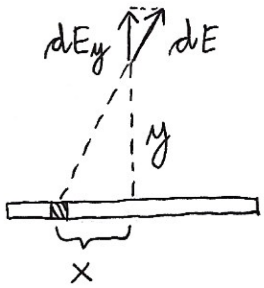
EMP – 1. KOLOKVIJ

1 – ena palica



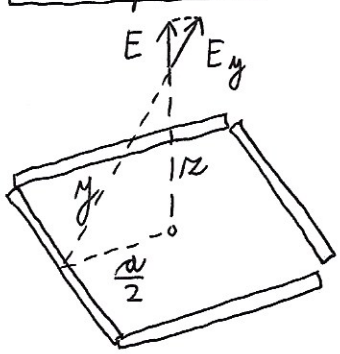
$$dE_y = dE\,\frac{y}{\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}}\,\frac{\overbrace{\mu\,dx}^{de}}{4\pi\varepsilon_0 (x^2+y^2)}$$

$$dE_y = \frac{\mu y}{4\pi\varepsilon_0}\,\frac{dx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$E_y = \frac{\mu y}{4\pi\varepsilon_0}\,\frac{x}{y^2\sqrt{x^2+y^2}}\Bigg|_{-\frac{a}{2}}^{\frac{a}{2}} = \frac{\mu y}{4\pi\varepsilon_0}\,\frac{a}{y^2\sqrt{\frac{a^2}{4}+y^2}}, \qquad \mu a = \frac{e}{4}$$

$$\underline{E_y = \frac{e}{16\pi\varepsilon_0}\,\frac{1}{y\sqrt{y^2+\frac{a^2}{4}}}}$$

– štiri palice



$$E = 4E_y\,\frac{z}{\sqrt{z^2+\frac{a^2}{4}}}\;, \qquad y = \sqrt{z^2+\frac{a^2}{4}}$$

$$E = 4\,\frac{e}{16\pi\varepsilon_0}\,\frac{1}{\sqrt{z^2+\frac{a^2}{4}}\sqrt{z^2+\frac{a^2}{2}}}\,\frac{z}{\sqrt{z^2+\frac{a^2}{4}}}$$

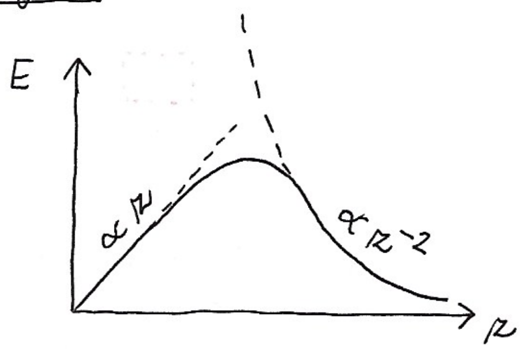
$$\boxed{E = \frac{e}{4\pi\varepsilon_0}\,\frac{z}{\left(z^2+\frac{a^2}{4}\right)\sqrt{z^2+\frac{a^2}{2}}}}$$

– limitna primera

$z \gg a$: $E = \frac{e}{4\pi\varepsilon_0}\,\frac{z}{z^2 z} = \underline{\frac{e}{4\pi\varepsilon_0 z^2}}$, kot točkasti naboj ✓

$z \ll a$: $E = \frac{e}{4\pi\varepsilon_0}\,\frac{z}{\frac{a^2}{4}\,\frac{a}{\sqrt{2}}} = \underline{\frac{e}{\sqrt{2}\,\pi\varepsilon_0 a^3}}\,z$, linearno

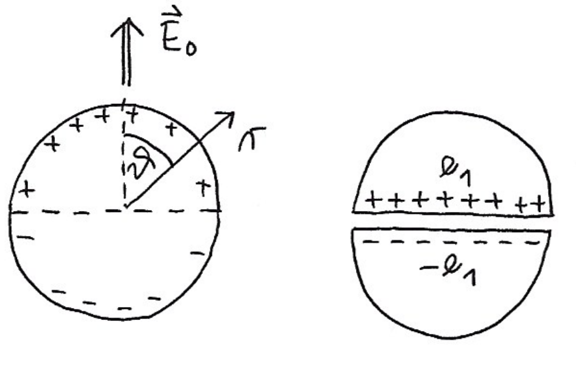
– graf



odvisnosti $\underline{\propto z}$ in $\underline{\propto z^{-2}}$ v obeh limitah morata biti ponazorjeni!

