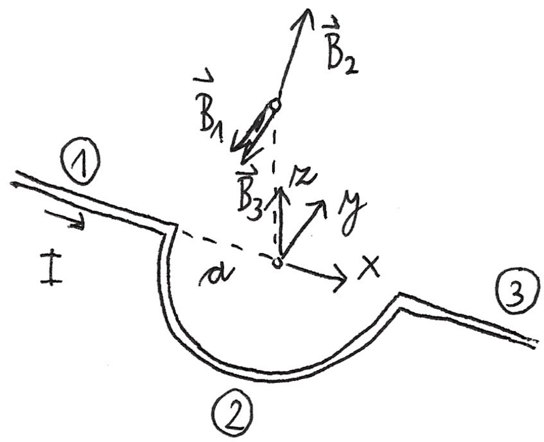
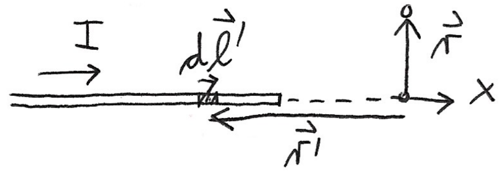
EMP, 1. KOLOKVIJ

1 VODNIK S POLKROŽNO IZBOKLINO



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{\ell}\,' \times (\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}$$

1



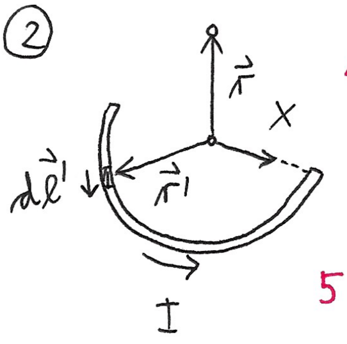
1+ $\left\{ \vec{r} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}, \quad \vec{r}\,' = \begin{bmatrix} -x' \\ 0 \\ 0 \end{bmatrix}, \quad d\vec{\ell}\,' = \begin{bmatrix} dx' \\ 0 \\ 0 \end{bmatrix} \right.$

2+ $\left\{ \begin{aligned} & d\vec{\ell}\,' \times (\vec{r}-\vec{r}\,') = \begin{bmatrix} 0 \\ -z\,dx' \\ 0 \end{bmatrix} \\ & |\vec{r}-\vec{r}\,'|^3 = (x'^2+z^2)^{\frac{3}{2}} \end{aligned} \right\} \Rightarrow \vec{B}_1 = \frac{\mu_0 I}{4\pi}(-\hat{e}_y)\, z \underbrace{\int_a^\infty \frac{dx'}{(z^2+x'^2)^{\frac{3}{2}}}}$

3+ $\left\{ \begin{aligned} & \frac{1}{z^2}\int_a^\infty \frac{d\left(\frac{x'}{z}\right)}{\left[1+\left(\frac{x'}{z}\right)^2\right]^{\frac{3}{2}}} = \frac{1}{z^2}\left.\frac{\frac{x'}{z}}{\sqrt{1+\left(\frac{x'}{z}\right)^2}}\right|_a^\infty = \frac{1}{z^2}\left(1-\frac{a}{\sqrt{a^2+z^2}}\right) \\ & \underline{\vec{B}_1 = -\hat{e}_y \frac{\mu_0 I}{4\pi z}\left(1-\frac{a}{\sqrt{a^2+z^2}}\right)} \end{aligned} \right.$

3 $\underline{\vec{B}_3 = \vec{B}_1}$

2



4+ $\left\{ \vec{r} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}, \quad \vec{r}\,' = \begin{bmatrix} a\cos\varphi' \\ a\sin\varphi' \\ 0 \end{bmatrix}, \quad d\vec{\ell}\,' = \begin{bmatrix} -\sin\varphi' \\ \cos\varphi' \\ 0 \end{bmatrix} a\,d\varphi' \right.$

