2. KOLOKVIJ

1

+ $$I = I_0 \sin\omega t \,, \quad e = \int I\, dt = -\frac{I_0}{\omega}(\cos\omega t - 1) = \frac{I_0}{\omega}(1-\cos\omega t)$$
$$E = \frac{e}{\varepsilon_0 S} = \underline{\frac{I_0}{\varepsilon_0 S\omega}(1-\cos\omega t)}$$

+ $$\vec{\nabla}\times\vec{B} = \varepsilon_0\mu_0 \frac{\partial\vec{E}}{\partial t} \,, \quad B\cdot 2\pi r = \varepsilon_0\mu_0 \pi r^2 \dot{E} = \varepsilon_0\mu_0 \pi r^2 \frac{I_0}{\varepsilon_0 S}\sin\omega t$$
$$\underline{B = \frac{\mu_0 I_0}{2S} r \sin\omega t}$$

$$P = \frac{1}{\mu_0} EB = \frac{I_0^2}{2\varepsilon_0 S^2 \omega} r \sin\omega t (1-\cos\omega t)$$

+ $$\int P\, dS = P(r=a)\, 2\pi a d = \frac{I_0^2 d}{\varepsilon_0 S^2\omega}\pi a^2 \sin\omega t(1-\cos\omega t) =$$
$$= \underline{\frac{I_0^2 d}{\varepsilon_0 S\omega}\sin\omega t(1-\cos\omega t)}$$

+ $$W_e = \frac{1}{2}\varepsilon_0 E^2 \cdot S d = \frac{I_0^2 d}{2\varepsilon_0 S\omega^2}(1-\cos\omega t)^2$$
$$\dot{W}_e = \frac{I_0^2 d}{2\varepsilon_0 S\omega^2}\, 2(1-\cos\omega t)(+\omega\sin\omega t) = \underline{\frac{I_0^2 d}{\varepsilon_0 S\omega}\sin\omega t(1-\cos\omega t)}$$ ✓

enako!

$\frac{1}{4}$ $$w_m = \frac{B^2}{2\mu_0} = \frac{\mu_0 I_0^2}{8S^2}\underline{r^2}\sin^2\omega t$$
$$W_m = \int w_m\, dV = \int w_m\, 2\pi r d \cdot dr = \frac{\mu_0 I_0^2}{8S^2} d\, 2\pi \sin^2\omega t \underbrace{\int_0^a r^3 dr}_{\frac{a^4}{4}} =$$
$$= \frac{\mu_0 I_0^2 d}{16\pi S^2}\underbrace{\pi^2 a^4}_{S^2}\sin^2\omega t = \underline{\frac{\mu_0 I_0^2 d}{16\pi}\sin^2\omega t}$$

+ $$\frac{W_{m0}}{W_{e0}} = \frac{\mu_0 I_0^2 d}{16\pi}\cdot\frac{2\varepsilon_0 S\omega^2}{I_0^2 d} = \frac{\overbrace{\mu_0\varepsilon_0}^{\frac{1}{c_0^2}} S\omega^2}{8\pi} = \frac{S}{8\pi}\left(\frac{\omega}{c_0}\right)^2 = \frac{S}{8\pi}\left(\frac{2\pi}{\lambda}\right)^2$$

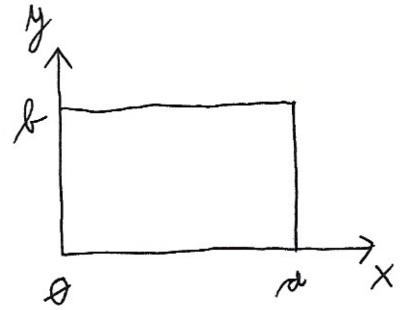
+ $$\boxed{\frac{W_{m0}}{W_{e0}} = \frac{\pi}{2}\frac{S}{\lambda^2}}$$

kvazistatični približek:

majhen $\omega \Rightarrow \lambda \gg \sqrt{S} \Rightarrow \underline{\frac{W_{m0}}{W_{e0}}} \ll 1$ ✓

