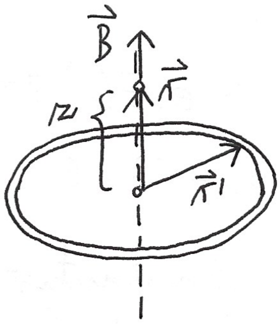
EMP : 1. KOLOKVIJ

1 GRADIENTNA TULJAVA

a)



$$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l}' \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

+(1) $\vec{r} = \begin{bmatrix} 0 \\ 0 \\ z \end{bmatrix}, \quad \vec{r}' = \begin{bmatrix} \cos\varphi' \\ \sin\varphi' \\ 0 \end{bmatrix} a, \quad d\vec{l}' = \begin{bmatrix} -\sin\varphi' \\ \cos\varphi' \\ 0 \end{bmatrix} a\, d\varphi'$

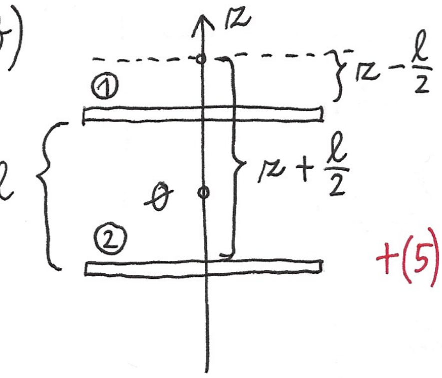
+(2) $$d\vec{l}' \times (\vec{r}-\vec{r}') = \begin{bmatrix} -\sin\varphi' \\ \cos\varphi' \\ 0 \end{bmatrix} a\, d\varphi' \times \begin{bmatrix} -a\cos\varphi' \\ -a\sin\varphi' \\ z \end{bmatrix} = \begin{bmatrix} z\cos\varphi' \\ z\sin\varphi' \\ a \end{bmatrix} a\, d\varphi'$$

$$|\vec{r}-\vec{r}'| = (a^2+z^2)^{\frac{3}{2}}$$

+(3) $$\vec{B}(\vec{r}) = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \begin{bmatrix} z\cos\varphi' \\ z\sin\varphi' \\ a \end{bmatrix} \frac{a\, d\varphi'}{(a^2+z^2)^{\frac{3}{2}}} = \frac{\mu_0 I}{4\pi} \hat{e}_z \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}} 2\pi$$

$$\boxed{B(z) = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}}}$$

b)



$$B_{1,2}(z) = \pm \frac{\mu_0 I}{2} \frac{a^2}{\left[a^2+\left(z \mp \frac{l}{2}\right)^2\right]^{\frac{3}{2}}}$$ +(4)

+(5) $$\frac{\partial B_{1,2}}{\partial z} = \pm \frac{\mu_0 I a^2}{2} \left(-\frac{3}{2}\right) \frac{2\left(z\mp\frac{l}{2}\right)}{\left[a^2+\left(z\mp\frac{l}{2}\right)^2\right]^{\frac{5}{2}}}$$

$$\left.\frac{\partial B}{\partial z}\right|_{z=0} = \left(\frac{\partial B_1}{\partial z} + \frac{\partial B_2}{\partial z}\right)_{z=0} = \frac{3\mu_0 I a^2}{2} \underbrace{\frac{l}{\left[a^2+\left(\frac{l}{2}\right)^2\right]^{\frac{5}{2}}}}_{g(l)}$$

maksimizacija $g(l)$:

+(6) $$\frac{dg}{dl} = \frac{\left(a^2+\left(\frac{l}{2}\right)^2\right)^{\frac{5}{2}} - l\cdot\frac{5}{2}\left(a^2+\left(\frac{l}{2}\right)^2\right)^{\frac{3}{2}}\cdot\frac{1}{2}l}{\left[a^2+\left(\frac{l}{2}\right)^2\right]^5} = 0$$

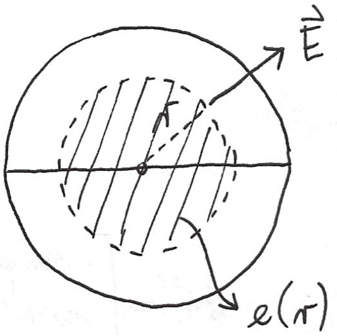
+(7) $$a^2 + \left(\frac{l}{2}\right)^2 - \frac{5}{4}l^2 = 0 \Rightarrow a^2 - 4\frac{l^2}{4} = a^2 - l^2 = 0 \Rightarrow \boxed{l = a}$$

+(8) $$\left.\frac{\partial B}{\partial z}\right|_{z=0} = \frac{3\mu_0 I a^2}{2} \cdot \frac{a}{\left(\frac{5}{4}a^2\right)^{\frac{5}{2}}} = \boxed{\left(\frac{4}{5}\right)^{\frac{5}{2}} \frac{3\mu_0 I}{2a^2}}$$

8 + → 1

\+ → $\frac{1}{8}$

2 ELEKTRIČNA SILA V NABITEM VALJU



+(1) $\left\{ e(r) = \int_0^r \rho(r)\,dV = \int_0^r \frac{\alpha}{r} \cdot 2\pi l r\,dr = 2\pi\alpha l r \right.$