5+ $\left\{ \begin{aligned} & \vec{r}-\vec{r}\,' = \begin{bmatrix} -a\cos\varphi' \\ -a\sin\varphi' \\ z \end{bmatrix}, \quad d\vec{\ell}\,' \times (\vec{r}-\vec{r}\,') = \begin{bmatrix} z\cos\varphi' \\ z\sin\varphi' \\ a \end{bmatrix} a\,d\varphi' \\ & |\vec{r}-\vec{r}\,'|^3 = (a^2+z^2)^{\frac{3}{2}} \end{aligned} \right.$

6+ $\left\{ \vec{B}_2 = \frac{\mu_0 I}{4\pi}\frac{a}{(a^2+z^2)^{\frac{3}{2}}}\int_\pi^{2\pi} \begin{bmatrix} z\cos\varphi' \\ z\sin\varphi' \\ a \end{bmatrix} d\varphi' = \underline{\frac{\mu_0 I}{4\pi}\frac{a}{(a^2+z^2)^{\frac{3}{2}}} \begin{bmatrix} 0 \\ -2z \\ \pi a \end{bmatrix}} \right.$

7+ $\left\{ \vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \vec{B}_2 + 2\vec{B}_1 = \right.$

8+ $\left\{ \begin{aligned} & \vec{B} = \frac{\mu_0 I}{4\pi}\left[\left\{\frac{2}{z}\left(1-\frac{a}{\sqrt{a^2+z^2}}\right) + \frac{2az}{(a^2+z^2)^{\frac{3}{2}}}\right\}(-\hat{e}_y) + \frac{\pi a^2}{(a^2+z^2)^{\frac{3}{2}}}\hat{e}_z\right] \\ & \boxed{B = \frac{\mu_0 I}{4\pi}\sqrt{\left\{\frac{2}{z}\left(1-\frac{a}{\sqrt{a^2+z^2}}\right) + \frac{2az}{(a^2+z^2)^{\frac{3}{2}}}\right\}^2 + \left\{\frac{\pi a^2}{(a^2+z^2)^{\frac{3}{2}}}\right\}^2}} \end{aligned} \right.$

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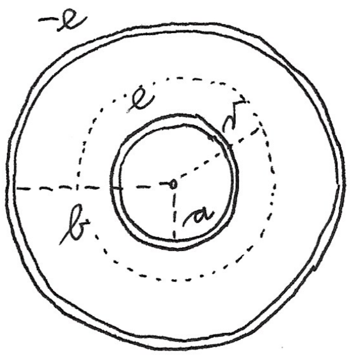
2 NAPETOST CEVI V VALJNEM KONDENZATORJU



- ZNOTRAJ $(a < r < b)$

1+
$$e = \varepsilon_0 E \cdot 2\pi r l \quad \text{Gauss}$$
$$E = \frac{e}{2\pi\varepsilon_0 l r}$$

2+
$$U_0 = \int_a^b E\,dr = \frac{e}{2\pi\varepsilon_0 l}\ln\frac{b}{a} \Rightarrow \frac{e}{2\pi\varepsilon_0 l} = \frac{U_0}{\ln\frac{b}{a}}$$
$$\underline{E = \frac{U_0}{\ln\frac{b}{a}}\cdot\frac{1}{r}}$$

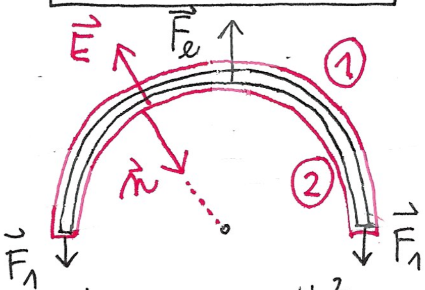
- ZUNAJ

$r > b$: $\quad e - e = \varepsilon_0 E \cdot 2\pi r l \Rightarrow \underline{E = 0}$

$r < a$: $\quad 0 = \varepsilon_0 E \cdot 2\pi r l \Rightarrow \underline{E = 0}$ +3

$$\vec{F}_e = \varepsilon_0 \oint \left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS$$