$\boxed{1}$

2



– valovna enačba

$$\left[\nabla_\perp^2 + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\right] H_z = 0$$

$$\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad H_z(x,y) = X(x)\,Y(y)$$

$$\to X''Y + XY'' + \varkappa^2 XY = 0$$

$$\frac{X''}{X} + \frac{Y''}{Y} + \varkappa^2 = 0, \quad \underbrace{\frac{X''}{X}}_{-\varkappa_x^2} = \underbrace{-\frac{Y''}{Y}}_{-\varkappa_y^2} - \varkappa^2$$

+

+

$$\begin{cases} X'' + \varkappa_x^2 X = 0 \Rightarrow X = A_x \cos\varkappa_x x + B_x \sin\varkappa_x x \\ Y'' + \varkappa_y^2 Y = 0 \Rightarrow Y = A_y \cos\varkappa_y y + B_y \sin\varkappa_y y \end{cases} \qquad \underline{\varkappa_x^2 + \varkappa_y^2 = \varkappa^2}$$

– robni pogoji za TE:

$$H_\perp\big|_\partial = 0 \Rightarrow \underline{\frac{\partial H_z}{\partial n}\Big|_\partial = 0}$$

+

$$x = 0, a: \quad \frac{\partial H_z}{\partial x} = X'Y = 0 \text{ za } \forall y \Rightarrow X' = 0$$

$$X'(0) = 0 \Rightarrow \underline{B_x = 0}$$

$$X'(a) = 0 \Rightarrow \varkappa_x a = m\pi, \quad \underline{\varkappa_x = \frac{m\pi}{a}}$$

$$y = 0, b: \quad \frac{\partial H_z}{\partial y} = XY' = 0 \text{ za } \forall x \Rightarrow Y' = 0$$

$$Y'(0) = 0 \Rightarrow \underline{B_y = 0}$$

$$Y'(b) = 0 \Rightarrow \varkappa_y b = n\pi, \quad \underline{\varkappa_y = \frac{n\pi}{b}}$$

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \frac{\omega^2}{c_0^2} - k^2 \Rightarrow \underline{\omega_{mn} = c_0\sqrt{k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

$$\underbrace{\begin{matrix} m = 0,1,2,\dots \\ n = 0,1,2,\dots \end{matrix}}_{\text{NE oba } 0!}$$

+

– pasovna širina

+

$$\left.\begin{aligned} \omega_1 &= \omega_{10}(k=0) = c_0\frac{\pi}{a} = \frac{2}{3}\,\frac{\pi c_0}{b} \\ \omega_2 &= \omega_{01}(k=0) = c_0\frac{\pi}{b} \end{aligned}\right\} \Delta\omega = \omega_2 - \omega_1 = \underline{\frac{1}{3}\,\frac{\pi c_0}{b}}$$

– plazma:

$$c_0 \to c = \frac{c_0}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} \Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \underbrace{\frac{\omega^2}{c_0^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)}_{\frac{\omega^2}{c_0^2} - \frac{\omega_p^2}{c_0^2} = \frac{\omega^2}{c_0^2} - \left(\frac{\pi}{a}\right)^2} - k^2$$

+

$$\underline{\omega_{mn} = c_0\sqrt{k^2 + \left(\frac{\pi}{a}\right)^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

1/4

$$\omega_1' = \omega_{10}(k=0) = c_0\sqrt{2\frac{\pi^2}{a^2}} = \sqrt{2}\,\frac{\pi c_0}{a} = \frac{2\sqrt{2}}{3}\,\frac{\pi c_0}{b}$$

$$\omega_2' = \omega_{01}(k=0) = c_0\sqrt{\frac{\pi^2}{a^2} + \frac{\pi^2}{b^2}} = \frac{\pi c_0}{b}\sqrt{\frac{4}{9} + 1} = \frac{\sqrt{13}}{3}\,\frac{\pi c_0}{b}$$

$$\Delta\omega' = \omega_2' - \omega_1' = \frac{\sqrt{13} - 2\sqrt{2}}{3}\,\frac{\pi c_0}{b} \Rightarrow \boxed{\frac{\Delta\omega'}{\Delta\omega} = \sqrt{13} - 2\sqrt{2} = 0.78}$$

