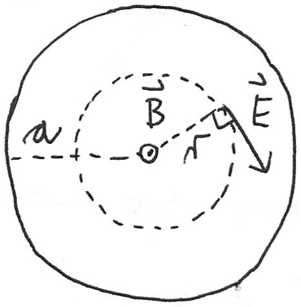
2. KOLOKVIJ

1 POYNTINGOV VEKTOR V TULJAVI



a) $B = \frac{\mu_0 N I}{l} = \boxed{\frac{\mu_0 N I_0}{l} \sin \omega t}$ } +

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{S}$ } +

$E \cdot 2\pi r = -\frac{\partial}{\partial t}\left[\frac{\mu_0 N I_0}{l} \pi r^2 \sin \omega t\right]$

$\boxed{E = \frac{\mu_0 N I_0 \omega}{2l} r \cos \omega t}$ } +

b) $\vec{P} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$, $\vec{E} \perp \vec{B} \Rightarrow P = \frac{1}{\mu_0} E B$

$P = \frac{\mu_0 N^2 I_0^2 \omega}{2l^2} r \sin \omega t \cos \omega t$

$\int \vec{P} \cdot d\vec{S} = P \cdot 2\pi a l = \underline{\frac{\mu_0 N^2 I_0^2 \omega}{l} \overbrace{\pi a^2}^{S} \sin \omega t \cos \omega t}$ } +

$W_m = \frac{B^2}{2\mu_0} \cdot V = \frac{\mu_0 N^2 I_0^2}{2l^2} \sin^2 \omega t \cdot S l = \frac{\mu_0 N^2 I_0^2 S}{2l} \sin^2 \omega t$ } +

$\underline{\frac{\partial W_m}{\partial t} = \frac{\mu_0 N^2 I_0^2 S}{l} \omega \sin \omega t \cos \omega t} \Rightarrow$ ENAKO kot $\int \vec{P} \cdot d\vec{S}$ ✓

c) $W_e = \int \frac{1}{2} \varepsilon E^2 \, dV = \frac{\varepsilon_0}{2} \frac{\mu_0^2 N^2 I_0^2 \omega}{4 l^2} \cos^2 \omega t \int_0^a r^2 \cdot \overbrace{l 2\pi r \, dr}^{dV} =$ } +

$= \frac{\pi \varepsilon_0 \mu_0^2 N^2 I_0^2 \omega^2}{4l} \cdot \frac{a^4}{4} \cos^2 \omega t$, $S = \pi a^2 \Rightarrow a^4 = \frac{S^2}{\pi^2}$ } +

$\underline{W_e = \frac{\varepsilon_0 \mu_0^2 N^2 I_0^2 \omega^2 S^2}{16 \pi l} \cos^2 \omega t}$

+ { $\frac{W_{e0}}{W_{m0}} = \frac{\varepsilon_0 \mu_0^2 N^2 I_0^2 \omega^2 S^2 2l}{16 \pi l \, \mu_0 N^2 I_0^2 S} = \frac{1}{8\pi} \frac{\omega^2 S}{c_0^2} = \frac{1}{8\pi} \frac{4\pi^2 \nu^2 S}{c_0^2} = \boxed{\frac{\pi}{2} \frac{S}{\lambda^2}}$

kvazistatika: $\lambda \gg \sqrt{S} \Rightarrow \boxed{\frac{W_{e0}}{W_{m0}} \ll 1}$

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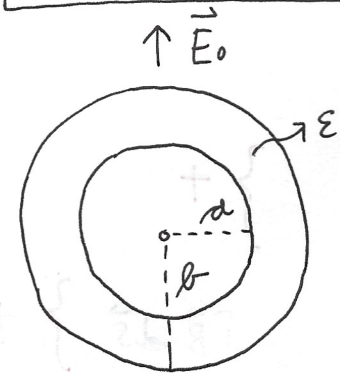
2 CEV IZ DIELEKTRIČNE SNOVI



$$
U(r,\varphi) = \begin{cases} -E_0\, r\cos\varphi + \frac{A}{r}\cos\varphi, & r>b \\ B\, r\cos\varphi + \frac{C}{r}\cos\varphi, & a<r<b \\ D\, r\cos\varphi, & r<a \end{cases}
$$

RP1 zveznost U pri a & b:

LEPŠE napisane enačbe:

$$
\begin{cases} -E_0 b + \frac{A}{b} = Bb + \frac{C}{b} \\ Ba + \frac{C}{a} = Da \end{cases}
\quad\Rightarrow\quad
\begin{aligned} -E_0 + \frac{A}{b^2} &= B + \frac{C}{b^2} \quad (1) \\ D &= B + \frac{C}{a^2} \quad (2) \end{aligned}
$$