- ZUNANJA CEV



① $E = 0 \Rightarrow \underline{\vec{F}_{e1} = 0}$

② $\vec{E}\cdot\vec{n} = -E$, $\quad \vec{E}(\vec{E}\cdot\vec{n}) = -E\vec{E} = E^2\vec{n}$
$$\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n} = \frac{1}{2}E^2\vec{n}$$ +4
$$E = \frac{U_0}{\ln\frac{b}{a}}\cdot\frac{1}{b}$$

5+
$$\vec{F}_{e2} = \frac{\varepsilon_0}{2}\,\frac{U_0^2}{\ln^2\frac{b}{a}}\,\frac{1}{b^2}\int \underbrace{\vec{n}}_{\begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix}}\,\underbrace{dS}_{b\,l\,d\varphi}$$

cev VLEČE NARAZEN

6+
$$\vec{F}_{e2} = \frac{\varepsilon_0}{2}\,\frac{U_0^2}{\ln^2\frac{b}{a}}\,\frac{l}{b}\int_0^\pi \begin{bmatrix}\cos\varphi\\ \sin\varphi\end{bmatrix} d\varphi = \underline{\frac{\varepsilon_0 U_0^2}{b\ln^2\frac{b}{a}}\cdot l\,\hat{e}_y}$$

7+
$$F_1 = \frac{F_e}{2} = \frac{F_{e1} + F_{e2}}{2}, \quad \boxed{\frac{F_1}{l} = \frac{\varepsilon_0 U_0^2}{2b\ln^2\frac{b}{a}}}$$

- NOTRANJA CEV → ENAK račun kot za zunanjo cev, le z dvema SPREMEMBAMA:

8+

1) $\vec{n} \rightarrow -\vec{n}$

2) $E = \frac{U_0}{\ln\frac{b}{a}}\cdot\frac{1}{b} \rightarrow \frac{U_0}{\ln\frac{b}{a}}\cdot\frac{1}{a}$

$$\Rightarrow \boxed{\frac{F_1}{l} = \frac{\varepsilon_0 U_0^2}{2a\ln^2\frac{b}{a}}}$$

1

3 PREVODNA KROGLA V KVADRUPOLNEM POTENCIALU

1+

$$U = V\left(z^2 - \frac{x^2}{2} - \frac{y^2}{2}\right) = Vr^2\left(\cos^2\vartheta - \frac{1}{2}\sin^2\vartheta\cos^2\varphi - \frac{1}{2}\sin^2\vartheta\sin^2\varphi\right) =$$

$$= Vr^2\left(\cos^2\vartheta - \frac{1}{2}\underbrace{\sin^2\vartheta}_{1-\cos^2\vartheta}\right) = Vr^2\underbrace{\left(\frac{3}{2}\cos^2\vartheta - \frac{1}{2}\right)}_{P_2(\cos\vartheta)}$$

$\underline{U(\vec{r}) = Vr^2 P_2(\cos\vartheta)}$ potencial zunanjega polja

a) $\nabla^2 U(\vec{r}) = 0$ ZUNAJ krogle $\Rightarrow U(\vec{r}) = \sum_{l=0}^{\infty}\left(A_l r^l + B_l r^{-(l+1)}\right)P_l(\cos\vartheta)$

2+

$\boxed{\text{RP1}}$: $U(r\to\infty,\vartheta) = Vr^2 P_2(\cos\vartheta) \Rightarrow \underline{A_2 = V,\ A_{l\neq 2} = 0}$

$\boxed{\text{RP2}}$: $U(r=a,\vartheta) = 0 \Rightarrow \underline{B_{l\neq 2} = 0}$

$A_2 a^2 + B_2 a^{-3} = 0 \Rightarrow \underline{B_2 = -Va^5}$

$\Downarrow$

3+

$$\boxed{U(r,\vartheta) = Vr^2 P_2(\cos\vartheta) - \frac{Va^5}{r^3}P_2(\cos\vartheta)}$$

b) 4+

$$\sigma = \varepsilon_0 E(a,\vartheta) = -\varepsilon_0 \left.\frac{\partial U}{\partial r}\right|_{r=a} = -\varepsilon_0(2Va + 3Va)P_2(\cos\vartheta)$$

