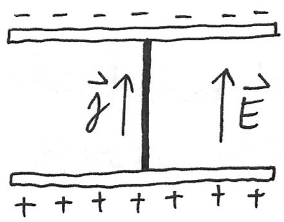
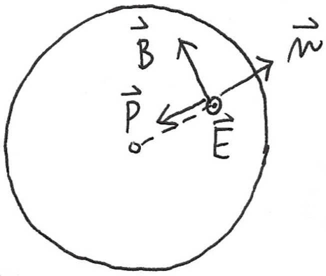
EMP: 2. KOLOKVIJ

1 PRAZNJENJE KONDENZATORJA





a) tokovni krog:

+(1) $\dot{U}_t = R\dot{I} + \frac{I}{C} = 0 \Rightarrow \frac{\dot{I}}{I} = -\frac{1}{RC} = -\frac{1}{\tau}$

$$\underline{I = I_0 e^{-\frac{t}{\tau}}}$$

+(2) $U(t) = RI(t) = RI_0 e^{-\frac{t}{\tau}} = U_0 e^{-\frac{t}{\tau}}$

$E(r,t) = \frac{U(t)}{l} = \boxed{\frac{U_0}{l} e^{-\frac{t}{\tau}}}$, $\underline{\tau = RC = \frac{\varepsilon_0 \pi a^2 R}{l}}$

$$\vec{\nabla} \times \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j} \quad / \int_0^r d\vec{S}$$

+(3) $B \cdot 2\pi r = \varepsilon_0 \mu_0 \dot{E}(t) \pi r^2 + \mu_0 \underbrace{\int \vec{j}\, d\vec{S}}_{I(t)}$

+(4) $B = \varepsilon_0 \mu_0 \frac{r}{2} \left(-\frac{1}{\tau}\right) \frac{U_0}{l} e^{-\frac{t}{\tau}} + \mu_0 \frac{I_0 e^{-\frac{t}{\tau}}}{2\pi r} =$

$= \frac{\mu_0 I_0 e^{-\frac{t}{\tau}}}{2\pi r} \left[1 - \varepsilon_0 \frac{r}{2} 2\pi r \underbrace{\frac{l}{\varepsilon_0 \pi a^2 R}}_{\frac{1}{\tau}} \frac{R}{l}\right] = \frac{\mu_0 I_0 e^{-\frac{t}{\tau}}}{2\pi r} \left(1 - \frac{r^2}{a^2}\right)$

$$\boxed{B(r,t) = \frac{\mu_0 U_0}{2\pi R r} e^{-\frac{t}{\tau}} \left(1 - \frac{r^2}{a^2}\right)}$$

+(5) b) $\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$, $\vec{P}$ kaže proti središču (vodniku)

$P = \frac{1}{\mu_0} EB = \underline{\frac{U_0^2}{2\pi l R} e^{-\frac{2t}{\tau}} \frac{1}{r} \left(1 - \frac{r^2}{a^2}\right)}$

8+ → 1

\+ → $\frac{1}{8}$

+(6) $\underline{P(a) = 0} \Rightarrow \boxed{\int_{r=a} \vec{P} \cdot d\vec{S} = 0}$

c) $r_0 \ll a$ → polmer vodnika $\Rightarrow P(r_0) = \frac{U^2(t)}{2\pi l R r_0}$, $U^2(t) \rightarrow U_0^2 e^{-\frac{2t}{\tau}}$

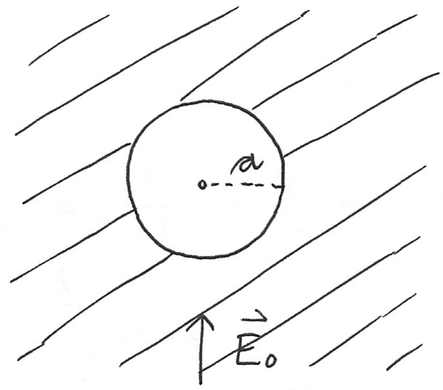
+(7) $\oint \vec{P} \cdot \underbrace{d\vec{S}}_{\parallel \vec{n}} = -P(r_0) 2\pi r_0 l = -\underline{\frac{U^2(t)}{R_0}}$, $\int \vec{j} \cdot \vec{E}\, dV = +j \overbrace{E l}^{U} \pi r_0^2 = UI = \underline{\frac{U^2(t)}{R}}$