RP2 zveznost $D_r = \varepsilon \frac{\partial E}{\partial r}$ pri a & b:

$$
\begin{cases} E_0 + \frac{A}{b^2} = \varepsilon\left(-B + \frac{C}{b^2}\right) \\ \varepsilon\left(-B + \frac{C}{a^2}\right) = -D \end{cases}
\quad\Rightarrow\quad
\begin{aligned} E_0 + \frac{A}{b^2} &= -\varepsilon B + \varepsilon \frac{C}{b^2} \quad (3) \\ D &= \varepsilon B - \varepsilon \frac{C}{a^2} \quad (4) \end{aligned}
$$

$$
\begin{aligned} (3)-(1):&\quad 2E_0 = -(\varepsilon+1)B + (\varepsilon-1)\frac{C}{b^2} \\ (4)-(2):&\quad (\varepsilon-1)B = (\varepsilon+1)\frac{C}{a^2} \end{aligned}
\quad\Rightarrow\quad C = \frac{(\varepsilon-1)}{(\varepsilon+1)} a^2 B
$$

$$
2E_0 = -(\varepsilon+1)B + \frac{\varepsilon-1}{b^2} a^2 \frac{\varepsilon-1}{\varepsilon+1} B = B(\varepsilon+1)\left[\left(\frac{a}{b}\right)^2\left(\frac{\varepsilon-1}{\varepsilon+1}\right)^2 - 1\right]
$$

$$
B = \frac{2E_0}{(\varepsilon+1)\left[\left(\frac{a}{b}\right)^2\left(\frac{\varepsilon-1}{\varepsilon+1}\right)^2 - 1\right]} = \frac{2E_0(\varepsilon+1)}{\left[\left(\frac{a}{b}\right)^2(\varepsilon-1)^2 - (\varepsilon+1)^2\right]}
$$

$$
\frac{C}{a^2} = \frac{(\varepsilon-1)}{(\varepsilon+1)} B = \frac{2E_0(\varepsilon-1)}{\left[\left(\frac{a}{b}\right)^2(\varepsilon-1)^2 - (\varepsilon+1)^2\right]}
$$

$$
\boxed{D} = B + \frac{C}{a^2} = \frac{4\varepsilon E_0}{\left(\frac{a}{b}\right)^2(\varepsilon-1)^2 - (\varepsilon+1)^2}
$$

→ do D je mogoče priti na različne načine

POLJE ZNOTRAJ: $E_1 = -\frac{\partial U}{\partial z} = -D$, $U_1 = Dz \Rightarrow$ HOMOGENO

$$
\boxed{E_1 = \frac{4\varepsilon E_0}{(\varepsilon+1)^2 - \left(\frac{a}{b}\right)^2(\varepsilon-1)^2}}
$$

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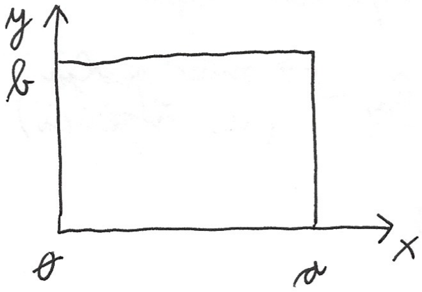
3 VALOVNI VODNIK S PRAVOKOTNIM PRESEKOM



– valovna enačba, TE način

$$\Big[\nabla_\perp^2 + \overbrace{\Big(\frac{\omega^2}{c_0^2} - k^2\Big)}^{\varkappa^2}\Big] H_z = 0$$

$$\nabla_\perp^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad H_z(x,y) = X(x)\,Y(y)$$

$$X''Y + XY'' + \varkappa^2 XY = 0$$

$$\frac{X''}{X} + \frac{Y''}{Y} + \varkappa^2 = 0, \quad \underbrace{\frac{X''}{X}}_{-\varkappa_x^2} = -\underbrace{\frac{Y''}{Y}}_{-\varkappa_y^2} - \varkappa^2$$

+

+ $X'' + \varkappa_x^2 X = 0 \Rightarrow X = A_x \cos\varkappa_x x + B_x \sin\varkappa_x x$

$Y'' + \varkappa_y^2 Y = 0 \Rightarrow Y = A_y \cos\varkappa_y y + B_y \sin\varkappa_y y$

$\varkappa_x^2 + \varkappa_y^2 = \varkappa^2$

– robni pogoji : $H_\perp\big|_\partial = 0 \Rightarrow \frac{\partial H_z}{\partial n}\Big|_\partial = 0$

+ $x = 0, a$: $\frac{\partial H_z}{\partial x} = 0 \Rightarrow X'(0) = 0 \Rightarrow B_x = 0$

$X'(a) = 0 \Rightarrow \sin\varkappa_x a = 0, \quad \varkappa_x = \frac{m\pi}{a}$

$y = 0, b$: $\frac{\partial H_z}{\partial y} = 0 \Rightarrow Y'(0) = 0 \Rightarrow B_y = 0$

$Y'(b) = 0 \Rightarrow \sin\varkappa_y b = 0, \quad \varkappa_y = \frac{n\pi}{b}$

+ $$\Big(\frac{m\pi}{a}\Big)^2 + \Big(\frac{n\pi}{b}\Big)^2 = \frac{\omega^2}{c_0^2} - k^2 \Rightarrow \omega_{mn} = c_0\sqrt{k^2 + \Big(\frac{m\pi}{a}\Big)^2 + \Big(\frac{n\pi}{b}\Big)^2}$$