1

$\boxed{3}$ ena prečka: $p_e = le$, $\dot{p}_e = l\dot{e} = lI$, $\ddot{p}_e = l\dot{I}$

$+$ – <u>križna antena</u>: $I_1 = I_0 \cos\omega t$, $\ddot{\vec{p}}_{e1} = l\dot{I}_1 \hat{e}_x = -lI_0\omega \sin\omega t\, \hat{e}_x$

$I_2 = I_0 \sin\omega t$, $\ddot{\vec{p}}_{e2} = l\dot{I}_2 \hat{e}_y = lI_0\omega\cos\omega t\,\hat{e}_y$

$$\vec{B} = -\frac{\mu_0}{4\pi c_0 r}\hat{e}_r \times \ddot{\vec{p}}_e(t_r) \qquad \ddot{\vec{p}}_e = lI_0\omega(\hat{e}_y\cos\omega t - \hat{e}_x \sin\omega t)$$

$+$
$$\vec{B} = -\frac{\mu_0 l I_0 \omega}{4\pi c_0 r}\left(\hat{e}_r\times\hat{e}_y \cos\omega t_r - \hat{e}_r\times\hat{e}_x \sin\omega t_r\right)$$

$$\hat{e}_r = \begin{bmatrix}\cos\varphi\sin\vartheta\\ \sin\varphi\sin\vartheta\\ \cos\vartheta\end{bmatrix},\quad \hat{e}_r\times\hat{e}_x = \begin{bmatrix}0\\ \cos\vartheta\\ -\sin\varphi\sin\vartheta\end{bmatrix},\quad \hat{e}_r\times\hat{e}_y = \begin{bmatrix}-\cos\vartheta\\ 0\\ \cos\varphi\sin\vartheta\end{bmatrix}$$

$$\vec{B} = -\underbrace{\frac{\mu_0 l I_0\omega}{4\pi c_0 r}}_{B_r}\begin{bmatrix}-\cos\vartheta\cos\omega t_r\\ -\cos\vartheta\sin\omega t_r\\ \sin\vartheta\,(\underbrace{\cos\varphi\cos\omega t_r + \sin\varphi\sin\omega t_r}_{\cos(\omega t_r - \varphi)})\end{bmatrix}$$

$+$
$$\vec{P} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = \frac{1}{\mu_0}(-\hat{e}_r\times c_0\vec{B})\times\vec{B} = -\frac{c_0}{\mu_0}\left[\vec{B}(\underbrace{\hat{e}_r\cdot\vec{B}}_{0}) - \hat{e}_r B^2\right] = \underline{\frac{c_0}{\mu_0}\hat{e}_r B^2}$$

$\Downarrow$

$\underline{P \propto B^2}$

$+$
$$B^2 = B_r^2\left[\cos^2\vartheta\cos^2\omega t_r + \cos^2\vartheta\sin^2\omega t_r + \sin^2\vartheta\cos^2(\omega t_r - \varphi)\right]$$

$$\langle B^2\rangle = B_r^2\left(\tfrac{1}{2}\cos^2\vartheta + \tfrac{1}{2}\cos^2\vartheta + \tfrac{1}{2}\sin^2\vartheta\right) = \underline{B_r^2\,\tfrac{1}{2}(1+\cos^2\vartheta)}$$