$$\boxed{\sigma = -\frac{5}{2}\varepsilon_0 Va\left(3\cos^2\vartheta - 1\right)}$$

c) $\underline{dS = 2\pi a^2\, d(\cos\vartheta)}$

5+

$$e = \int \sigma\, dS = -\frac{5}{2}\varepsilon_0 Va \cdot 2\pi a^2 \underbrace{\int_{\cos\vartheta=-1}^{1}\left(3\cos^2\vartheta - 1\right)d(\cos\vartheta)}_{\left.\left(\frac{3}{3}\cos^3\vartheta - \cos\vartheta\right)\right|_{-1}^{1} = 0} = \boxed{0}$$

$$p_e = \int \sigma \underbrace{z}_{\to a\cos\vartheta} dS = -\frac{5}{2}\varepsilon_0 Va \cdot 2\pi a^2 \cdot a \underbrace{\int_{\cos\vartheta=-1}^{1}\left(3\cos^2\vartheta - 1\right)\cos\vartheta\, d(\cos\vartheta)}_{\left.\left(\frac{3}{4}\cos^4\vartheta - \frac{1}{2}\cos^2\vartheta\right)\right|_{-1}^{1} = 0} = \boxed{0}$$

$$Q_{ij} = \int \sigma \left(3 r_i r_j - \delta_{ij} r^2\right) dS \quad , \quad dS = a^2\, d\varphi\, d(\cos\vartheta)$$

$$\vec{r} = a \begin{bmatrix} \cos\varphi \sin\vartheta \\ \sin\varphi \sin\vartheta \\ \cos\vartheta \end{bmatrix}$$

6+ $\left\{ \begin{array}{l} \underline{i \neq j} \Rightarrow r_i r_j \text{ vsebuje} \\ \underline{\cos\varphi, \sin\varphi \text{ ali } \cos\varphi\sin\varphi} \Rightarrow \int_0^{2\pi} d\varphi = \boxed{0} \end{array} \right.$

$\underline{i = j}$

7+

$$i = 1, \quad Q_{11} = -\frac{5}{2}\varepsilon_0 V a^5 \int_0^{2\pi}\int_{-1}^{1} \left(3\cos^2\varphi \sin^2\vartheta - 1\right)(3\cos^2\vartheta - 1)\, d\varphi\, d(\cos\vartheta) =$$

$$= -\frac{5}{2}\varepsilon_0 V a^5 \int_{-1}^{1} \underbrace{\left(\frac{3}{2}\sin^2\vartheta - 1\right)}_{\frac{1}{2} - \frac{3}{2}\cos^2\vartheta} (3\cos^2\vartheta - 1)\, 2\pi\, d(\cos\vartheta) =$$

$$= \frac{5}{2}\varepsilon_0 V a^5\, 2\pi \frac{1}{2} \int_{-1}^{1} \left(3\cos^2\vartheta - 1\right)^2 d(\cos\vartheta) =$$

$$= \frac{5}{4}\varepsilon_0 V a^5 \cdot 2\pi \underbrace{\left(9\cdot\frac{2}{5} - 6\cdot\frac{2}{3} + 1\cdot 2\right)}_{\frac{18}{5} - 4 + 2 = \frac{8}{5}} = \boxed{4\pi\varepsilon_0 V a^5}$$

$i = 2$, $\boxed{Q_{22} = Q_{11}}$ zaradi simetrije $x \leftrightarrow y$

8+

$$i = 3, \quad Q_{33} = -\frac{5}{2}\varepsilon_0 V a^5 \int_0^{2\pi}\int_{-1}^{1} \underbrace{\left(3\cos^2\vartheta - 1\right)}_{-2\text{-krat več kot pri } Q_{11}} (3\cos^2\vartheta - 1)\, 2\pi\, d(\cos\vartheta)$$

$$Q_{33} = -2\, Q_{11} = \boxed{-8\pi\varepsilon_0 V a^5}$$

lahko tudi iz $Q_{11} + Q_{22} + Q_{33} = 0$

$$\boxed{\underline{Q} = 4\pi\varepsilon_0 V a^5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}}$$

$\boxed{1}$