$m = 0, 1, 2, \ldots \quad n = 0, 1, 2, \ldots$ NE oba 0!

a) $b = a$ $\quad \omega_{mn} = c_0\sqrt{k^2 + \frac{\pi^2}{a^2}(m^2+n^2)}, \quad s = 4a \Rightarrow a = \frac{s}{4}$

$\frac{1}{4}$ najnižja : $\omega_{01} = c_0\frac{\pi}{a} = \boxed{4\pi\frac{c_0}{s}}$

2\. najnižja : $\omega_{\boxed{11}} = c_0\frac{\pi}{a}\sqrt{2} = 4\pi\sqrt{2}\,\frac{c_0}{s}$

$$\boxed{\Delta\omega = 4\pi(\sqrt{2}-1)\frac{c_0}{s}}$$

b) $a = 2b$ $\quad \omega_{mn} = c_0\sqrt{k^2 + \frac{\pi^2}{a^2}(m^2 + 4n^2)}, \quad s = 2a + 2b = 3a \Rightarrow a = \frac{s}{3}$

$\frac{1}{4}$ najnižja : $\omega_{10} = c_0\frac{\pi}{a} = \boxed{3\pi\frac{c_0}{s}}$

2\. najnižja : $\omega_{\boxed{20}} = c_0\frac{2\pi}{a} = 6\pi\frac{c_0}{s}$

$$\boxed{\Delta\omega = 3\pi\frac{c_0}{s}}$$

$\boxed{1}$

$\boxed{4}$ VALOVANJE V IONOSFERI $\qquad \omega_B = \frac{eB}{m}, \quad \omega_p = \sqrt{\frac{ne^2}{m\varepsilon_0}}$

+ a) $\vec{E}_\pm = \vec{E}_0 (\hat{e}_x \pm i\hat{e}_y) e^{i(kz-\omega t)}$ → krožna polarizacija

$m\ddot{\vec{r}} = -eE_0 (\hat{e}_x + i\hat{e}_y) e^{i(kz-\omega t)} - eB\dot{\vec{r}} \times \hat{e}_z$ → smer polja (in širjenja)

$\Downarrow$

+ $\vec{r}_\pm = r_0 (\hat{e}_x \pm i\hat{e}_y) e^{i(kz-\omega t)}$ → nastavek

$\dot{\vec{r}}_\pm \times \hat{e}_z = -i\omega r_0 e^{i(kz-\omega t)} \underbrace{(\hat{e}_x \pm i\hat{e}_y) \times \hat{e}_z}_{-\hat{e}_y \pm i\hat{e}_x \;=\; \pm i(\hat{e}_x \pm i\hat{e}_y)}$

+ $(-\omega^2 m r_0 + eE_0 \pm eB\omega r_0)(\hat{e}_x \pm i\hat{e}_y) = 0$ → nastavek deluje!

+ b) $eE_0 = (m\omega^2 \mp eB\omega) r_0 \Rightarrow r_0 = \frac{eE_0}{m\omega^2 \mp m\omega_B\omega} = \frac{eE_0}{m\omega(\omega \mp \omega_B)}$

+ $\vec{P}_\pm = -en\vec{r}_\pm = \varepsilon_0(\varepsilon_\pm - 1)\vec{E} \Rightarrow \varepsilon_\pm = 1 - \frac{en}{\varepsilon_0}\frac{r_0}{E_0}$

($\vec{r}_\pm \to r_0$, $\vec{E} \to E_0$)

$\varepsilon_\pm = 1 - \frac{e^2 n}{\varepsilon_0 m\omega(\omega \mp \omega_B)} = \boxed{1 - \frac{\omega_p^2}{\omega(\omega \mp \omega_B)}}$

+ $\omega \ll \omega_B$: $\varepsilon_\pm = 1 \pm \frac{\omega_p^2}{\omega_B\omega}$

$\omega \ll \omega_p$: $\boxed{\varepsilon_\pm = \pm \frac{\omega_p^2}{\omega_B\omega}}$ ⇒ NEGATIVNA krožna polarizacija se NE ŠIRI!

+ c) $k = \frac{\omega}{c_0}\sqrt{\varepsilon_+} = \frac{\omega}{c_0}\frac{\omega_p}{\sqrt{\omega_B\omega}} = \frac{\omega_p}{c_0\sqrt{\omega_B}}\sqrt{\omega} \Rightarrow \boxed{\omega = \frac{\omega_B c_0^2}{\omega_p^2}k^2}$

KVADRATIČNA disperzija

+ $v = \frac{\omega}{k} = \frac{\omega_B c_0^2}{\omega_p^2}k = \frac{\omega_B c_0^2}{\omega_p^2}\frac{\omega_p}{c_0\sqrt{\omega_B}}\sqrt{\omega}$

$v = \frac{c_0\sqrt{\omega_B}}{\omega_p}\sqrt{\omega} \Rightarrow \boxed{v \propto \sqrt{\omega}}$ FREKVENČNA odvisnost

$\boxed{1}$