1

$\boxed{2}$



- zunanje polje:
$U_0(r,\vartheta) = -E_0 r \underbrace{\cos\vartheta} \Rightarrow$ SAMO $\underbrace{l=1}$ $\}+$

- nastavek:
$U(r,\vartheta) = -E_0 r\cos\vartheta + \frac{B}{r^2}\cos\vartheta$ $\}+$

- robni pogoj: $U(a,\vartheta) = 0 \Rightarrow B = E_0 a^3$ $\}+$

$$\underbrace{U(r,\vartheta) = -E_0 r\cos\vartheta + \frac{E_0 a^3}{r^2}\cos\vartheta} \quad \}+$$

- inducirani površinski naboj na obodu krogle:

$$\sigma = \varepsilon_0\left(-\frac{\partial U}{\partial r}\right)_{r=a} = \varepsilon_0 E_0\cos\vartheta + 2\varepsilon_0 E_0\cos\vartheta = 3\varepsilon_0 E_0\cos\vartheta \quad \}+$$

$$e_1 = \int \sigma \cdot a\,d\vartheta \cdot 2\pi a\sin\vartheta = 2\pi a^2\, 3\varepsilon_0 E \int_0^1 \cos\vartheta\, d(\cos\vartheta) = \underbrace{\pi a^2 \cdot 3\varepsilon_0 E_0}_{+}$$

↳ naboj na polovici krogle

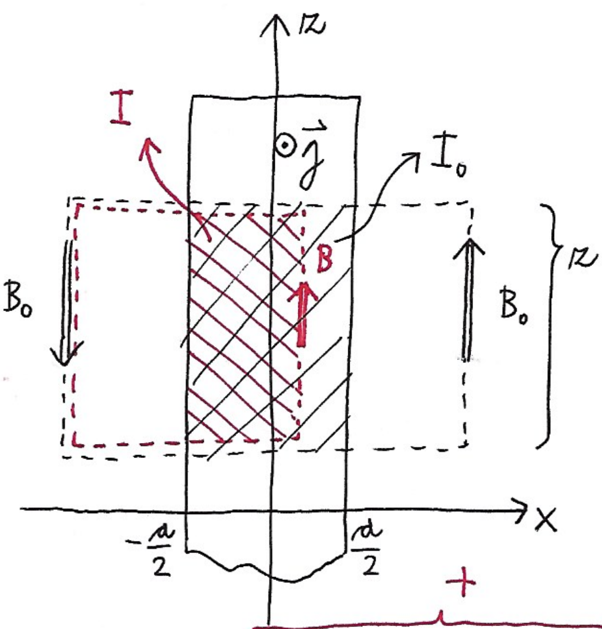
- špranja kot ploščati kondenzator:

$$\sigma_1 = \frac{e_1}{\pi a^2} = 3\varepsilon_0 E_0 \quad \}+$$

$$E_1 = \frac{\sigma_1}{\varepsilon_0} = \boxed{3E_0} \rightarrow \text{kaže navzdol!} \quad \}+$$

$\boxed{1}$

$\boxed{3}$



simetrija : $\underbrace{\vec{B} \parallel \text{površino plošče}}_{+}$

- večja zanka:

$$\mu_0 I_0 = \mu_0 j z a = 2B_0 z$$

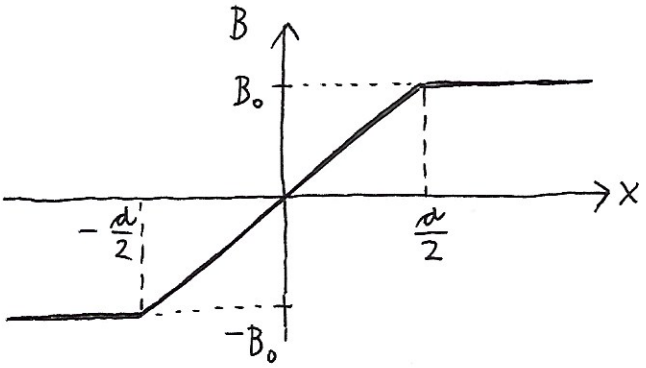
$$\underbrace{B_0 = \frac{1}{2}\mu_0 j a} \rightarrow \text{HOMOGENO!} \quad \}+$$

