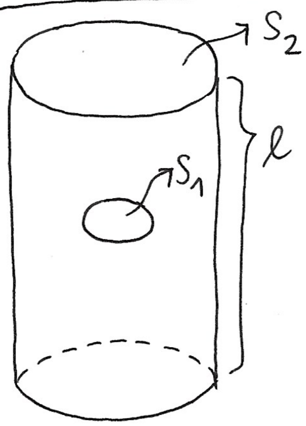
EMP, 2. KOLOKVIJ

1 MEDSEBOJNA INDUKCIJA TULJAVE IN ZANKE



– izračun MEDSEBOJNE induktivnosti

1+
$$B_2 = \frac{\mu_0 N I_2}{\ell}$$
$$\Phi_1 = B_2 S_1 = \frac{\mu_0 N S_1}{\ell} I_2 \Rightarrow \underline{L_{12} = \frac{\mu_0 N S_1}{\ell}}$$

– splošni izračun INDUKCIJE v tuljavi

2+ $\Phi_2 = L_{21} I_1 = L_{12} I_1$ (pretok v tuljavi)

3+
$U_2 = -\dot{\Phi}_2 = -L_{12}\dot{I}_1$ (indukcija)

$U_2 = L_2 \dot{I}_2$ (tokovni krog)

$$\underline{\dot{I}_2 = -\frac{L_{12}}{L_2}\dot{I}_1} \Leftarrow$$

4+

– vstavitev podane tokovne odvisnosti

5+
$$I_1 = I_0 e^{-\frac{t}{\tau}} \rightarrow \dot{I}_1 = -\frac{I_0}{\tau} e^{-\frac{t}{\tau}}$$
$$L_2 = \frac{\mu_0 N^2 S_2}{\ell}$$
$$\dot{I}_2 = -\frac{\mu_0 N S_1}{\ell} \cdot \frac{\ell}{\mu_0 N^2 S_2}\left(-\frac{I_0}{\tau}\right) e^{-\frac{t}{\tau}} = \frac{S_1}{N S_2} \cdot \frac{I_0}{\tau} e^{-\frac{t}{\tau}}$$

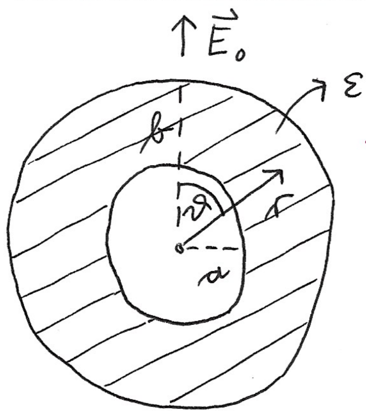
6+
$$I_2(t) - \underbrace{I_2(0)}_{0} = \frac{S_1}{N S_2} \cdot I_0 \left(-e^{-\frac{t}{\tau}}\right)\Big|_0^t = \frac{S_1}{N S_2} I_0 \left(1 - e^{-\frac{t}{\tau}}\right)$$

7+
$$\boxed{I_2(t) = \frac{S_1}{N S_2} I_0 \left(1 - e^{-\frac{t}{\tau}}\right)}$$ (časovni potek)

8+
$$\underline{I_2(t \to \infty) = \frac{S_1}{N S_2} I_0}$$ (končna vrednost)

1

2 DIELEKTRIČNA KROGELNA LUPINA



1+ nastavek

$$U(r,\vartheta) = \begin{cases} -E_0 r\cos\vartheta + \dfrac{A}{r^2}\cos\vartheta, & r > b \\ B r\cos\vartheta + \dfrac{C}{r^2}\cos\vartheta, & a < r < b \\ D r\cos\vartheta, & r < a \end{cases}$$

2+ RP1 zveznost U pri a & b

$$-E_0 b + \frac{A}{b^2} = Bb + \frac{C}{b^2} \rightarrow$$

$$Da = Ba + \frac{C}{a^2} \rightarrow$$

ustrezneje zapisane enačbe:

$$-2E_0 + \frac{2A}{b^3} = 2B + \frac{2C}{b^3} \quad (1)$$

$$D = B + \frac{C}{a^3} \quad (2)$$

3+ RP2 zveznost $D_r = -\varepsilon \dfrac{\partial U}{\partial r}$ pri a & b

$$E_0 + 2\frac{A}{b^3} = \varepsilon\left(-B + 2\frac{C}{b^3}\right) \rightarrow$$

$$-D = \varepsilon\left(-B + 2\frac{C}{a^3}\right) \rightarrow$$

$$E_0 + \frac{2A}{b^3} = -\varepsilon B + \varepsilon\frac{2C}{b^3} \quad (3)$$

$$D = \varepsilon B - \varepsilon\frac{2C}{a^3} \quad (4)$$

4+ $(4)-(2)$: $(\varepsilon-1)B = (2\varepsilon+1)\dfrac{C}{a^3} \Rightarrow B = \dfrac{2\varepsilon+1}{\varepsilon-1}\dfrac{C}{a^3}$ → 4 zveza med DVEMA (lahko tudi kakšna druga