$r_0 \ll a \Rightarrow W = 0$

$$\boxed{\oint \vec{P} \cdot d\vec{S} + \int \vec{j} \cdot \vec{E}\, dV = 0}$$

+(8)

2 KROGELNA VOTLINA V DIELEKTRIKU



a) +(1)

$$U(r) = \begin{cases} -E_0 r\cos\vartheta + \dfrac{B}{r^2}\cos\vartheta, & r > a \\ A r\cos\vartheta, & r \leq a \end{cases}$$

Ker je polje daleč stran od votline $-E_0 r\cos\vartheta$, so v rešitvi mogoči le členi s $P_1(\cos\vartheta) = \cos\vartheta$.

+(2) RP1 zveznost $U(r,\vartheta)$ pri $r = a$

$$\underbrace{-E_0 a + \frac{B}{a^2} = A a} \Rightarrow A = -E_0 + \frac{B}{a^3}$$

+(3) RP2 zveznost $D_r(r,\vartheta)$ pri $r = a$ $\Leftarrow$ NI zunanjih nabojev

$$D_r = -\varepsilon_0 \varepsilon \frac{\partial U}{\partial r}$$

$$\underbrace{-A = \varepsilon\left(E_0 + \frac{2B}{a^3}\right)} \Rightarrow E_0 - \frac{B}{a^3} = \varepsilon E_0 + 2\varepsilon \frac{B}{a^3}$$

$$(\varepsilon - 1)E_0 = -(2\varepsilon+1)\frac{B}{a^3}$$

+(4)

+(5)

$$A = -E_0 - \frac{\varepsilon-1}{2\varepsilon+1}E_0 = \underbrace{-\frac{3\varepsilon}{2\varepsilon+1}E_0} \Leftarrow \underbrace{B = -\frac{\varepsilon-1}{2\varepsilon+1}E_0 a^3}$$

$$\boxed{U(r,\vartheta) = \begin{cases} -\dfrac{3\varepsilon}{2\varepsilon+1}E_0 r\cos\vartheta, & r \leq a \\ -E_0 r\cos\vartheta - \dfrac{\varepsilon-1}{2\varepsilon+1}E_0 a^3 \dfrac{\cos\vartheta}{r^2}, & r > a \end{cases}}$$

b) polje znotraj votline, +(6)

$$U_{NOT}(r,\vartheta) = -\frac{3\varepsilon}{2\varepsilon+1}E_0 z$$

$$\underbrace{\vec{E}_{NOT} = \frac{3\varepsilon}{2\varepsilon+1}\vec{E}_0}, \quad \vec{E}_{NOT} = \vec{E}_r + \vec{E}_0$$

$$\vec{E}_r = \left(\frac{3\varepsilon}{2\varepsilon+1} - 1\right)\vec{E}_0$$

$$\boxed{\vec{E}_r = \frac{\varepsilon-1}{2\varepsilon+1}\vec{E}_0}$$

+(7)

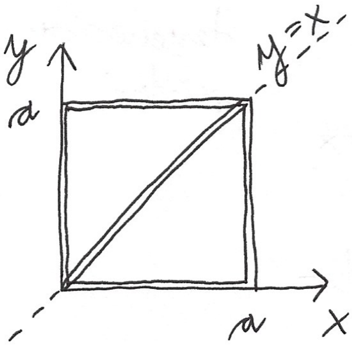
+(8)

$$U_{DIP} = \frac{p_e\cos\vartheta}{4\pi\varepsilon_0 r^2} = -\frac{\varepsilon-1}{2\varepsilon+1}E_0 a^3 \frac{\cos\vartheta}{r^2}$$

$$\boxed{p_e = -4\pi\varepsilon_0 \frac{\varepsilon-1}{2\varepsilon+1}E_0 a^3}$$ dipolni moment vezanih nabojev