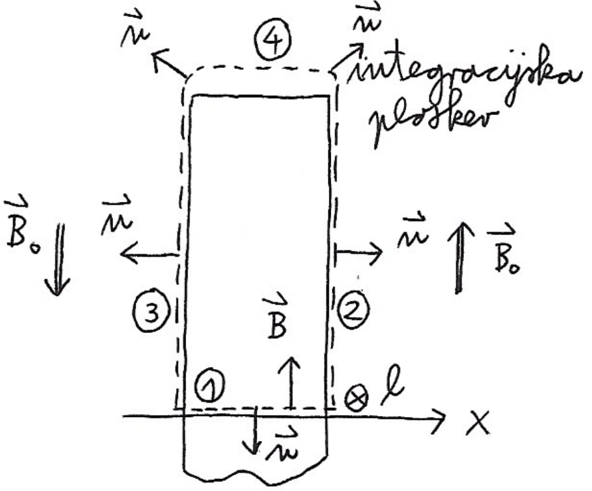
- manjša zanka:

$$\mu_0 I = \mu_0 j z\left(\frac{a}{2} + x\right) = B_0 z + B z \quad \}+$$

$$B = \underbrace{\frac{1}{2}\mu_0 j a - B_0}_{0} + \mu_0 j x$$

$$\underbrace{B = \mu_0 j x} \rightarrow \text{LINEARNO narašča!} \quad \}+$$





$$① \quad \vec F_m = \frac{1}{\mu_0}\int\Big[\overbrace{\vec B(\vec B\cdot\vec n)}^{B^2\vec n} - \frac12 B^2\vec n\Big]dS = \frac{1}{2\mu_0}\vec n\int B^2\, \underbrace{dS}_{l\,dx} = \quad \Big\}+$$

$$= \frac{1}{2\mu_0}\vec n\, \mu_0^2 j^2 l\int_{-\frac d2}^{\frac d2} x^2 dx = \underline{\frac{1}{24}\mu_0 j^2 l d^3 \vec n} \quad \Big\}+$$

② & ③ se odštejeta, saj je $\vec B_0 \perp \vec n$ in NASPROTNA $\vec n$ }+

④ pozabimo

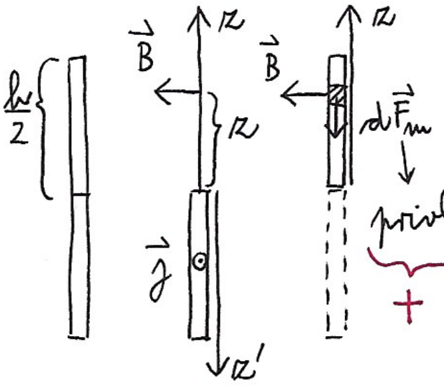
⇓

$$\boxed{\frac{\vec F_m}{l} = \frac{\mu_0}{24} j^2 d^3 \vec n} \rightarrow \text{PRIVLAČNA} \quad \Big\}+$$

Rezultat je žal NAPAČEN, saj ima ④ nezanemarljiv prispevek. Tam namreč NI $\vec B_0 \perp \vec n$ in se obe strani NE ODŠTEJETA!

→ Vseeno priznam za pravilno rešitev! $\boxed{1}$

–PRAVILNA REŠITEV



$\underbrace{\text{privlačna}}_{+}$

– magnetno polje SPODNJE polovice plošče:

$$\underbrace{dB = \frac{\mu_0 j d\, dz'}{2\pi(z+z')}}_{+} \Big\}\ \underbrace{dI = j d\, dz'}_{+},\quad B = \frac{\mu_0 j d}{2\pi}\ln(z'+z)\Big|_{z'=0}^{\frac h2}$$

$$\underline{B = \frac{\mu_0 j d}{2\pi}\ln\left(1+\frac{h}{2z}\right)} \quad \Big\}+$$

– sila na ZGORNJO polovico plošče:

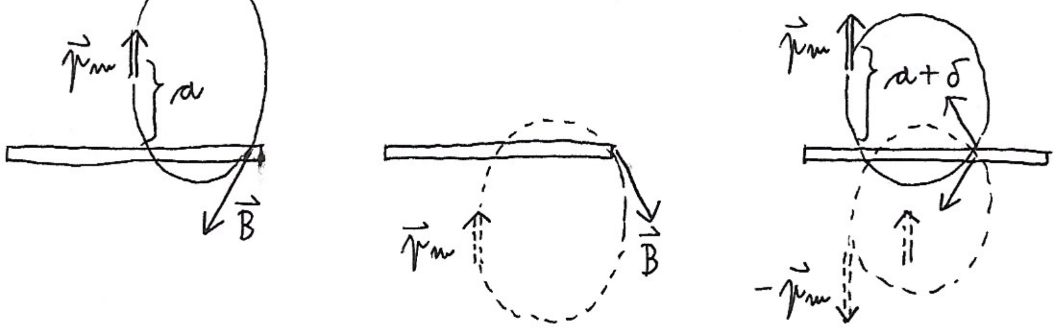
$$dF_m = dI\cdot l\cdot B = j d\, dz\cdot l\,\frac{\mu_0 j d}{2\pi}\ln\left(1+\frac{h}{2z}\right) \quad \Big\}+$$

$$F_m = \frac{\mu_0}{2\pi}j^2 d^2 l\,\underbrace{\underbrace{\int_0^{\frac h2}\ln\left(1+\frac{h}{2z}\right)dz}_{\frac h2\, 2\ln 2}}_{\frac14} = \frac{\ln 2}{2\pi}\mu_0 j^2 d^2 h l$$

⇓

$$\boxed{\frac{F_m}{l} = \frac{\ln 2}{2\pi}\mu_0 j^2 d^2 h} \quad \Big\}+ \quad \boxed{1}$$

4 $\vec{p}_m$ pri a $\rightarrow$ odstranimo $\vec{p}_m$ $\rightarrow$ $\vec{p}_m$ na $a+\delta$



antizrcalni dipol, ki OHRANI $B_\perp$ in s tem PRETOK

$\frac{1}{4}$

ZRCALNI dipol, ki skupaj z originalnim poskrbi, da se PRETOK spet OHRANI, se pravi, da je skupni $B_\perp = 0$

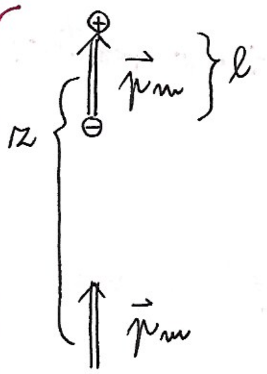
$\Downarrow$

izračunati moramo silo med dipolom $\vec{p}_m$ in njegovima DVEMA slikama!

$\frac{1}{4}$

– izračun sile med DVEMA kolinearnima DIPOLOMA

$\frac{1}{4}$



$$\vec{B} = \frac{\mu_0}{4\pi}\,\frac{3(\vec{p}_m\cdot\vec{r})\vec{r} - \vec{p}_m r^2}{r^5}$$

$$B = \frac{\mu_0}{4\pi}\,\frac{2p_m}{r^3}$$ na mestu zgornjega

$$F = g\,B\left(r+\frac{l}{2}\right) - g\,B\left(r-\frac{l}{2}\right) = g\,l\,\frac{\partial B}{\partial r} = p_m\frac{\partial B}{\partial r}$$

$$F = \frac{\mu_0}{4\pi}(-3)\frac{2p_m^2}{r^4} = \underbrace{-\frac{3\mu_0}{2\pi}\,\frac{p_m^2}{r^4}}$$ privlačna ✓

– izračun sile na končni $\vec{p}_m$ v našem primeru:

$\frac{1}{4}$

$$F = +\frac{3\mu_0}{2\pi}\,\frac{p_m^2}{[2(a+\delta)]^4} - \frac{3\mu_0}{2\pi}\,\frac{p_m^2}{[a+a+\delta]^4} = \frac{3\mu_0}{32\pi}p_m^2\frac{1}{a^4}\left[\left(1+\frac{2\delta}{2a}\right)^{-4} - \left(1+\frac{\delta}{2a}\right)^{-4}\right]$$

$$= \frac{3\mu_0}{32\pi}\,\frac{p_m^2}{a^4}\left[\underbrace{1-4\frac{\delta}{a}-1+4\frac{\delta}{2a}}_{-4\frac{\delta}{2a}}\right] = \boxed{-\frac{3\mu_0}{16\pi}\,\frac{p_m^2}{a^5}\,\delta}$$

↳ LINEARNO naraščajoča POVRATNA sila

$\Downarrow$

$\boxed{\text{dipol LEVITIRA}}$ $\Leftarrow$ STABILNA lega dipola