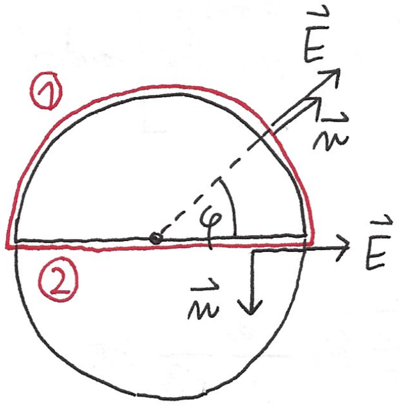
+(2) Gauss:

$$e(r) = \varepsilon_0 E S = \varepsilon_0 E(r) \cdot 2\pi r l$$

$$2\pi\alpha l r = 2\pi \varepsilon_0 E(r)\, r l$$

$$\underline{E(r) = \frac{\alpha}{\varepsilon_0}}$$

konstantno polje v radialni smeri



$$\vec{F}_e = \varepsilon_0 \oint \left[\vec{E}(\vec{E}\cdot\vec{n}) - \frac{1}{2}E^2\vec{n}\right] dS$$

① $\vec{E} \parallel \vec{n} \Rightarrow \vec{E}(\vec{E}\cdot\vec{n}) = E^2\vec{n}$ +(3)

$$\vec{F}_{e1} = \varepsilon_0 \int \frac{1}{2} E^2 \vec{n}\, dS =$$

$$= \varepsilon_0 \frac{1}{2}\left(\frac{\alpha}{\varepsilon_0}\right)^2 \int \vec{n}\, a l\, d\varphi = \quad +(4)$$

$$= \frac{\alpha^2}{2\varepsilon_0} a l \underbrace{\int_0^\pi \begin{bmatrix}\cos\varphi \\ \sin\varphi\end{bmatrix} d\varphi}_{2\hat{e}_y}$$

$$\underline{\vec{F}_{e1} = \frac{\alpha^2 a}{\varepsilon_0} l\, \hat{e}_y} \quad +(5)$$

② $\vec{E} \perp \vec{n} \Rightarrow \vec{E}(\vec{E}\cdot\vec{n}) = 0$ +(6)

$$\vec{F}_{e2} = -\frac{\varepsilon_0}{2} \int E^2 \vec{n}\, dS = -\frac{\varepsilon_0}{2} \underbrace{\left(\frac{\alpha}{\varepsilon_0}\right)^2}_{E} \underbrace{(-\hat{e}_y)}_{\vec{n}} \underbrace{2al}_{S} \quad +(7)$$

$$\underline{\vec{F}_{e2} = \frac{\alpha^2 a}{\varepsilon_0} l\, \hat{e}_y}$$

$$\boxed{\frac{F_e}{l} = \frac{F_{e1}}{l} + \frac{F_{e2}}{l} = \frac{2\alpha^2 a}{\varepsilon_0}} \quad +(8)$$

8+ → $\boxed{1}$

+ → $\frac{1}{8}$

3 NABITA KROGELNA LUPINA

a) $\sigma = \sigma_0 \cos\vartheta = \sigma_0 P_1(\cos\vartheta) \Rightarrow$ rešitev vsebuje le $P_1(\cos\vartheta)$ +(1)

$$U(r,\vartheta) = \begin{cases} A_1 r \cos\vartheta, & r < a \\ \dfrac{B_1}{r^2}\cos\vartheta, & r \geq a \end{cases} \quad +(2)$$

RP1 : $U(r,\vartheta)$ je ZVEZEN pri $r = a$

$$A_1 a = \frac{B_1}{a^2} \quad +(3)$$

RP2 : $\dfrac{\sigma}{\varepsilon_0} = E_r^{ZUN} - E_r^{NOT}$ +(4) $\Leftarrow$ $de = \sigma dS = \varepsilon_0 (E_r^{ZUN} - E_r^{NOT}) dS$ Gauss

$$\frac{\sigma_0}{\varepsilon_0}\cos\vartheta = \left(-\frac{\partial U}{\partial r}\right)^{ZUN}_{r=a} - \left(-\frac{\partial U}{\partial r}\right)^{NOT}_{r=a} = \left(\frac{2B_1}{a^3} + A_1\right)\cos\vartheta$$

+(5) $\dfrac{\sigma_0}{\varepsilon_0} = \dfrac{2B_1}{a^3} + A_1 \xrightarrow{RP1} \dfrac{3B_1}{a^3} \Rightarrow$ $B_1 = \dfrac{\sigma_0 a^3}{3\varepsilon_0}$, $A_1 = \dfrac{B_1}{a^3} = \dfrac{\sigma_0}{3\varepsilon_0}$

$$\boxed{U(r,\vartheta) = \begin{cases} \dfrac{\sigma_0}{3\varepsilon_0} r \cos\vartheta, & r < a \\ \dfrac{\sigma_0 a^3}{3\varepsilon_0}\dfrac{1}{r^2}\cos\vartheta, & r \geq a \end{cases}} \quad +(6)$$

b) $r < a$: +(7) $U = \dfrac{\sigma_0}{3\varepsilon_0} r\cos\vartheta = \dfrac{\sigma_0}{3\varepsilon_0} z \Rightarrow \vec{E} = \left(-\dfrac{\partial U}{\partial z}\right)\hat{e}_z = \boxed{-\dfrac{\sigma_0}{3\varepsilon_0}\hat{e}_z}$

NI enako NIČ! $\leftarrow$ HOMOGENO polje ZNOTRAJ krogelne lupine

$V_{ij} = \dfrac{\partial^2 U}{\partial r_i \partial r_j} = \boxed{0}$ +(8), saj je potencial U LINEARNA funkcija samo z $\Rightarrow$ drugi odvodi so vsi NIČ

8 + → $\boxed{1}$

\+ → $\frac{1}{8}$