$+$
$$\left\langle\int\vec{P}\,d\vec{S}\right\rangle \propto B_r^2 r^2\int\tfrac{1}{2}(1+\cos^2\vartheta)\,2\pi\, d(\cos\vartheta) = B_r^2 r^2\, 2\pi\,\tfrac{1}{2}\left(2+\tfrac{2}{3}\right) =$$
$$= \boxed{B_r^2 r^2\, 2\pi\cdot\tfrac{4}{3}}$$

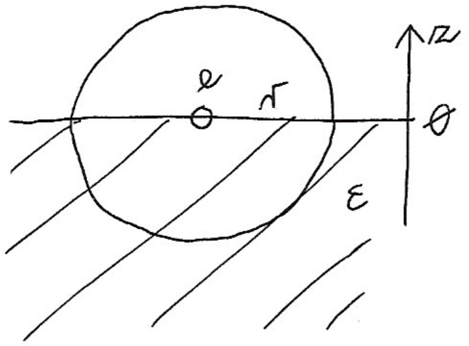
– <u>ena sama rama antena</u>

$\frac{1}{4}$
$$\ddot{\vec{p}}_e = \underline{2l}\, I_0\,\omega\cos\omega t\,\hat{e}_z, \quad \vec{B} = -\underline{2}B_r\,\hat{e}_r\times\hat{e}_z\cos\omega t \qquad \boxed{1}$$

$$\hat{e}_r\times\hat{e}_z = \begin{bmatrix}\sin\varphi\sin\vartheta\\ -\cos\varphi\sin\vartheta\\ 0\end{bmatrix},\quad B^2 = 4B_r^2\sin^2\vartheta\cos^2\omega t,\quad \underline{\langle B^2\rangle = 2B_r^2\sin^2\vartheta}$$

$+$
$$\langle\vec{P}\,d\vec{S}\rangle \propto 2B_r^2 r^2\overbrace{\int\sin^2\vartheta\, 2\pi\, d(\cos\vartheta)}^{2\pi(2-\frac{2}{3})} = \boxed{B_r^2 r^2\, 2\pi\cdot\tfrac{8}{3}} \Rightarrow \boxed{\text{KRIŽNA seva 2-krat MANJ}}$$

$\boxed{4}$



$\frac{1}{4}$

$\nabla^2 U = 0$ razen v naboju

$\Downarrow$

$\underbrace{U(r) \propto \frac{1}{r} P_0(\cos\vartheta) = \frac{1}{r}}$ $(l = 0)$

zaradi robnih pogojev v $r = 0$ in $r \to \infty$

$+$

$$U(r) = \begin{cases} -\dfrac{e_1}{4\pi\varepsilon_0 r}, & z > 0 \\ -\dfrac{e_2}{4\pi\varepsilon_0 r}, & z < 0 \end{cases}$$

predfaktorja (konstanti) postavimo kar na $\frac{e_{1,2}}{4\pi\varepsilon_0}$

$\hookrightarrow$ ju IŠČEMO!

$\frac{1}{4}$

– robni pogoj na meji: $\underbrace{(E_1)_\parallel = (E_2)_\parallel \text{ ali } V_1 = V_2}$

$\hookrightarrow$ $e_1 = e_2$

$\frac{1}{4}$

– povezava z originalnim nabojem $\to$ Gaussov izrek:

$$e = \int \vec{D} \cdot d\vec{S} = \frac{e_1}{4\pi\varepsilon_0 r^2}\varepsilon_0 r^2 2\pi + \frac{e_2}{4\pi\varepsilon_0 r^2}\varepsilon_0 \varepsilon r^2 2\pi =$$

$$= \frac{1}{2} e_1 + \frac{\varepsilon}{2} e_2 = \frac{1+\varepsilon}{2} e_1$$

$+$

$$\underbrace{e_1 = \frac{e}{\frac{\varepsilon+1}{2}} = e_2} \Rightarrow \boxed{U(r) = -\frac{e}{4\pi\varepsilon_0 \frac{\varepsilon+1}{2} r^2}}$$ POVSOD! ✓

$\boxed{1}$