5+ $(3)-(1)$:

$$3E_0 = -(\varepsilon+2)B + 2(\varepsilon-1)\frac{C}{b^3} = \frac{-(\varepsilon+2)(2\varepsilon+1) + 2(\varepsilon-1)^2\left(\frac{a}{b}\right)^3}{(\varepsilon-1)a^3}\,C$$

$$\frac{C}{a^3} = -\frac{3(\varepsilon-1)}{(\varepsilon+2)(2\varepsilon+1) - 2\left(\frac{a}{b}\right)^3(\varepsilon-1)^2}E_0$$

→ 5 ENA neznanka

6+ (2) : $D = B + \dfrac{C}{a^3} = \left(\dfrac{2\varepsilon+1}{\varepsilon-1} + 1\right)\dfrac{C}{a^3} = \dfrac{3\varepsilon}{\varepsilon-1}\dfrac{C}{a^3}$ → 6 D s to neznanko

7+

$$\boxed{D = -\frac{9\varepsilon}{(\varepsilon+2)(2\varepsilon+1) - 2\left(\frac{a}{b}\right)^3(\varepsilon-1)^2}E_0}$$

Koraki 4, 5, 6 so lahko tudi drugačni! $(2\to1\to D)$!

$$\underbrace{\varepsilon = 3,\ \frac{a}{b} = \frac{1}{2}}$$

$$\Downarrow$$

$$E_1 = \frac{27}{34}E_0$$

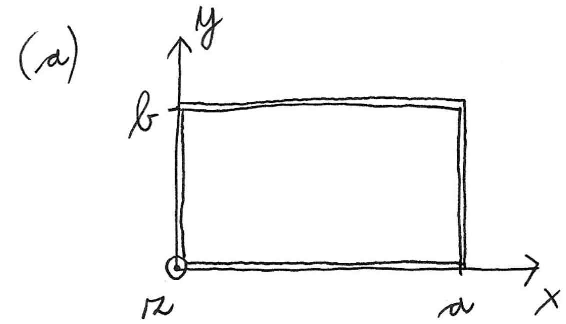
8+ POLJE ZNOTRAJ: HOMOGENO

$$E_1 = -\frac{\partial U}{\partial z} = \boxed{-D}\ \checkmark, \quad U_1 = Dr\cos\vartheta = Dz$$

$\boxed{1}$

3 ENERGIJSKI TOK V VALOVNEM VODNIKU

(a)



separacija +1

$\underline{TE \Rightarrow E_z = 0, \; H_z \neq 0}$

$$\Big[\underbrace{\nabla_\perp^2}_{\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}} + \underbrace{\left(\frac{\omega^2}{c_0^2} - k^2\right)}_{\varkappa^2}\Big] H_z(x,y) = 0$$

$$H_z(x,y) = X(x)Y(y)$$

$$X''Y + XY'' + \varkappa^2 XY = 0 \Rightarrow \underbrace{\frac{X''}{X}}_{-\varkappa_x^2} = \underbrace{-\frac{Y''}{Y}}_{-\varkappa_y^2} - \varkappa^2 \rightarrow \varkappa_x^2 + \varkappa_y^2 = \varkappa^2$$

rešitev z izločitvijo sin

2+
$$X'' + \varkappa_x^2 X = 0 \Rightarrow \underline{X \propto \cos \varkappa_x x}$$
$$Y'' + \varkappa_y^2 Y = 0 \Rightarrow \underline{Y \propto \cos \varkappa_y y}$$
upoštevaje že $\boxed{RP1}$,
$$\frac{\partial H_z}{\partial \vec{n}}(x=0 \text{ ali } y=0) = 0$$

3+
$\boxed{RP2}$ $\frac{\partial H_z}{\partial \vec{n}}(x=a \text{ ali } y=b) = 0 \Rightarrow \underline{\varkappa_x = \frac{m\pi}{a}, \; \varkappa_y = \frac{n\pi}{b}}$

$$\varkappa_x^2 + \varkappa_y^2 = \varkappa^2 \Rightarrow \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \frac{\omega^2}{c_0^2} - k^2$$

končna rešitev in disperzijska relacija

$$\underline{\omega_{mn} = c_0 \sqrt{k^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

$m = 0, 1, 2, \ldots \quad n = 0, 1, 2, \ldots \quad$ NE oba 0!