8+ → $\boxed{1}$, + → $\frac{1}{8}$

3 KVADRATNI IN TRIKOTNI VALOVNI VODNIK



a) KVADRATNI vodnik

$$TM \rightarrow H_z = 0, \quad E_z \neq 0$$

$$\Big[\underbrace{\nabla_\perp^2}_{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}} + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\Big] \underbrace{E_z(x,y)}_{X(x)Y(y)} = 0$$

$$X''Y + XY'' + \varkappa^2 XY = 0 \Rightarrow \underbrace{\frac{X''}{X}}_{-\varkappa_x^2} = -\underbrace{\frac{Y''}{Y}}_{-\varkappa_y^2} - \varkappa^2 \rightarrow \varkappa_x^2 + \varkappa_y^2 = \varkappa^2$$

+(1) separacija

rešitev z izločitvijo cos

+(2)

$$X'' + \varkappa_x^2 X = 0 \Rightarrow X \propto \sin \varkappa_x x$$
$$Y'' + \varkappa_y^2 Y = 0 \Rightarrow Y \propto \sin \varkappa_y y$$

upoštevaje že $\boxed{RP1}$ $E_z(x=0 \text{ ali } y=0) = 0$

+(3)

$\boxed{RP2}$ $E_z(x=a \text{ ali } y=a) = 0 \Rightarrow \varkappa_x = \frac{m\pi}{a}, \quad \varkappa_y = \frac{n\pi}{a}$

$m, n = 1, 2, 3, \ldots$

$$\boxed{E_z(x,y) \propto \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{a}}$$

$$\varkappa_x^2 + \varkappa_y^2 = \varkappa^2 \rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{a}\right)^2 = \frac{\omega^2}{c_0^2} - k^2$$

končna rešitev in disperzijska relacija

$$\omega_{mn} = c_0 \sqrt{k^2 + \frac{\pi^2}{a^2}(m^2 + n^2)}$$

+(4)

$$\omega_{MIN} = \omega_{11}(k=0) = c_0 \frac{\pi}{a}\sqrt{2} \quad \text{(najnižja veja)}$$

$$\omega_{MIN}' = \omega_{21}(k=0) = c_0 \frac{\pi}{a}\sqrt{5} \quad \text{(druga najnižja veja)}$$

$$\Delta\omega = \omega_{MIN}' - \omega_{MIN} = \boxed{c_0 \frac{\pi}{a}\left(\sqrt{5} - \sqrt{2}\right)} \quad \text{(pasovna širina)}$$

pasovna širina

b) TRIKOTNI vodnik

+(5) – (m,n) in (n,m) sta DEGENERIRANI rešitvi

degenerirani rešitvi

$\hookrightarrow \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{a}$ in $\sin\frac{m\pi y}{a}\sin\frac{n\pi x}{a}$

+(6) – $\boxed{\text{RP3}}$ $E_z(y=x)=0$

končna rešitev

$$\Downarrow$$

$$\boxed{E_z(x,y)\propto \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{a}-\sin\frac{m\pi y}{a}\sin\frac{n\pi x}{a}}$$

– zaradi degeneriranosti obeh členov je disperzijska relacija za to linearno kombinacijo ENAKA!

$$\omega_{mn}=c_0\sqrt{k^2+\frac{\pi^2}{a^2}(m^2+n^2)}$$

+(7) – možni indeksi: $m\neq n$, sicer je rešitev enaka nič

indeksi

$\boxed{m<n}$, $m,n=1,2,3,\dots$ pokrije vse različne rešitve

+(8) – najnižji veji

$$\omega_{MIN}=\omega_{12}(k=0)=c_0\frac{\pi}{a}\sqrt{5}$$

$$\omega_{MIN}'=\omega_{13}(k=0)=c_0\frac{\pi}{a}\sqrt{10}$$

$$\Delta\omega=\omega_{MIN}'-\omega_{MIN}=\boxed{c_0\frac{\pi}{a}\left(\sqrt{10}-\sqrt{5}\right)}$$ (pasovna širina)

pasovna širina

$8+\rightarrow\boxed{1}$

$+\rightarrow\frac{1}{8}$