4+
$\underline{b \leq a} \Rightarrow \omega_{MIN} = \omega_{10}(k=0) = c_0 \frac{\pi}{a}$ (najnižja veja)

$$\omega'_{MIN} = \begin{cases} \omega_{01}(k=0) = c_0 \frac{\pi}{b}, & b \geq \frac{a}{2} \\ \omega_{20}(k=0) = c_0 \frac{2\pi}{a}, & b \leq \frac{a}{2} \end{cases}$$
(druga najnižja veja)

Ko b ZNIŽUJEMO z a, se ω_{01} POVEČUJE, vse dokler ne doseže vrednosti ω_{20} pri $b \leq \frac{a}{2}$, ki je hkrati NAJVEČJA vrednost pri DANEM a.

$\Downarrow$

maksimizacija širine pasu

$\boxed{b \leq \frac{a}{2}}$, $\Delta\omega = \omega_{20}(k=0) - \omega_{10}(k=0) = \underline{c_0 \frac{\pi}{a}}$

(ŠIRINA uporabnega pasu)

(b) UPORABNI pas $\rightarrow$ $\underbrace{m=1,\ n=0} \rightarrow \underbrace{\varkappa = \frac{\pi}{a}}$

5+

$$\boxed{H_z} = H_0 \cos\frac{\pi X}{a}, \quad E_z = 0$$

podane enačbe $\rightarrow$ $\boxed{H_x} = \frac{ik}{\varkappa^2} H_0 \left(-\frac{\pi}{a}\right) \sin\frac{\pi X}{a} = \underbrace{-i H_0 \frac{ka}{\pi} \sin\frac{\pi X}{a}}$

$$H_y = 0$$

FAZA $\frac{\pi}{2}$!

$$E_x = 0$$

$$\boxed{E_y} = \frac{-i\omega\mu_0}{\varkappa^2} H_0\left(-\frac{\pi}{a}\right)\sin\frac{\pi X}{a} = \underbrace{i \underbrace{H_0 \frac{\omega\mu_0 a}{\pi}}_{\boxed{E_0}} \sin\frac{\pi X}{a}}$$

komponente

$$\boxed{E_0} \Rightarrow \frac{H_0 a}{\pi} = \frac{E_0}{\omega\mu_0}$$

6+

$$\vec{P} = \vec{E}\times\vec{H} = \begin{bmatrix} 0 \\ E_y \\ 0 \end{bmatrix} \times \begin{bmatrix} H_x \\ 0 \\ H_z \end{bmatrix} = \begin{bmatrix} E_y H_z \\ 0 \\ -E_y H_x \end{bmatrix} \rightarrow \underline{\text{smer } z}$$

časovno povprečje

časovno povprečje: $\langle E_y H_z \rangle \rightarrow 0$ (ste $\frac{\pi}{2}$ iz faze)

$\langle E_y H_x \rangle \rightarrow$ faktor $\frac{1}{2}$ (povprečje $\sin^2\omega t$)

7+

$$\langle P_z \rangle = \frac{1}{2}|E_y||H_x| = \frac{1}{2} E_0 \cdot H_0 \frac{ka}{\pi} \sin^2\frac{\pi X}{a} =$$

$$= \frac{1}{2} E_0 \frac{k E_0}{\omega\mu_0} \sin^2\frac{\pi X}{a} = \frac{1}{2} E_0^2 \left(\frac{k}{\omega\mu_0}\right) \sin^2\frac{\pi X}{a}$$

$$\frac{\omega^2}{c_0^2} = k^2 + \underbrace{\frac{\pi^2}{a^2}}_{\frac{\omega_{MIN}^2}{c_0^2}} \Rightarrow \frac{k}{\omega\mu_0} = \frac{1}{c_0}\frac{\sqrt{\omega^2-\omega_{MIN}^2}}{\mu_0\omega} = \underbrace{\underbrace{\sqrt{\frac{\varepsilon_0}{\mu_0}}}_{\frac{1}{Z_0}}\sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}}}$$

upoštevanje disperzijske relacije in E_0

$\Downarrow$

8+

$$\langle P_z \rangle = \frac{E_0^2}{2Z_0}\sin^2\frac{\pi X}{a}$$

$$\int \langle P_z \rangle \, dS_z = \underbrace{\frac{E_0^2}{2Z_0}}_{\sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}}} \overbrace{\int_0^a\int_0^b \sin^2\frac{\pi X}{a}\, dx\, dy}^{b\frac{a}{2}} = \boxed{\frac{E_0^2}{4Z_0} ab \sqrt{1-\frac{\omega_{MIN}^2}{\omega^2}}}$$

integral

$\boxed{1}$